BIANCHI TYPE I DUST FILLED UNIVERSE WITH DECAYING VACUUM ENERGY (Λ) IN C-FIELD COSMOLOGY

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ABSTRACT

Bianchi Type I dust filled universe (p = 0) with time dependent vacuum energy density (Λ) in the presence of creation field (C-field) is investigated. Bianchi Type I metric is taken as $ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2$. A,B, C are functions of t-alone. To get the deterministic solution, we assume that $\sigma_1^1 \alpha \theta$ where σ_1^1 is the eigen value of σ_i^j (shear tensor) and θ is the expansion in the model,

$$\sigma_1^1 = \frac{1}{3}\left(\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right), \theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}.$$

The condition $\sigma_1^1 \alpha \theta$ leads to A = $(BC)^n$. To get the solution in terms of cosmic time t, we have assumed n = 1. The solution so obtained satisfies conservation equation $(8\pi G\, T_i^j + \Lambda g_i^j)_{;j} = 0$ where $\underset{(m)}{T}{}_i^j = \rho v_i v^j$ and $\underset{(c)}{T}{}_i^j = -f(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha)$ are the energy momentum tensors for matter and C-field. We find that C-field increases with time which agrees with C-field cosmology. The deceleration parameter (q) > 0, however, if we assume A = B = C then q < 0. Thus the universe have decelerating and accelerating phase. The cosmological constant (Λ) decays with time which agrees with the present day observations. The other physical aspects of the model related with astronomical observations are also discussed.

Key words: *Bianchi I, Dust, Universe, Decaying vacuum energy, C-field.*

1. INTRODUCTION

All the investigations dealing with physical process use a model of the universe, usually called a 'Big bang' model. The 'Big bang' models have the following problems:

(i) the model has singularity in the past and one in future.
(ii) the conservation of energy is violated.
(iii) it leads to a very small particle horizon in the early epoch of the universe.
(iv) No consistent scenario exists within the frame work of Big bang model that explain the origin and evolution of the universe.
(v) it has flatness problem.

If a model explains successfully the creation of positive energy matter without violating the conservation of energy, then it becomes necessary to have some degrees of freedom which acts as a negative energy mode. Thus negative energy provides a natural way for creation of matter. By introducing a massless and chargeless scalar field, Hoyle and Narlikar [10] adopted a field theoretic approach for creation of matter. There is no Big bang type singularity in C-field (Creation field theory). Narlikar and Padmanabhan [13] have investigated the solution of Einstein's field equations which admit radiation (ρ= 3p) and negative-energy massless scalar creation field as a source where ρ is energy density and p the isotropic pressure. They have shown that the cosmological model based on this solution satisfies all the observational tests and thus is a viable alternative to the Big bang model. Bali and Tikekar [6] have investigated C-field cosmological model for dust distribution (p = 0) in flat FRW (Friedmann-Robertson-Walker) model with variable gravitational constant. Bali and Kumawat [7] have investigated C-field cosmological models for dust distribution using FRW space-time for positive and negative curvature with variable gravitational constant.

The non-trivial role of vacuum generates a cosmological constant (Λ) term in Einstein's field equations which leads to the inflationary scenario (Abers and Lee [2]) which predicted that during an early exponential phase, the vacuum energy is treated as large cosmological constant which is expected by Glashow-Salam-Weinberg and by Grand Unified Theory as mentioned by Langacker [12]. Therefore, the present day observations of smallness of cosmological constant $(\Lambda \le 10^{-56}\,\mathrm{cm}^{-2})$ support to assume that cosmological constant is time dependent. Gibbons and Hawking [9] investigated that cosmological models with positive cosmological constant leads to de-Sitter space-time asymptotically. Therefore, the cosmological models linking the variation of cosmological constant having the form of Einstein's field equations unchanged and preserving the energy-momentum tensor of matter content, have been studied by several authors viz. Berman [8], Abdussattar and Vishwakarma [1], Singh and Chaubey [16], Pradhan et al. [14], Bali and Jain [3], Bali and Singh [4], Bali and Tinker [5], Ram and Verma [15].

Motivated by aforesaid investigations, we have investigated Bianchi Type I dust filled universe with decaying vacuum energy (Λ) in C-field cosmology. We find that Creation field increases with time. The universe have decelerating then accelerating phase. The cosmological constant (Λ) decays with time which agrees with present day observations. The other physical aspects of the model related with astronomical observations are also discussed.

2. THE METRIC AND FIELD EQUATION

We consider the Bianchi Type I metric given by

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \quad \ldots(1)$$

where A, B, C are functions of t alone and $\sqrt{-g} = ABC.$

Einstein's field equation by introduction of C-field is modified by Hoyle and Narlikar [11] as

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G\,[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j] - \Lambda g_i^j \quad \ldots(2)$$

The energy momentum tensor $\underset{(m)}{T}{}_i^j$ for perfect fluid and $\underset{(c)}{T}{}_i^j$ for creation field are given by

$$\underset{(m)}{T}{}_i^j = (p+\rho)v_i v^j - p\,g_i^j \quad \ldots(3)$$

$$\underset{(c)}{T}{}_i^j = -f\,(C_i C^j - \frac{1}{2}g_i^j C^\alpha C_\alpha) \quad \ldots(4)$$

where f > 0 is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$. The field equation (2) for the metric (1) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = 8\pi G\left(-p + \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad \ldots(5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = 8\pi G\left(-p + \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad \ldots(6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = 8\pi G\left(-p + \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad \ldots(7)$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} = 8\pi G\left(\rho - \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad \ldots(8)$$

3. SOLUTION OF FIELD EQUATIONS

The conservation equation

$$[8\pi G\, T_i^j + \Lambda g_i^j]_{;j} = 0 \qquad \text{…(9)}$$

leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right) + \frac{\dot{\Lambda}}{4\pi Gf} \qquad \text{…(10)}$$

p being isotropic pressure.

Following Hoyle and Narlikar [11], we have taken p = 0. The source equation of C-field $C^i_{;i} = \frac{n}{f}$ leads to C = t for large r. Thus $\dot{C} = 1.$

Using $p = 0, \dot{C} = 1,$ equation (5), (6), (7) and (8) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = 4\pi Gf + \Lambda \qquad \text{…(11)}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = 4\pi Gf + \Lambda \qquad \text{…(12)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = 4\pi Gf + \Lambda \qquad \text{…(13)}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} = 8\pi G\rho - 4\pi Gf + \Lambda \qquad \text{…(14)}$$

Using equations (12) and (13), we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = -\frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) \qquad \text{…(15)}$$

Equation (15) leads to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{L}{A} \qquad \text{…(16)}$$

where L is constant of integration.
Equations (11) and (13) lead to

$$\frac{C_{44}}{C} - \frac{A_{44}}{A} = -\frac{B_4}{B}\left(\frac{C_4}{C} - \frac{A_4}{A}\right) \qquad \text{...(17)}$$

which leads to

$$C^2\left(\frac{A}{C}\right)_4 = \frac{M}{B} \qquad \text{…(18)}$$

where M is constant of integration.
To get the deterministic value of B and C, we assume

$$BC = \mu \qquad \text{…(19)}$$

and

$$B/C = \nu \qquad \text{…(20)}$$

Using $B^2 = \mu\nu$, equations (16) and (18) lead to

$$\frac{d}{dt}(\mu^2) = N \qquad \dots(21)$$

where N = 4M – 2L and A = BC
which leads to

$$\mu^2 = Nt + \alpha \qquad \dots(22)$$

where α is constant of integration.
Equations (19) and (20) lead to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^2} \qquad \dots(23)$$

Thus equations (22) and (23) lead to

$$\nu = \beta(Nt + \alpha)^{L/N} \qquad \dots(24)$$

where β is constant of integration.
Equations (19), (20), (22) and (24) lead to

$$A = (Nt + \alpha)^{1/2} \qquad \dots(25)$$

$$B = \beta^{1/2}(Nt + \alpha)^{(L/2N+1/4)} \qquad \dots(26)$$

and

$$C = \frac{1}{\beta^{1/2}}(Nt + \alpha)^{(1/4-L/2N)} \qquad \dots(27)$$

Equation (12) leads to

$$\Lambda = \frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - k \qquad \dots(28)$$

where $4\pi G f = k$.

Thus equation (28) leads to

$$\Lambda = -\frac{\gamma}{2}\frac{N^2}{(Nt + \alpha)^2} - k \qquad \dots(29)$$

where $\gamma = \frac{5}{8} - \frac{L^2}{2N^2}$ and $\gamma > 0$.

Equations (14), (25), (26), (27) and (29) lead to

$$\rho = \frac{1}{8\pi G}\left[\frac{\gamma N^2}{(Nt + \alpha)^2}\right] + \frac{f}{2} \qquad \dots(30)$$

Thus the metric (1), after using (25), (26) and (27) leads to

$$ds^2 = dt^2 - (Nt + \alpha)dx^2 - \beta(NT + \alpha)^{(L/N+1/2)}dy^2 - \frac{1}{\beta}(NT + \alpha)^{(1/2-L/N)}dz^2 \qquad \dots(31)$$

Using equations (25), (26), (27), (29) and (30) into Equation (10), we have

$$\frac{d}{dt}\dot{C}^2 + \frac{2N}{(Nt + \alpha)}\dot{C}^2 = \frac{2N}{(Nt + \alpha)} \qquad \dots(32)$$

From equation (32), we have

$$\dot{C}^2 = 1 \qquad \ldots(33)$$

Thus we have

$$\dot{C} = 1 \qquad \ldots(34)$$

Taking α^2 as constant of integration, we find $\dot{C} = 1$, which agrees with the value used in the source equation. Thus creation field C is proportional to time t.

4. PHYSICAL AND GEOMETRICAL ASPECTS

The homogeneous mass density (ρ), the creation field (C), cosmological constant (Λ) and spatial volume (R^3), the deceleration parameter (q) for the model (31) are given by

$$\rho = \frac{1}{8\pi G}\left[\frac{\gamma N^2}{(Nt+\alpha)^2}\right] + \frac{f}{2} \qquad \ldots(35)$$

C = t

$$\Lambda = -\frac{\gamma N^2}{2(Nt+\alpha)^2} \qquad \ldots(36)$$

$$R^3 = (Nt+\alpha) \qquad \ldots(37)$$

$$q = -\tan h^2 \alpha t \qquad \ldots(38)$$

where $\gamma > 0$, $\alpha > 0$.

The co-ordinate distance to the horizon $\gamma_H(t)$ is the maximum distance a null ray could have traveled at time t starting from infinite past i.e.

$$\gamma_H(t) = \int_{-\infty}^{t} \frac{dt}{R^3(t)}$$

where R^3 is a scale factor. We could extend the proper time t to $-\infty$ in the past because of non-singular nature of space-time. Thus

$$\gamma_H(t) = \int_0^t \frac{dt}{R^3(t)} = \int_0^t \frac{dt}{(Nt+\alpha)}$$

$$= \left[\frac{\log(Nt+\alpha)}{N}\right]_0^t$$

The integral at lower limit is infinite which shows that the model is free from particle horizon.

5. CONCLUSION AND DISCUSSION

If we assume A = B = C in metric (1) then the results also satisfies conservation equation. The deceleration parameter $q < 0$ shows that the universe is accelerating. The creation field (C) increases with time which supports the result obtained by Hoyle and Narlikar [11]. The energy density $\rho > 0$. $|\Lambda| \sim \frac{1}{t^2}$ which agrees with latest astronomical observations. The spatial volume (R^3) increases with time and deceleration parameter (q) < 0 shows that the model represents accelerating universe. Thus Bianchi Type I dust filled universe in creation field cosmology satisfies astronomical tests.

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