ON THE RATEOF RETURNS, THE RISK AND THE DISTRBUTIONOF F0REX MARKET INVESTMENT

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ABSTRACT
The foreign exchange market (currency market, F_X or forex) is a form of exchange for the global decentralized trading of international currencies. The foreign exchange market determines the relative values of different currencies. One of the characteristics that make the F_X unique is its huge volume representing the largest asset class in the world leading to high liquidity. This liquidity is independent of regional trading sessions. Fixed income traders are active in the markets, buying and selling securities to influence prices, and thereby interest rates. This means that the trader is either paying out or receiving interest on their position, depending on whether the interest rate differential is for or against them. The interest rate differential is what we refer to as *rate of returns*. In this paper we consider the rate of return, defined here as the net accumulation of the forex broker (an investor) on investing on this market and we suggest a formula for determining this rate of return. Risk is always involved financial markets, so we also consider risk involved in this market and develop a risk measure and an index to determine the risk type when occurred. Furthermore we show that the distribution of this return rate follows Logistic distribution.

Keywords: *Forex Trading, Risk, Rate of Returns, Distribution of return, Logistic distribution,*
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1. INTRODUCTION

Forex trading also known as F_X, foreign exchange is an International (Interbank) financial market for trading currencies.To make it easier to understand which positions are opened a concept of currency pair at Forex market was introduced. Instead of saying "I have bought euro for dollar" or "I have sold euro for dollar" the professional language used by traders in shorter phases is"To buy euro-dollar or To sell euro-dollar". The first currency in the pair is the BASIC and the second is the QUOTED currency respectively, for example USDCAD (USD is the Basic while CAD is the Quoted).

There are some pairs of currencies called major, that is any one paired with USD.They are highly liquid. Liquidity influences the attractiveness of an investment and market efficiency. If the market is efficient then given two currencies A and B, say, and a corresponding forward exchange rate we have

$$1 + i_A = \frac{r_{p,\ AB}(1 + i_\beta)}{r_{f,AB}},$$

wherei_A and i_β are interest rates of the two different currencies and r_{pAB} and r_{fAB} are the present and the forward exchange rates (Nguyen [8]).

The market efficiency characterizes how fast and precise market quotes reflect in the exchange rate on the market. Virtually any financial asset can be bought or sold within seconds during trading sessions.

Financial instruments that are characterized by significant volatility and ability to change greatly have potential for profit making.The lack of an exchange and the direct trade with the broker creates a difference between stock and forex trading. In the stock market brokers will generally charge a commission for each buy and sell transaction you do. In forex, though, most brokers do not charge any commissions. Since they are taking the other side of all the customer trades, they profit by making the spread between the bid and offer prices.

Some traders do not like the structure of the spot forex market. They are not comfortable with their broker being on the other side of their trades as they feel it presents a type of conflict of interest. They also question the safety of their funds and the lack of overall regulation. There are some worthwhile concerns, certainly, but the fact of the matter is that the majority of forex brokers are very reliable and ethical. Those that are not reliable do not stay in business very long. Fixed income traders know that central bankers, like the Federal Reserve, are active in the markets, buying and selling securities to influence prices, and thereby interest rates. This is not something which happens in stocks, but it does in the forex markets. This is known as intervention. It happens when a central bank or other national monetary authority buys or sells currency in the market with the objective of influencing exchange rates.

Prices are the corner stone of trading.In Forex the goods are money, but it does not change the sense of trading. Here we take into consideration the value of one currency against another, and define spread as the difference between the lowest price of supply (bid) and the highest price or demand (ask).When trading forex, one is essentially borrowing one currency, converting it in to another, and depositing it. This is all done on an overnight basis, so the trader is paying the overnight interest rate on the borrowed currency and at the same time earning the overnight rate on the currency being held. This means the trader is either paying out or receiving interest on their position, depending on whether the interest rate differential is for or against them.
The size of the difference constantly changes as the market forges ahead. Sometimes an arbitrage trading consisting of borrowing currency generated by low interest rate and invested in high yielding currency are made, assuming the exchange rate does not change over time or at least change less than necessary to turn a potential profit into a realized loss. Figure 1 shows how profitable trading conditions of one of forex brokers can be via arbitrage.

Currency	Sell	Buy	High	Low	IntrS	Intr B	Pip Cost	MMR	Time
EUR/USD	1.3332	1.3335	1.3349	1.3253	0.43	-0.48	0.75	50.00	19:28:15
USD/JPY	117.94	117.97	118.44	117.62	-1.01	0.99	0.65	50.00	19:28:15
GBP/USD	1.9695	1.9699	1.9725	1.9581	-0.05	0.03	0.75	50.00	19:28:33
USD/CHF	1.2147	1.2151	1.2230	1.2116	-0.68	0.64	0.62	50.00	19:28:15
EUR/CHF	1.6197	1.6201	1.6220	1.6169	-0.47	0.45	0.62	50.00	19:28:00
AUD/USD	0.8101	0.8104	0.8109	0.8027	0.17	0.13	0.75	50.00	19:24:52
USD/CAD	1.1620	1.1625	1.1640	1.1592	-0.23	0.21	0.65	50.00	19:28:12
NZD/USD	0.7172	0.7172	0.7184	0.7088	-0.35	0.34	0.75	50.00	19:28:43
EUR/GBP	0.6769	0.6772	0.6778	0.6753	0.40	-0.52	1.48	50.00	19:28:01
EUR/JPY	157.26	157.30	157.41	156.28	-0.87	0.84	0.64	50.00	19:28:46
GBP/JPY	232.31	232.40	232.84	230.83	-1,98	1.87	0.64	50.00	19:28:33
CHF/JPY	97.08	97,12	97,18	96.53	-0.27	0.24	0.64	50.00	19:28:45
GBP/CHF	2.3926	2.3932	2.4011	2.3882	-1.37	1.22	0.62	50.00	19:28:33
EUR/AUD	1.6453	1.6462	1.6525	1.6425	0.68	-0.73	0.61	50.00	19:28:21
EUR/CAD	1.5493	1.5501	1.5516	1.5390	0.14	-0.18	0.65	50.00	19:28:12
AUD/CAD	0.9414	0.9421	0.9430	0.9332	-0.35	0.29	0.65	50.00	19:28:40
AUD/JPY	95.55	95.61	95.73	94.68	-0.95	0.90	0.64	50.00	19:28:37
CAD/JPY	101.47	101.54	101.99	101.28	-0.68	0.64	0.64	50.00	19:28:17
AUD/NZD	1.1298	1.1302	1.1377	1.1270	0.23	-0.25	0.54	50.00	19:28:43
EUR/NZD	1.8593	1.8605	1.8765	1.8540	1.05	-1.16	0.54	50.00	19:28:43

Figure 1: Overnight rates changed by FXCM broker. Source www.forexfactory.com, 2009.

Columns 6 and 7 on the figure above show interest rates charged for overnight position on the respective currency pair, i.e. the difference in overnight exchange rates. These interest rates are simply meant to reflect the difference between interest rates for each of two exchanged currencies, and thus are meant to be symmetric. In practice, negative rates have higher absolute value which constitutes another part of trading costs.
In a certain time period the rate of pair EURUSD is equal to 1.37513/1.37541. This means that in this time period it is possible to buy euro at the price of 1.37541 (it is the lowest price at which you can buy it) and it is possible to sell it at the price of 1.37513 (it is the highest you can sell it) in this case spread is equal to 28 points (2.8 standard point).
Each trader always want to buy at the lowest prices and to sell at the highest one so there is a need to always ascertain the rate of change of each currency in a pair.
In Forex trading, traders are always interested on how the prices will change. The questions are:
(i) Can we determine the rate of returns (or the portfolio) of a Forex market investor whose aim is to make profit from this currency exchange?(ii) What are the parameters that influence the market? (iii).What kinds of risks are involved in this market? (iv) What is the distribution of this market growth rate? Our aim in this paper is to address these questions. The nature of the market is affected by the basic component of values of currencies (example: Basic and Quoted). We investigate herein the nature of market equilibrium points in two, three or four components – models as in (Bonhoeffer et al [3]). Formulas for determining the rate of return and that measuring the risk involved in this trade are derived. We further shows that the portfolio of and investor in this trade follows the logistic distribution.

2. PROBLEM FORMULATION

The Forex market is the most liquid and volatile of all financial markets, this makes it most risk intensive.We define herein our portfolio behaviour or growth rate of the market behaviour (based on the Forex market) to be;

$$N(t) = B(t) + Q(t) + P(t) + S(t), \quad (1)$$

where

$B(t)$=the value of the Basic currency
$Q(t)$= the value of the Quoted currency
$S(t)$= spread of currency
$P(t)$= Price of currency.

Assuming in a given trade entered with a certain pair of currency at time t, say, we use the following set of differential equations;

$$\frac{dB(t)}{dt} = -vB(t)Q(t) - \bar{O}B(t), \quad (2)$$

$$\frac{dQ(t)}{dt} = vB(t)Q(t) - \delta Q(t) - \alpha Q(t) - \mu Q(t) - CQ(t), \quad (3)$$

$$\frac{dP(t)}{dt} = \mu Q(t) - \delta P(t) - \delta P(t) - \gamma P(t), \quad (4)$$

$$\frac{dS(t)}{dt} = CQ(t) + \gamma S(t) - \delta S(t), \quad (5)$$

where

v =rate of increase in the value of the Basic currency
μ =rate of increase in the value of the Quoted currency
σ =rate of decrease in the value of the Basic currency
α =rate of decrease in the value of the quoted currency
γ =rate of increase in the general market price
δ =rate of decrease in the general market price
c =rate of spread,

to describe the rate of increase (decrease) of the value or price of the Basic or Quoted currency and the rate of increase (decrease) in the general market price.

Assumption of the Model: The following assumptions are taken into consideration:

1) The value of the Basic currency in one way or the other influences the Quoted Currency.
2) The Quoted currency influences the market price.
3) By law of transitivity Basic currency influences the price too.
4) In general market price is been influence by spread rate.

2.1.1 The Rate Of Returns Of Forex Market Investmen

The arbitrary price at time $t \leq T$ of a call option on the forex which plays return at a constant rate k during the option's lifetime is given by risk-neutral formula.

$$C_t^k = B_t E_Q \left(B_T^{-1}(N_T - K)^t \Big/ F_{\leftarrow} \right), \forall\, t \in [0, T]. \quad (6)$$

Where Q is the unique Martingale measure or explicitly,

$$C_t^k = \bar{N}_t G\big(d_1(\bar{N}_t, T-t)\big) - Ke^{-r(T-t)} G\big(d_2(\bar{N}_t, T-t)\big), \quad (7)$$

where $B_t = e^{rt}$, $\forall\, t \in [0, T]$ B is the accumulation factor corresponding to a Forex broker's account also known as trading account, $\bar{N}_t = N_t e^{-t(T-t)}$,and d_1, d_2 are as in

$$C_t^k = e^{-t(T-t)} N_t G\left(\hat{d}_1(N_t, T-t)\right) - Ke^{-r(T-t)} G\left(\hat{d}_2(\bar{N}_t, T-t)\right), \quad (8)$$

with

$$\hat{d}_{1,2}(N, t) = \frac{\ln\left(N/K\right) + \left(r - K \pm 1/2\, \sigma^2\right)t}{\sigma\sqrt{t}}, \quad (9)$$

and $\bar{N}_t = N_t e^{Kt}$ is and auxiliary process whose dynamics is a single risky stock and therefore driven by the stochastic differential equation

$$d\bar{N}_t = \mu_K \bar{N} dt + \sigma \bar{N}_t dW_t \quad (10)$$

where $\mu_K = \mu + K$. The solution of (10) by Ito's formula is well known to be

$$\bar{N}_t = \bar{N}_o exp\left\{\sigma W_t + \left(\mu_K - \frac{\sigma^2}{2}\right) t, \right\}, \forall\, t \in [0,1].$$

Lemma 1: Assume that $N(t)$is characterised by more than a single risky stock, then the rate of returns of a Forex broker's investment is;

$$N(t) = \frac{N(0)exp\left\{\left(a-\frac{1}{2}c^2\right)t+cW_t\right\}}{1+N(0)b\int_0^t exp\left\{\left(a-\frac{1}{2}c^2\right)s+cW_s\right\}ds}, \qquad (11a)$$

and

$$N(t) = \frac{N(0)exp\{at\}}{1+N(0)b\int_0^t exp\{as\}ds}, \quad c = 0. \qquad (11b)$$

Proof:

Suppose instead that $N(t)$ evolves according to the more general stochastic differential equation (Browne [2])

$$dN = [aN - bN^2]dt + cNdW_t; \quad a > 0, b > 0 \; and \; c \geq 0. \qquad (12)$$

Put $Z = N^{-1}$. Then Z evolves (by the application of Ito's formula) according to the following linear differential equation

$$dZ = [b + (c^2 - a)Z]dt - cZdW_t, \qquad (13)$$

with solution (Karatzas and shreve [6]) given by

$$Z = exp\left\{-\left(a - \frac{1}{2}c^2\right)t - cW_t\right\}\left[Z(0) + b\int_0^t exp\left\{\left(a - \frac{1}{2}c^2\right)s + cW_s\right\}ds\right]. \qquad (14)$$

By putting $N = Z^{-1}$, we have (11) as required. By (11), the Forex broker can measure the rate of returns of his total investment. Notice that as $N \uparrow \infty$, the investor (broker) accumulates gain at a faster rate. On the hand, if $N \downarrow \infty$, the broker accumulates gain at a slow rate.

2.2 The Stability For The Market Trading Price Of The Forex-Trading Model

The studying of the above system for the market requires the knowledge of the stability about it equilibrium points [where the value of both Basic and Quoted currencies are equal in price which also implies no spread at that point.

At the equilibrium point the result of the trade should be close to zero ,that is investing in one currency should not produce more return than investing in another, taking into account trading costs, the result might be expected to be below zero approximately equal to the encountered trading cost.

The equilibrium points for the Forex-model can be obtained by solving

$$\frac{dB(t)}{dt} = \frac{dQ(t)}{dt} = \frac{dP(t)}{dt} = \frac{dS(t)}{dt} = 0. \qquad (15)$$

Let

$$P_n = (B'(t), Q'(t), P'(t), S(t)$$

be the equilibrium points of the system above, we employ the method in Elbab et al [4] to obtain $P_1 = (0, 0, 0, 0)$equilibrium of the occurrence at zero. When there is no rate of change on the general market price, then rate of change in spread in Forex market as at the stipulated time t implies $C = \delta = \alpha, \ \gamma = 0$, so that second point of equilibrium becomes

$$P_2 = \left(\frac{\delta + \mu}{v}, -\frac{\delta}{v}, -\frac{\mu}{v}, 0\right)$$

This implies that the Forex market remain also at equilibrium where there is no spread or when spread is zero. Here the result of trading of the currencies in the pair does not make any difference hence; it is advisable to ascertain the point at which the market will be at its equilibrium to enable the trader (investor) to know the right time to invest.

3. THE KINDS OF RISKS INVOLVED IN THE FOREX MARKET

The risk of a portfolio comprises of the undiversifiable (systematic) risk and diversifiable (unsystematic or idiosyncratic) risk. Each of these risks can be further divide into two broad types: the hard (catastrophic) and the soft (degradation) risks.While hard risk involves abrupt, extreme and complete financial ruin of an investment, soft risk involves mild and partial ruin of an investment. Some risky financial processes (like the Forex trading) may involve one distinct type of risk, while others may involve both.

For the hard risk, let the rate of returns $N(t)$ be modelled as a Weibull distributed random variable, with the probability density defined as follows:

$$f_n\big(N(t)\big) = \frac{\gamma}{\beta}\left(\frac{(N(t)-\omega}{\beta}\right)^{\gamma-1} \exp\left\{-(\frac{N(t)-\omega}{\beta}\right\}, \qquad (16)$$

where;

$f_h\big(N(t)\big) \geq 0, -\infty < \omega < \infty, \ N(t) \geq 0, \ \gamma > 0, \ \beta > 0.$

γ, the shape parameter provides more information about the properties of the rate of returns. $\beta > 0$, the scale parameter is directly proportional to mean time-to-failure and ω is the location parameter (Mustafa [7]).

On the other hand, for the soft risk, we assume without loss of generality that the return rate of $N(t)$ can be described by the dynamics:

$$dN(t) = \beta N(t)dW_t + \omega N(t)dt, \qquad (17)$$

where W_t is a standard Weiner process defined on a complete filtered probability space $(\Omega, f, \{f_t\}, P)$ $with\ f_i = \sigma\{W_s: s \leq t\}$.
Then $N(t)$ follows an inverse Gaussian distribution with the Lebesgue function;

$$f_s\big(N(t)\big) = \frac{N(t)}{\sigma\sqrt{2\pi}}\exp\left\{-{}^{1}\!/_{2}\left(\frac{N(t)-\omega}{\sigma}\right)^2\right\}, \qquad (18)$$

$f_s\big(N(t)\big) \geq 0, \sigma > 0, -\infty < \omega < \infty.$
Consider a finite set of returns $i = 1, \dots, d$, of the spot market trading with portfolio position $N = (N_1, \dots, N_d)$, associated with factor ϑ, which describes any kind of risk (i.e. hard, soft risk or both). Associate with

$$V(N, \vartheta) = \sum_{i=0}^{n} N_i \vartheta_i, \qquad (19)$$

the portfolio position with respect to risk factors ϑ.Thenthe risk measure of $N(t)$, the Forex market returns modelled as hard or soft risk with density f_ϑ is given in principle as;

$$H(V(N, \vartheta) = \int_0^\infty f_\vartheta(N)dN. \qquad (20)$$

It has been shown in Osu [10] that the behaviour of the portfolio $N(t)$ with soft risk at time t is given as

$$H(V(N, \vartheta) = \frac{1}{\sqrt{2\pi}}\left[\sigma + \omega\left(\frac{\sigma}{2-\omega}\right)\right]. \qquad (21)$$

On the other hand $N(t)$ with hard risk at time t has its portfolio behaviour as (Okoroafor and Osu [9]):

$$H(V(N, \vartheta)) = {}^{\beta^\gamma}\!/_{\gamma}\,(2-\omega)^{1-\gamma} \qquad (22)$$

Lemma 2: The measure of risk of growth rate of the Forex market returns given the combination of both risk components is;

$$H\big(V(N, \vartheta)\big) = \left\{\left\{\left(\frac{N-\omega}{\sigma^2}\right) + \frac{\gamma}{\beta}\left(\frac{N-\omega}{\beta}\right)\gamma - 1\right\}\right\}^{\delta(f_h, f_s)}, \qquad (23)$$

with the index define

$$\delta(f_h, f_s) = \begin{cases} \frac{N-\tilde{N}}{\varphi(\sigma,p,\gamma)}, & if\ soft\ risk\ occured \\ \frac{|v-|\tilde{v}}{\varphi(\sigma,\beta,\gamma)}, & if\ hard\ risk\ occured \end{cases}. \qquad (24)$$

Where $\varphi(\sigma, \rho, \gamma) = \sigma\pi - \frac{\beta}{\gamma} \mp \left(\frac{1}{\gamma}\right)$, is the integral over the difference between the survival functions of f_h and f_s. $\tilde{N} = (\prod_{i=1}^{j} N_i)^{1/j}$ and $\tilde{v} = (\sum_{i=1}^{j} \frac{(N_i - \bar{N})^\gamma}{n})^{1/\gamma}$ are the arithmetic mean and variance respectively of $N_i, \dots, N$, (Osu [10])
Proof of (23) is by taking the reliabilities for a mission of $N(t)$ at time t for the Weibull and Gaussian distributions and applying equation (20) with a little calculation.

4. THE DISTRIBUTION OF THE MARKET GROWTH RATE

It is important to note that investors in financial market want to be compensated for taking risk that is they want to earn a return high enough to make them comfortable with the level of risk they are assuming. The variance of returns provides a quantification of incurred risks. In what follows we show that the returns $N(t)$ are distributed according to the logistic distribution.

Theorem 1: Let

$$f_n(N\colon b, \beta) = \frac{\beta^b}{\Gamma(b)} exp\{-bN\} exp\{-\beta \exp(-N)\},\ -\infty < N < \infty, b, \beta > 0, \qquad (25)$$

be a two parameter Gumbel probability density function of a continuous random variable N (where b, β are the location and scale parameters, respectively). Suppose that β has a Gamma distribution with density:

$$h(\beta\colon \varepsilon, b) = \frac{\varepsilon^b}{\Gamma(b)} \beta^{b-1} exp\{-\varepsilon\beta\}, b > o, \varepsilon > 0, \qquad (26)$$

then compound distribution of (25) and (26) is the logistic density function.

Proof: Put

$$f_n(N|\epsilon, \beta) = \int_0^\infty f_n(N|b, \beta) h(\beta|\varepsilon, b) d\beta$$

$$= \int_0^\infty \frac{\beta^b}{\Gamma(b)} exp\{-bN\} exp\{-\beta exp(-N)\} \frac{\varepsilon^b}{\Gamma(b)} \beta^{b-1} exp\{-\varepsilon\beta\} d\beta$$

$= \frac{\epsilon^b exp\{-bN\}}{[\Gamma(b)]^2} \int_0^\infty \beta^{2b-1} exp\{-\beta(\varepsilon + \exp(-N))\} d\beta.$

Let

$X = exp\{-\beta(\varepsilon + exp(-N))\}.$ (27)

Then:

$\beta = \frac{N}{(\varepsilon + exp(-N))}$ and $\frac{dX}{d\beta} = (\varepsilon + exp(-N)).$

So that:

$$f_n(N|\epsilon,\beta) = \frac{\epsilon^b exp\{-bN\}}{[\Gamma(b)]^2} \int_0^\infty X^{2b-1} exp\{-\beta X\} dX$$

$= \frac{\Gamma(2b)\epsilon^b exp\{-bN\}}{[\Gamma(b)]^{2(\varepsilon + exp(-N))^{2b}}}, \quad -\infty < N < \infty, (\varepsilon,\beta) > 0.$ (28)

Equation (28) is a probability density function of a random variable with the type 3 generalized logistic distribution (Balakrishman and Leung [1]).

We now derive the distribution of the portfolio of a forex market investor whose rate of return is characterize by two independent interest rate differentials $N_i(t)$ and $N_j(t)$ with different risk density functions as in (16) and (18).

Let $N_i(t)$ be the returns with soft risk and density function $f_s(N(t)) = \frac{N(t)}{\sigma\sqrt{2\pi}} \exp\left\{-{}^1/_2 \left(\frac{N(t)-\omega}{\sigma}\right)^2\right\}$ and $N_j(t)$ the returns with hard risk and density function $f_n(N(t) = \frac{\gamma}{\beta}\left(\frac{(N(t)-\omega}{\beta}\right)^{\gamma-1} \exp\left\{-(\frac{N(t)-\omega}{\beta}\right\}.$

Then the net accumulation or portfolio of the forex broker is;

$$f_{s,h}(N_i, N_j) = \frac{\gamma N_i}{\sigma\beta\sqrt{2\pi}} (\frac{N_j - \omega_2}{\beta})^{\gamma-1} exp\left\{-(\frac{N_j - \omega_2}{\beta})^\gamma\right\} exp\left\{-(\frac{N_i - \omega_1}{\sigma})^2\right\}$$

$$= \frac{\gamma N_i (\frac{N_j}{\beta_2})^{\gamma-1}}{\sigma\beta\sqrt{2\pi}} (1 - \frac{\omega_2}{N_j})^{\gamma-1} exp\left\{-(\frac{N_j}{\beta})^\gamma (1 - \frac{\omega_2}{N_j})^\gamma\right\} exp\left\{-\frac{1}{2}(\frac{N_i - \omega_2}{\sigma})^2\right\}$$

$$= \frac{\gamma N_i N_j^\gamma}{\sigma\beta^\gamma N_2\sqrt{2\pi}} (1 - \frac{\omega_2}{N_j})^{-1} exp\left\{\log(1 - \frac{\omega_2}{N_j})^\gamma\right\} exp\left\{-(\frac{N_j}{\beta})^\gamma exp\left(log(1 - \frac{\omega_2}{N_j}\right)\right\}$$

$$\times exp\left\{-\frac{1}{2\sigma}(N_i - \omega_1\right\}, for \left(1 - \frac{\omega}{N}\right) = exp\left(\gamma log(1 - \frac{\omega}{N}\right)$$

$$\le \frac{\gamma N_i N_j^\gamma}{\sigma\beta^\gamma N_2\sqrt{2\pi}} \exp(\frac{\omega_2}{N_j}) exp\left\{-\gamma\left(\frac{\omega_2}{N_j}\right)\right\} exp\left\{-\frac{N_j}{\beta}\left(exp(-\gamma\frac{\omega_2}{N_j}\right)\right\} \times exp\left\{-\frac{N_i^2}{2\sigma}\left(exp\left(-2\frac{\omega_2}{N_i}\right)\right)\right\}$$

$\le \frac{\gamma N_i N_j^\gamma}{\sigma\beta^\gamma N_2\sqrt{2\pi}} \exp\left\{-\frac{\omega}{N_j}\gamma - 1\right\} \exp\left\{-\left[(\frac{N_j}{\beta})^\gamma exp - \gamma\frac{\omega}{N_j} + \left(\frac{N_i^2}{2\sigma}\right) exp - 2\frac{\omega_1}{N_i}\right]\right\}.$

Let

$V = f(N_i) + f(N_j)$(29)

and

$U = \frac{f(N_i)}{f(N_j)}.$ (30)

Where $f(N_i) = \left(\frac{N_i^2}{2\sigma}\right) exp\left(-2\frac{\mu}{N_i}\right)$ $and\ f(N_j) = \left(\frac{N_j}{\beta}\right) exp\left(-\gamma\frac{\omega}{N_j}\right)$

Then, $f(N_i) = \frac{UV}{U+1}$ $and\ f(N_j) = \frac{V}{U+1}$ and the Jacobian of the transformation from $(f(N_i), f(N_i)$ to $(U,V)$is $\frac{V}{(U+1)^2}$.

From (29), we have $\frac{\partial V}{\partial N_j} = 1 + \frac{\omega_2}{N_j} = 0 \to N_j = -\omega_2$, similarly,

$\frac{\partial V}{\partial N_i} = \frac{N_i}{\omega_1} + 1 = 0 \to N_i = -\omega_1.$

So that

$f_{s,h}(U,V) = \rho(\gamma)\frac{V}{(1+U)^2} e^{-v}, \ 0 < U, V < \infty,$ (31)

where $\rho(\gamma) = \frac{\gamma\beta^{-\gamma}\omega_1\lambda_2^{\gamma-1}}{\sigma\sqrt{w\pi}} e^{-(\gamma-1)}$

Proposition 1: Let Y be as in (25). Then the behaviour of the portfolio follows a logistic distribution.
Proof: From (30) we note that V and U are independent of each other and that V is the sum of two independent functions whereas the probability density function of U is (Kapadia et al [5]),
$f_{s,h}(u) = \rho(\gamma)\frac{1}{(1+u)^2}, 0 < u < \infty.$
Define $Y = \log U$. Then

$$F(f_{s,h}) = \frac{d}{dY}\{P(u > e^{-y})\}$$
$$= \frac{d}{dY}\left\{\int_{e^{-\gamma}}^{\infty} \rho(\gamma)\frac{du}{(1+u)^2}\right\}$$
$$= \frac{d}{dY}\left\{\rho(\gamma)\frac{1}{(1+u}\Big|_{e^{-\gamma}}^{\infty}\right\}$$

$$= \rho(\gamma)\frac{e^{-\gamma}}{1+e^{-\gamma}}, -\infty < y < \infty, \qquad (32)$$
as required.

5. OVERALL CONCLUSIONS

The forex market is exceptionally volatile, especially on the high frequent data. This paper explores the effect of the interest rate differentials on the portfolio of the investors in the market. There is always some level of risks in the financial markets. We have examined these risks types and develop an index to ascertain the kind of risk that has just occurred in the market transaction. Given a probability density function and portfolio position with respect to risk factors, we are able to show that the behaviour of the portfolio of the investor follows the logistic distribution.

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