

# A MODEL REFERENCE-BASED FUZZY ADAPTIVE PI CONTROLLER FOR NON-LINEAR LEVEL PROCESS SYSTEM

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## ABSTRACT

This paper proposes a model reference tracking based fuzzy adaptive PI (MRFAPI) controller for non-linear process plant. The second order reference model is proposed for the system. The conical-tank level process system is used as a process plant. The proposed MRFAPI controller is combined with fuzzy logic and conventional variable PI controllers. The on-line estimation of the controller parameters is based on the error between reference model and time-varying parameter of the process plant. The model of the process plant is formulated based on the real laboratory-scale system and the control of the process plant is done using MATLAB simulink. The controller can produce the appropriate control signals to control the plant in presence of plant nonlinearity, disturbance and measurement noise.

**Keywords:** *Model Reference Fuzzy Adaptive PI (MRFAPI), Conical-Tank, SISO, V-MAT.*

## 1. INTRODUCTION

The industrial process control systems present many challenging control problems due to their non-linear dynamic behavior. Because of the inherent non-linearity, most of the process industries are in need of traditional control techniques. Control of level in conical tank is a challenging problem due to its constantly changing the cross section. The primary task of the controller is to maintain the process at the desired operating conditions and to achieve the optimum performance when facing various types of disturbances.

There is a rapid growth in the use of fuzzy logic controllers for plants those are complex and ill-defined. Control algorithms based on fuzzy logic have been implemented in many processes. The application of such control techniques has been motivated by the following reasons: 1) improved robustness over the conventional linear control algorithms; 2) simplified control design for difficult system models; 3) simplified implementation. In most applications of fuzzy logic controllers, the rule base of the fuzzy controller is constructed from expert knowledge; the need may arise to tune the controller parameters if the plant dynamics change. In an attempt to overcome this problem, researchers have introduced adaptive control techniques. In such techniques, the functional approximation capability and on-line learning ability of fuzzy based variable PI systems are exploited.

In this paper we introduce MRAFPI controller for SISO nonlinear systems with unknown dynamics and disturbance. We use standard fuzzy systems to estimate on-line the plant dynamics. The control law is synthesized based on these estimates and a robustifying control term is added to cancel out the effect of approximation errors and disturbance. Adaptive laws to adjust the controller parameter with help of fuzzy systems and methods to deal with reconstruction errors and disturbance problems. The proposed control scheme avoids the possible controller singularity problem, does not need the knowledge of some bounding parameter values, guarantees the boundedness of all the states and the signals of the closed-loop system, and the convergence of the tracking error to zero.

## 2. CONICAL-TANK LEVEL PROCESS SYSTEM

The mathematical model of the conical-tank level control system in the simulation is formulated based on the real laboratory-scale conical-tank system. Fig.1 shows the experimental setup which consists of a conical process tank, a pneumatic control valve, a storage tank, a pump, a I/P converter, a Differential Pressure Level Transmitter (DPLT), VMAT interfacing card and I/V & V/I converter. Water from storage tank is pumped continuously to the conical tank through a pneumatic control valve. The DPLT transmits a current signal (4-20mA) to the I/V converter. The output of the I/V converter (1-5V) is given to the VMAT interfacing hardware consisting of multifunction high speed ADC and DAC. The onboard data converters of the VMAT can be directly linked with the Simulink tool of MATLAB thus forming a complete closed loop system. The signal from the PC is transmitted to the I/P converter through V/I converter. The output from the I/P converter is the pressured air in the range of 3-15 psi for the actuating the control valve, which regulates the flow of liquid into the conical tank.

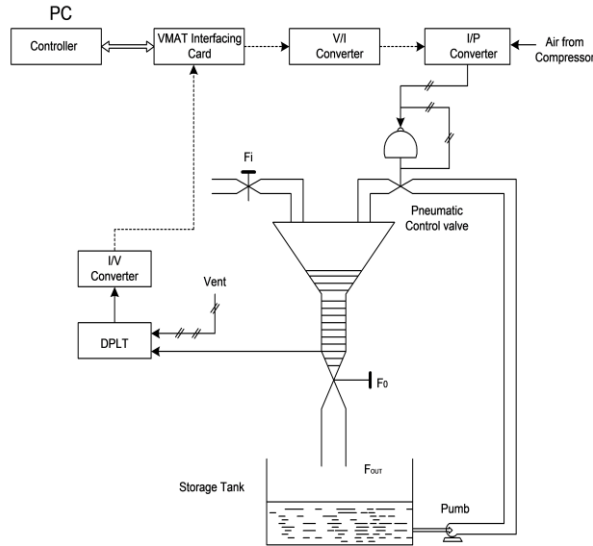


Figure1. Functional Block diagram of Conical-Tank System

**2.1. Description of the Conical-Tank Level Process**

The tank is made up of stainless steel body and is mounted over a stand vertically. Water enters the tank from the top and leaves the bottom to the storage tank. The System specifications of the tank are as follows:

Table 1. System Specifications of Conical tank

PART NAME	DETAILS
Conical tank	Stainless steel body, height– 65 cm, Top diameter–33.5 cm Bottom diameter – 3.5 cm
Differential Pressure Level Transmitter	Capacitance type, Range 2.5-250mbar, output 4-20 mA
Pump	Centrifugal 0.5 HP
Control Valve	Size ¼ Pneumatic actuated Type: Air to open, Input 3-15PSI
Rota meter	Range 0-460 LPH

**2.2. Mathematical Model of the Conical-Tank Level Process**

Here  $F_i$  is the inlet flow rate to the tank,  $F_o$  be the outlet flow rate from the tank,  $F_L$  be the disturbance applied to the tank.

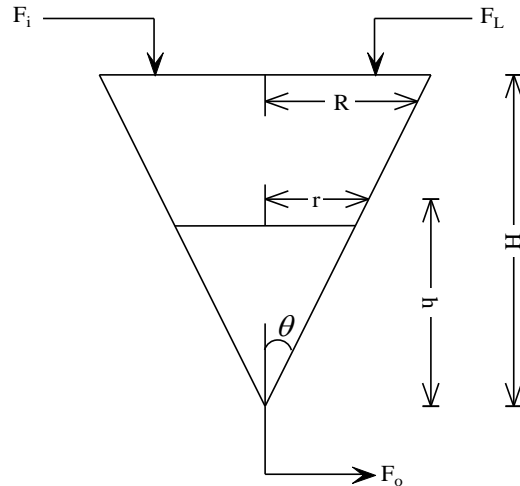


Figure2. Schematic of Conical-Tank Level System

The dynamic model of the conical-tank system when nominal operating level h is given by

$$A \frac{dh}{dt} = F_i - F_o \tag{1}$$

$$F_o = b\sqrt{2gh} \tag{2}$$

$$A = \frac{\pi R^2 h^2}{H^2} \tag{3}$$

Substitute the equation (2) and (3) in (1)

$$\frac{dh}{dt} = \frac{F_i - b\sqrt{2gh}}{\frac{\pi R^2 h^2}{H^2}} \tag{4}$$

Where h is the height of the liquid level in the conical tank, H is the height of the conical tank, r is the radius of the conical vessel at a particular level of height h, R is the top radius of the conical tank, A is the area of the conical tank, outlet valve ratio b, inflow rate to the tank Fi and outflow rate of the tank Fo.

**2.3. Transfer Function Model of the Conical-Tank Level Process**

In order to find the open loop response, the step input of 2.0v is applied to the ADC input in simulink tool of MATLAB platform directly with the fixed inflow rate and outflow rate. For the given step input the system attains the steady state at 15cm. After that a step increment of 0.25v is given and various readings are noted till the process becomes stable. The same procedure is repeated to take different operating regions in conical tank system. The experimental data are approximated to be a FOPDT model, the model parameters of the transfer functions for the above mentioned are tabulated in Table II.

Table 2. Mathematical Model of the Conical-Tank Level Process for various operating region

Models	Input ADC Voltage	Level in cm	Transfer Function Model
Model_1	2.0v to 2.25v	15 – 30	$\frac{5.415 e^{-100s}}{3000s+1}$
Model_2	2.25v to 2.50v	30 – 38	$\frac{2.999 e^{-350s}}{3500s+1}$
Model_3	2.5v to 2.75v	38 – 43	$\frac{1.705 e^{-405s}}{4000s+1}$

**2.4 Reference Model**

The reference model can be formulated with different types; each model governs different response specifications. The transient response of reference models depends on the plant to be controlled which is in this case, a conical-tank. Therefore, the selection of reference model is usually preceded by a careful study on the plant’s boundary and

physical limits of which it can operate. In this simulation, nominal reference model is formulated according to the nominal operation of the conical-tank equipment as outlined in the equipment's manual. In this study, the order of the formulated reference model is first order and it is converted to second order using pade approximation technique. If a second order model is used, the velocity state component should be set to zero. The second consideration is that all initial plant and model states would be set to zero if the plant starts from rest. The initial conditions of the reference model would have to be set such that the initial plant reference model output vectors have the same value.

The second order state space representation of the reference model\_1 is decomposed (similarly for model\_2 and model\_3) as follows

$$\begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \end{bmatrix} = \begin{bmatrix} -0.02033 & -0.003413 \\ 0.001953 & 0 \end{bmatrix} \begin{bmatrix} x_{m1} \\ x_{m2} \end{bmatrix} + \begin{bmatrix} 0.125 \\ 0 \end{bmatrix} u_m \quad (5)$$

$$y_m = [-0.01444 \quad 0.1479] \begin{bmatrix} x_{m1} \\ x_{m2} \end{bmatrix} \quad (6)$$

### 3. MODEL REFERENCE ADAPTIVE CONTROL

The Lyapunov's stability theory is normally used to develop algorithms for adjusting the parameters of an adaptive system. In order to develop such algorithms, derived a differential equation for the error,  $e(t) = y(t) - y_m(t)$  containing adjustable parameters is essential.

The state space representation of the reference model is

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= c_m(t) x_m(t) \end{aligned} \quad (7)$$

The controller is

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \quad (8)$$

Introduce the error

$$e(t) = y(t) - y_m(t) \quad (9)$$

If the parameters are updated as

$$\begin{aligned} \frac{d\theta_1}{dx} &= -\gamma u_c e \\ \frac{d\theta_2}{dx} &= \gamma y e \end{aligned} \quad (10)$$

The control law is

$$u(t) = \left\{ \frac{d}{dt} (-\gamma u_c(t) [y(t) - y_m(t)]) u_c(t) - \frac{d}{dt} (\gamma y(t) [y(t) - y_m(t)]) y(t) \right\} \quad (11)$$

Where

- $\theta_1, \theta_2$  are the updating parameters
- $\gamma$  is the adaptation gain
- $y$  is the plant output
- $y_m$  is the model output

### 4. MODEL REFERENCE BASED FUZZY ADAPTIVE PI

The proposed model reference based fuzzy adaptive PI (MRFAPI) control uses an adaptive control structure is shown in figure 3. It consists of command inputs, reference model states, fuzzy logic control, variable PI control and the feedback of the output errors. In an ideal situation, the outputs of the system track the outputs of the known reference model. This proposed design methodology to adjust controller parameter using MRFAPI technique for solving the problem of conical tank non-linear process. The fuzzy tuning PI controller is an auto-adaptive controller that is designed by using an incremental fuzzy logic controller in place of the proportional term in the conventional PI controller to tune the parameters of PI controller on-line by fuzzy control rules. The controller uses the error and the rate of change of error as its inputs and can meet desire of self-tuning parameters based on time-varying  $e$  and  $\dot{e}$ . There are many studies on determining the parameters of controllers and finding new values of controller parameters according to changing situations.

$$u_p(t) = \Delta K_p e(t) + \Delta K_i \int_0^t e(t) dt \quad (12)$$

Where  $K_p$  is proportional gain;  $K_i = K_p(T/T_i)$ ;  $T$  is sample-time;  $T_i$  is integral time parameter. Because the proposed fuzzy tuning PI controller aims to improve the control performance yielded by a PI controller, it keeps the simple structure of the PI controller and it is not necessary to modify any hardware parts of the original control system for implementation.

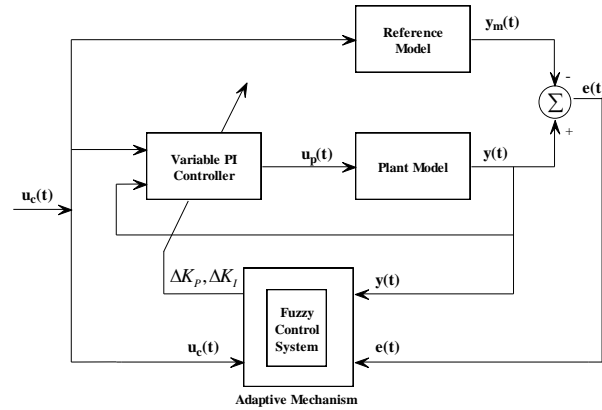


Figure3. Structure of Model Reference based Fuzzy Adaptive PI Controller

**4.1. Adaptation Mechanism**

The fuzzy adaptive system in accordance with the desired response we shall here use the term training for the adaptation activity to align with the terminology used in fuzzy modeling. At the sampling instant  $n$ . It reads the error signal  $e_f(t)$  is defined as  $e_f(t) = u_c(t) - u(t)$  and rate of change of error signal, to find the new PI parameters  $\Delta K_p, \Delta K_I$  using fuzzy rule base system. A typical structure of a PI control system, where it can be seen that in a PI controller, the error signal  $e(t)$  is used to generate the proportional, integral actions, with the resulting signals weighted and summed to form the PI controller is where  $u(t)$  is the input signal to the plant model. The self-tuning PI-type fuzzy controller is an auto adaptive controller that is designed by using an incremental fuzzy logic controller in place of the proportional term in the conventional PI controller to tune the parameters of PI controller on line by fuzzy control rules. The controller uses the error and change in error as its inputs and can meet desire of self-tuning parameters based on time-varying  $e$  and  $ce$ .

**4.2. Fuzzy Systems**

Mathematically, a fuzzy logic system is a function mapping from  $\mathfrak{R}^n$  to  $\mathfrak{R}^m$ . In this paper we consider multi-input single-output (MISO) fuzzy logic systems mapping from an input vector  $x = [x_1, \dots, x_n]^T \in U \subseteq \mathfrak{R}^n$  to an output  $y \in \mathfrak{R}$ . Let  $F_i^{k_i}$ ,  $k_i = (1, \dots, p_i)$ , be the fuzzy sets defined on the  $i$ th input. The fuzzy logic system is characterized by a set of simplified if-then rules expressed in the following form

$$R^k : \text{If } x_1 \text{ is } F_i^{k_i} \text{ and } \dots \text{ and } x_n \text{ is } F_i^{k_n} \text{ Then } y \text{ is } y^k \quad (k = 1, \dots, p)$$

Where  $P = \prod_{i=1}^n P_i$  is the total number of rules, and  $y^k$  is the crisp output for the  $k$ th rule.

The final output of the fuzzy system is calculated as follows

$$y(x) = \frac{\sum_{k=1}^p \mu_k(x) y^k}{\sum_{k=1}^p \mu_k(x)} \tag{13}$$

Where  $\mu_k(x) = \prod_{i=1}^n \mu_{F_i^{k_i}}(x_i)$ ,  $k_i \in \{1, 2, \dots, p_i\}$

Where  $\mu_{F_i^{k_i}}(x_i)$  is the membership function of fuzzy set  $F_i^{k_i}$ .

The membership function used by fuzzy controller is triangular membership function and Gaussian function. The fuzzy subsets are Negative Big, Negative small, Zero, Positive small, and Positive Big respectively termed as NB, NS, ZO, PS, PB.

The quantization factor and the scaling factor play an important role in the performance of the fuzzy controller. The control rules are framed to achieve the best performance of the fuzzy controller. These rules are given in the tables III and IV.

*Table 3.  $K_p$  Fuzzy control rule*

E \ CE	NB	NS	ZO	PS	PB
NB	PB	PB	PS	ZO	ZO
NS	PS	PB	PS	ZO	NS
ZO	PS	PS	ZO	NS	NB
PS	PS	ZO	NS	NS	NB
PB	ZO	ZO	NS	NB	NB

*Table 4.  $K_i$  Fuzzy control rule*

E \ CE	NB	NS	ZO	PS	PB
NB	PS	NS	ZB	NS	PS
NS	PS	NS	NS	NS	ZO
ZO	ZO	NS	NS	NS	ZO
PS	PB	NS	PS	PS	PB
PB	PB	PS	PS	PS	PB

Using this control rules conic\_kp.fis and conic\_ki.fis are created. This control rules are framed using the fuzzy logic toolbox available in MATLAB. The above said membership function with the mentioned fuzzy subsets and the control rules form the fuzzy controller. This .fis file is called in the simulink environment and the connection is established between them. The inference engine used here is the Mamdani inference engine.

**4.3. Rule viewer of the Fuzzy Controller**

Based on the established fuzzy rules, the rule viewer of  $K_p$ ,  $K_i$  are shown in Figure 4, and 5 respectively

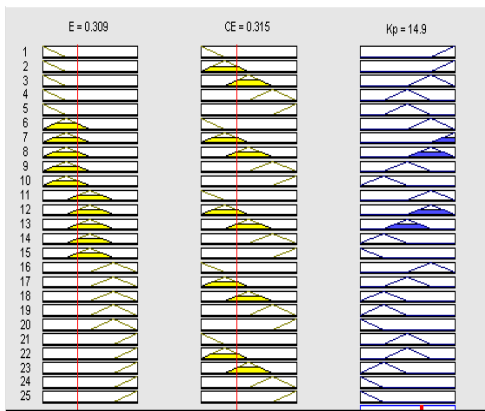


Figure 4. Surface view of  $K_p$

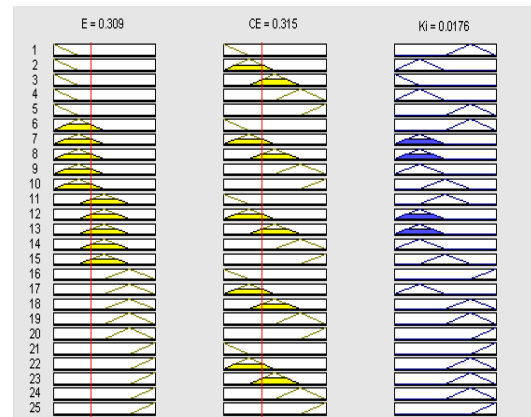


Figure 5. Surface view of  $K_i$

**5. RESULTS AND COMPARISON**

The MRAC and proposed MRFAPIC controller scheme have been simulated for various set points using MATLAB simulink software and the results are displayed in figures 6 to 9. The simulation results clearly show that the MRFAPIC controller possesses minimal overshoot and exhibits faster response as compared to the MRAC. The performance indices comparisons for various set points to the plant models with the designed controllers are presented.

Figure 6. Comparison of MRFAPIC and MRAC Controller for the set point of 20cm

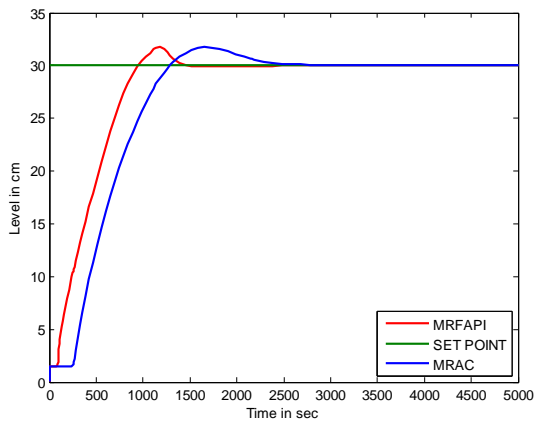
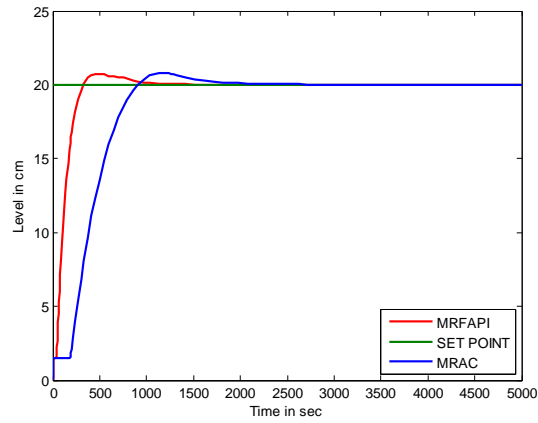


Figure 7. Comparison of MRFAPI and MRAC Controller for the set point of 30cm

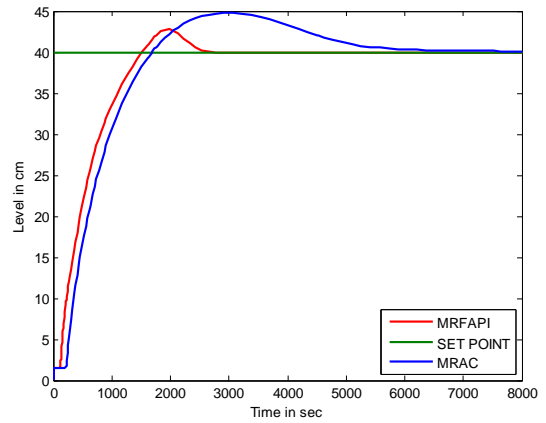


Figure 8. Comparison of MRFAPI and MRAC Controller for the set point of 35cm

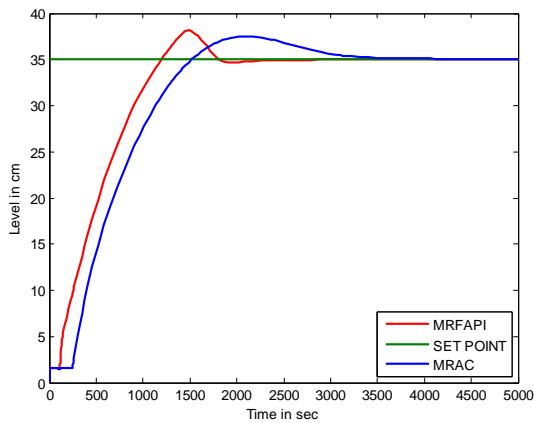


Figure 9. Comparison of MRFAPI and MRAC Controller for the set point of 40cm

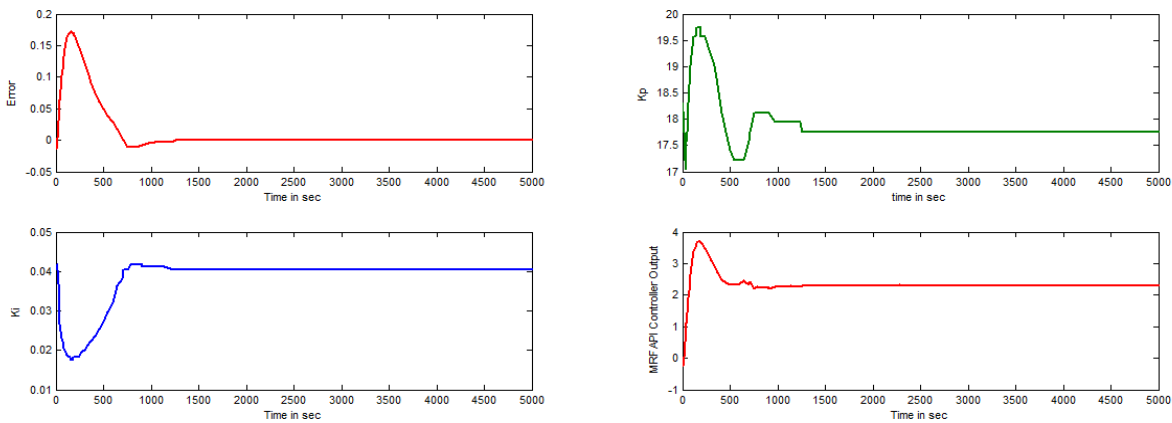


Figure 10. MRFAPI Controller parameters output  $K_p$ ,  $K_i$  and controller output for the Set Point of 20 cm

**5.1. Set point variation**

The set point variations of the level from 22cm to 27cm for both the controllers are shown in figure11.a.while figure 11.b highlightsthe set point changes from 4000 sec onwards. It is observed that the MRAC has minimum overshoot but it takes more time to track the change in set point compared to MRFAPI controller. The reaction of the MRFAPI controller for the set point change is also shown in figure 12.

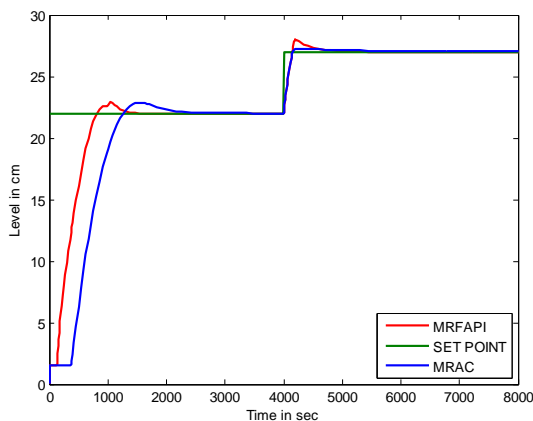


Figure 11.a. Set point changes from 22cm to 27 cm forMRFAPI and MRAC Controller

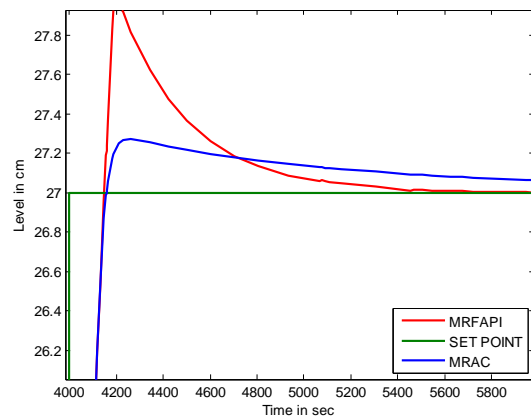


Figure 11.b. Highlight the Set point changes from 22cm to 27 cm forMRFAPI and MRAC Controller

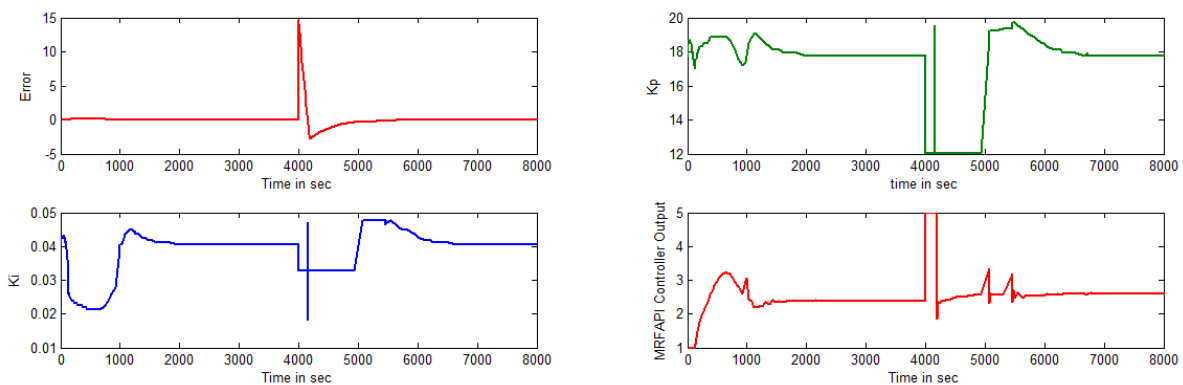


Figure 12. MRFAPI Controller parameters output  $K_p$ ,  $K_i$  and controller output for set point changes

**5.2. Load variation**



Once the process settles down at the desired set point, one more inflow is introduced at the rate of 200 lph to the conical tank. For the applied load disturbance the proposed MRFAPI controller takes faster action and without any overshoot as compared to the MRAC is shown in figure 13. while figure 13.b highlights the load variation from 4000 sec onwards. Figure 14 shows the proposed MRFAPI controller output for the applied load variation.

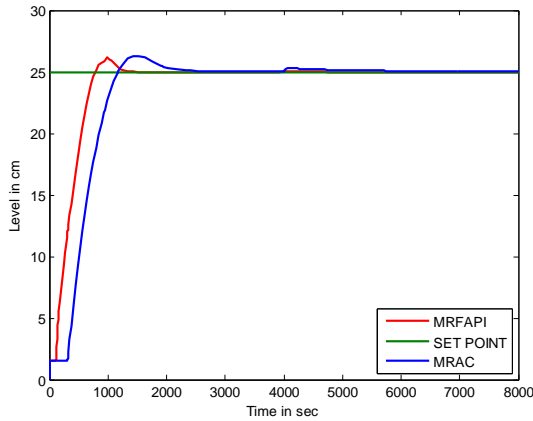


Figure 13.a. Load disturbance response for MRFAPI and MRAC Controller

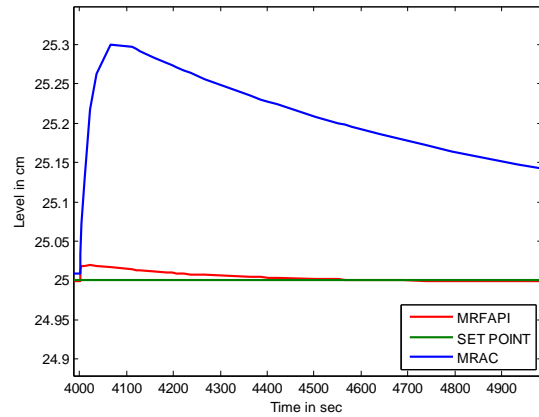


Figure 13.b. Highlight the Load disturbance response for MRFAPI and MRAC Controller

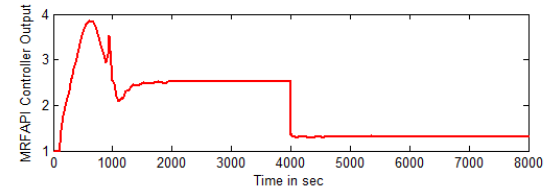
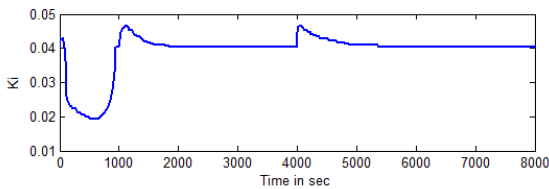
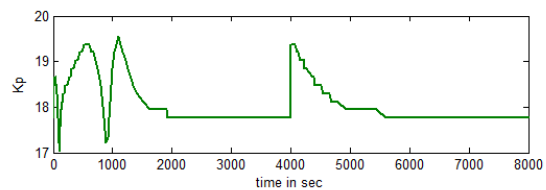
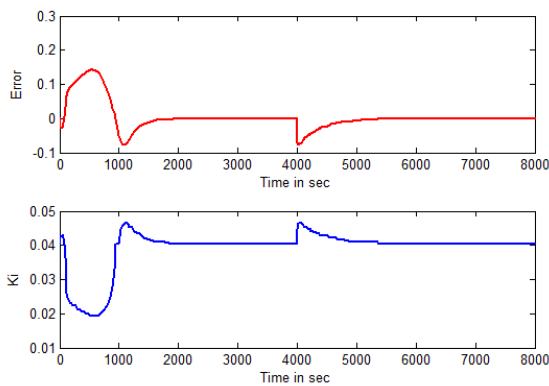


Figure 14. MRFAPI Controller parameters output  $K_p$ ,  $K_i$  and controller output for Load disturbance

### 5.3. Performance Index

The major error criterion techniques such as Integral of Absolute Error (IAE), Integral Square of Error (ISE) have been compared for various set points. The performance indices of both the proposed scheme as well as the MRAC scheme for various set points applied to the chosen level process are presented in the tables V. From these tables it is clearly seen that the proposed control strategy performs fairly satisfactorily then the MRAC scheme.

Table 5. Comparison of the performance indexes for different set point

SET POINT	METHOD	MRFAPI	MRAC
20 cm	IAE	63.2	86.68
	ISE	7.27	11.54
30 cm	IAE	171.40	195.40
	ISE	20.91	39.28
35 cm	IAE	256.9	262.4
	ISE	39.31	56.93
40 cm	IAE	365.6	392.60
	ISE	56.76	76.95

## 6. CONCLUSION

The practical feasibility of a MRFAPI controller for a highly non-linear level process has been presented in this work. The performance of the proposed control strategy is found to be quite satisfactory for set point as well as load variations under different operating levels. Table VI compares the time domain performance indices of MRFAPI and MRAC and from the table it is clearly seen that the proposed control strategy performs satisfactorily well over MRAC.

*Table 6. Comparison the time domain specification for different set point*

SET POINT	SPECIFICATION	MRFAPI	MRAC
20 cm	Rise Time ( $t_r$ )	325	900
	% Overshoot	3.75	3.94
	Settling Time ( $t_s$ )	1500	2700
30 cm	Rise Time ( $t_r$ )	940	1288
	% Overshoot	5.78	5.85
	Settling Time ( $t_s$ )	1850	2800
35 cm	Rise Time ( $t_r$ )	1190	1515
	% Overshoot	23.1	21.4
	Settling Time ( $t_s$ )	2650	4800
40 cm	Rise Time ( $t_r$ )	1525	1680
	% Overshoot	6.88	12.5
	Settling Time ( $t_s$ )	2750	6620

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