CLASSIFICATION OF THE THEORIES OF ELASTIC MEDIA WITH MICRO STRUCTURE

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ABSTRACT
In this paper we discuss that the couple – stress theories can be considered as a long wave – length approximation of a more general non local theory of elasticity closely connected with the theory of crystal lattice. Theories of media with microstructure are considerably more complicated than the classical theory of elasticity, although we reduce to this certain limits. Our application is justifiable only when we describe qualitatively new effect s which are not derivable from the local theory.
Here we give a brief classification of the theories of elastic media with micro structure. Explicit or implicit non locality is the characteristic feature of all classification of the theories of elastic media. We follow an explicit consideration of microstructure effects with the simultaneous consideration of non locality.

Keywords: *non local theory oy elasticity, Explicit and implicit non locality , Crystal lattice, local theory of elasticity.*

1. INTRODUCTION
It was recognized that the couple – stress theories can be considered as a long wave . the contributors to the non local theory of elasticity are Eriengen, Green, Kroner, Kunin, Rogula and others .Recent work on the couple-stress theory and non local elasticity is given in (43) (for a survey of contributions to the couple stress theory before 1960 (87).
The theories of media with microstructure are considerably more complicated than the classical theory of elasticity, although they reduce to this in certain limits . we give a brief classification of the theories of elastic media with microstructure . Explicit or implicit non locality is the characteristic feature of all classification of the theories of elastic media . The non locality displays itself in that the theories contain parameters which have the dimension of length. These scale parameter can have different physical meaning: a distance between particles in discrete structures , the dimension of a grain or a cell , a characteristic radius of correlation or axtion at a distance force
One has to distinguish the cases of weak and strong nonlocality, if the resolving power of the model has the order of the scale parameter, if in the corresponding theory , it is physically acceptable to consider wave length comparable with the scale parameter. Then we call the theory non local or strongly non local (when intending to emphasize this). In such model , one can consider the elements of the medium of the order of the scale parameter , but as a rule, distances much smaller than the parameter has no physical meaning . The equations of motion of a consistently non local theory necessarily contain integral , integro differential , or finite difference operator in the spatial variables , in non local models the velocity of wave propagation depends on wave length; therefore , the term medium with spatial dis persion is also used frequently.
In the present work, analysis of wave propagation is carried out in a micropolar elastic plate permeated by a constant magnetic field $\vec{H}$ .Dispersion equations of symmetric vibration has been obtained and the phase velocity of long wave is deduced, which gives modified longitudinal wave velocity.

2. BASIC EQUATION
on the assumption that the elastic deformations are infentesimal and that the displacement currents are negligible compared with the conductivity currents, the general form of the linearized magneto elastic equation for a perfectly conducting homogenious isotropic symmetric body in the absence of body forces are

$$\Box_2\vec{u}+\left(\lambda+\mu-\mu\right)grad\left(div\vec{u}\right)+2\alpha\cot\phi+\frac{1}{4\pi}curl\left[curl\left(\vec{u}\times\vec{H}\right)\right]\times\vec{H}=0 \text{ ......(1)}$$

$$\Box_4\phi+\left(\beta+\gamma-\upsilon\right)grad\ \ div\cdot\vec{\phi}+2\alpha\cot\vec{u}=0 \text{ ....(2)}$$

$$\vec{E} = -\frac{1}{c}\left(\frac{\partial \vec{u}}{\partial t} \times \vec{H} \right)$$

$\vec{h}$ = **curl** $\left(\vec{u} \times \vec{H}\right)$ **……………………………………………….(3)**

Where $\vec{H}$ = Steady magnetic field

$\vec{h}, \vec{E}$ = Perturbed magnetic and electric field intensities

$\vec{u}$ = displacement field

$\vec{\phi}$ = rotation vector

λ , μ = Lames constant

α , β , γ and υ =constants of cosserat medium.

ρ = density

C = velocity of light

$\square_2$ = $\left(\mu+\alpha\right)\nabla^2 - \rho\delta_t^2$

$\square_4$ = $\left(\gamma+\upsilon\right) - 4\alpha - J\delta_t^2$

Since the elastic medium is supposed to be in contact with vacuum the electro dynamics equation in vacuum are

$$\left(\frac{\delta^2}{\delta_t^2} - c^2\nabla^2 \right)\vec{E} = 0 \quad , \quad \left(\frac{\delta^2}{\delta_{t^2}} - c^2\nabla^2 \right) \vec{h} \quad \text{………………… (4)}$$

3. PROBLEM FORMULATION

In rectangular co ordinate system the displacement vector $\vec{u}$ and the magnetic field $\vec{H}$ have components $\vec{u}$ = $\left(u, v, w\right)$, $\vec{H}$ = $\left(H_1, H_2, H_3\right)$ **.**If $H_2 = 0$, the motion represented by (3.1) and (3.2) can be separated into a purely horizontally polarized movement corresponding to the SH motion and a motion with zero vertical rotation corresponding to the P and SV type of movement .

When $H_2 = V = H_2$ **= V =** $\frac{\partial}{\partial y}$ **= 0**

$\vec{u}$ = $\left(u, 0, w\right)$, $\vec{\phi}$ = $\left(0, \phi_2, 0\right)$

div $\vec{u}$ = $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$

grad **(** div $\vec{u}$ **)** = $\vec{i} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right)$ + $\vec{k} \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right)$

rot $\vec{\phi}$ = -- $\vec{i}\ \frac{\partial \phi_2}{\partial z} + \vec{k}\ \frac{\partial \phi_2}{\partial x}$

$\vec{u}\times\overrightarrow{H} = \vec{j}\ \left(H_1W - H_3u\right)$

Curl $\left\{curl\left(\overline{u\times H}\right)\right\}\times\overrightarrow{H} = \vec{i}\ H_3\nabla^2\left(H_3u - H_1w\right) - \overrightarrow{K}\ H_1\ \nabla^2\left(H_3u - H_1u\right)$

Grad (div $\vec{\phi}$) = **0**

Rot $\vec{u}$ = **j** $\left(\dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial x}\right)$

The equations $\square_2\vec{u} + \left(\lambda+\mu-\alpha\right)grad\left(div\vec{u}\right) + 2\alpha\cot\phi + \dfrac{1}{4\pi}\left[curl\left(\vec{u}\times\overrightarrow{H}\right)\right]\times\overrightarrow{H}$ =**O**

And $\square_4\phi + \left(\beta+\gamma-\vartheta\right)grad$ **div** $\vec{\phi}$ **+ 2** $\alpha\cot\vec{u}$ **= 0 reduce to**

$$\square_2 u + \left(\lambda+\mu-\alpha\right)\frac{\partial\ u}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) - 2\alpha\frac{\partial\phi_2}{\partial z} + \nabla^2\left(\frac{H^2{}_3}{4\pi}u - \frac{H_3H_1w}{4\pi}\right) = 0 \quad \text{……….(5)}$$

$$\square_2 w + \left(\lambda+\mu-\alpha\right)\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) + 2\alpha\frac{\partial\varphi_2}{\partial x} + \nabla^2\left(\frac{H^2{}_1w}{4\pi} - \frac{H_3H_1u}{4\pi}\right) = 0 \quad \text{………..(6)}$$

$$\square_4\phi_2 + 2\alpha\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0 \quad \text{……………………………..(7)}$$

Expressing the displacements by potentially

u = $\dfrac{\partial\phi}{\partial x} - \dfrac{\partial w}{\partial z}$, **w** = $\dfrac{\partial\phi}{\partial z} + \dfrac{\partial\psi}{\partial x}$

equations (5) , (6) and (7) are reduced to

$$\square_2\left(\frac{\partial\phi}{\partial x}\right) + \left(\lambda+\mu-\alpha\right)\frac{\partial}{\partial x}\left(\nabla^2\phi\right) + \nabla^2\left[\gamma_3\frac{\partial\phi}{\partial x} - \sqrt{\gamma_1\gamma_3}\frac{\partial\phi}{\partial z}\right] = 0 \quad \text{,,,,,,,,,,,,,,(8)}$$

$$\square_2\left(\frac{\partial\phi}{\partial z}\right) + 2\alpha\frac{\partial\phi_2}{\partial x} + \nabla^2\left[\gamma_3\frac{\partial\psi}{\partial z} + \sqrt{\gamma_1\gamma_3}\frac{\partial\psi}{\partial x}\right] = 0 \quad \text{…………..(9)}$$

$$\square_2\left(\frac{\partial\phi}{\partial z}\right) + \left(\lambda+\mu-\alpha\right)\frac{\partial}{\partial z}\left(\nabla^2\phi\right) + \nabla^2\left[\gamma_1\frac{\partial\phi}{\partial z} - \sqrt{\gamma_1\gamma_3}\frac{\partial\phi}{\partial x}\right] = 0 \quad \text{,,,,,,,,,,,,,,,(10)}$$

$$\square_2\left(\frac{\partial\psi}{\partial x}\right) + 2\alpha\frac{\partial\phi_2}{\partial x} + \nabla^2\left[\gamma_1\frac{\partial\psi}{\partial x} + \sqrt{\gamma_1\gamma_3}\frac{\partial\psi}{\partial z}\right] = 0 \quad \text{…………………..(11)}$$

$$\square_4\phi_2 + 2\alpha\left[\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z}\right) - \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x}\right)\right] = 0 \quad \text{…………………..(12)}$$

We examine the following two cases of applied magnetic field

Case (i) $\overrightarrow{H} = \left(0,0,H_3\right)$

Case (ii) $\overrightarrow{H} = (H, 0, 0)$

Here we discuss case (ii)

In case (ii) $\gamma_2 = 0$ and equations (.8) ,(9) (10) and (11) reduce to equations

$$\left[(\lambda + 2\mu + \gamma_1)\right]\nabla^2 - \rho\frac{\partial^2}{\partial t}\ ]\phi = 0 \text{ ....}\mathbf{(13)}$$

$$\left[(\mu + \alpha + \gamma_1)\nabla^2 - \rho\frac{\partial^2}{\partial t}\right]\psi + 2\alpha\phi_2 = 0 \quad \text{.....................}\mathbf{(14)}$$

$$\left|(\gamma + \vartheta)\nabla^2 - j\frac{\partial^2}{\partial t} - 4\alpha\right|\phi_2 - 2\alpha\nabla^2\psi = 0 \text{ ....}\mathbf{(15)}$$

assuming

$$\phi = f_1(x)\exp\{i(kz - wt)\}$$

$$\psi = g_1(x)\exp\{i(kz - wt)\}$$

Where w = kv

Solution of (13),(14) and (15)are given by

$$\phi = \left[A_1 \sinh \delta'_1 x + B_1 \cosh \delta'_1 x\right]\exp\{i(kz - wt)\}$$

$$\psi = \left[C_1 \cosh \lambda''_1 x + D_1 \sinh \lambda''_1 x + E_1 \cosh \lambda''_2\, x + F_1 \sinh \lambda''_2 x\ \right]\exp\{i(kz - wt)\}$$

Where

And $\phi_2 = \left[C'_1 \cosh \lambda''_1 x + D'_1 \sinh \lambda''_1 x + E'_1 \cosh \lambda''_2 x + F'_1 \sinh \lambda''_2 x\right] x \exp\{i(kz - wt)\}$

where

$$\lambda''^2_{1,2} = k^2 + \tfrac{1}{2}\left(\upsilon^2_0 - \eta''^2_0 - \sigma''^2_2 - \sigma'^2_4\right) \pm \left\{\sigma''^2_2 + \sigma'^2_4 + \eta''^2_0 - \rho^2{}_0\right\}^2 + 4\sigma''^2_2\left(\rho^2_0 - \sigma'^2_1\right)\ \}$$

$$C''^2_2 = \frac{\mu + \alpha + \gamma_1}{\rho}$$

$$\sigma_2''^2 = \frac{w^2}{c_2''^2}$$

$$C_1''^2 = \frac{\lambda + 2\mu + \gamma_1}{\rho}$$

$$\sigma_1''^2 = \frac{\omega^2}{c_1''^2}$$

$$\delta_1'^2 = k^2 - \sigma_1''^2$$

And $\eta_0^{''2} = \dfrac{4\mu^2}{(\mu+\alpha+\gamma_1)(\gamma+\upsilon)}$

The boundary conditions are same as in case1

In this case for symmetric vibration we have, $A_1 = C_1 = E_1 = C_1^{'} = E_1^{'} = 0$

In view of coupling as before we have

$$D_1^{'} = \chi_1^{''} D_1 \quad . \quad F_1^{'} = \chi_2^{''} F^{'}_1$$

Where $\chi_j^{''} = \dfrac{1}{s^{''}}\left(\sigma_2^{''2} + \lambda_j^{''2} - k^2\right)$ **, j =1,2**

$$s^{''} = \frac{2\alpha}{\mu+\alpha+\gamma_1}$$

Applying the boundary condition $\sigma_{xx} = \sigma_{xz} = \mu_{xy} = 0$ on x= $\pm h$ the frequency equation is

$$\frac{\tan\delta_1^{''}h}{\tan\lambda_1^{''}h} = \left\{a_1^{''}\chi_2^{''} - a_2^{''}\chi_1^{''}\frac{\lambda_1^{''}}{\lambda_2^{''}}\frac{\tanh\lambda_2^{''}h}{\tanh\lambda_1^{''}h}\right\} \times \frac{(\lambda+2\mu)\delta_1^{'2} - \lambda k^2}{4\mu^2 k^2 \lambda_1^{''}\left(\chi_2^{''} - \chi_1^{''}\right)\delta^{'}_1}$$

For long waves compared to the thickness of the plate ,we have

$$4\mu^2 k^2 \delta_1^{'2} = \left\{2\mu k^2 - (\mu+\alpha)\sigma_2^{''2}\right\}\left\{(\lambda+2\mu)\delta_1^{'2} - \lambda k^2\right\}$$

We get,

$$v^2 = \frac{(\mu+\alpha)(\lambda+2\mu+\lambda_1) + (\mu+\alpha+\gamma_1)}{(\lambda+2\mu)(\mu+\alpha)}$$

In classical isotropic elastic medium without magnetic field $\left(\alpha \to 0, (\mu+\alpha)\right)$

$v^2 = 2\beta\left(1 - \dfrac{\beta^2}{\alpha_1^2}\right)^{1/2}$ where $\beta^2 = \dfrac{\mu}{\rho}$ and $\alpha_1^2 = \dfrac{\lambda+2\mu}{\rho}$

4. CONCLUSION

It can be readily shown that the only boundary conditions involved in determining the frequency of vibration of the plate are the continuity of the normal and shear stresses across the plate vacuum interface.

Here we found that the continuity of the magnetic field vector across the interface reduce the continuity of the stress so that of vanishing of σ_{xx} and μ_{xx} on the faces of the plate.

5. REFERENCES

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