MICROSCOPIC OPTICAL POTENTIAL FOR P-12C ELASTIC SCATTERING AT INTERMEDIATE ENERGIES

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ABSTRACT

Analytical formula is constructed for nucleon nucleus microscopic optical potential based on the Glauber-Sitenko microscopic theory of multiple scattering at high energies. In the present work the elastic scattering of proton on ^{12}C is studied and analyzed at proton incident energies 120 MeV, 160 MeV and 200 MeV. The microscopic Eikonal phase of scattering in the optical limit has been used with the usual energy dependent NN scattering amplitude. The microscopic form of the real, imaginary and spin orbit of the obtained optical potential gives a good description of the experimental data of the considered reaction. The theoretical calculations of the differential cross sections as well as analyzing power are in good agreement with the experimental data. In order to get confidence and generalization, the same model should be applied to greater range of energies and other types of target nuclei.

Keyword: *Nuclear Reactions, Optical Model, Proton Elastic Scattering, Glauber Sitenko Theory, 120 MeV, 160 MeV 200 MeV and 400 MeV*

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1. INTRODUCTION

In medium energy nucleus-nucleus scattering ,the theoretical works suggested the elastic scattering of such reaction for incident proton energy greater than 100MeV, provided a powerful means for probing nuclear structure [1]. These studies indicated that the microscopic analysis of single nucleons interactions with nucleus are related to a fundamental problem in nuclear physics. It is the description of the complicated many body scattering process solely in terms of the underlying nucleon-nucleon (NN) dynamics and the structure of the target nucleus. The corrected description of the hadrons nucleus interactions, which is given and studied by multiple scattering theory and the optical model approach, is meant as an approximate scheme [2].

The nucleon optical potential is expected to have an energy dependence which arises from a number of different sources[3]-[5].The free nucleon-nucleon (NN) scattering amplitude has an intrinsic energy dependence. An additional energy dependence arises from exchange effects in the NN scattering amplitude and from the effects of Pauli principle and Hund's rule in nuclear medium. An additional source of energy dependence arises from the projectile, propagating between scattering in the mean field generated by the target nucleus [6]. Phenomenological analysis of nucleon-nucleus (NA) scattering in the energy greater than 100 MeV has shown that the real part of the central potential deviates considerably from the Fermi distribution shape associated with target density [7]. The need for such a unorthodox real central potential shape in this energy region was first noted by Elton [8].

A convincing microscopic theory needs to incorporate an energy dependence which can predict this behavior of the optical potential. The concept of alpha-clustering has found many applications in nuclear reactions and nuclear structure [9].

In 1965, Brink has developed an alpha cluster model (BM).This model has been considered and applied to describe different configuration mixing for several 4N light nuclei from ^{12}C to ^{28}Si. Brink's alpha cluster model [10] is a simple model for describing the spatial four nucleon correlations. It has been successful in describing the elastic form factor of electrons[11],[12] and in calculating the binding energy of light nuclei [13]. In particular, the ^{12}C nucleus in its ground state is assumed to consists of three clusters in an equilateral triangle configuration whose (0S) shell model cluster orbitals are only considered. The form factor corresponding to the spherical part of the wave function gives a good fit to the high energy electron scattering data[14] showing that (BM) is good approximation to the ground state of ^{12}C.

In the present work, an analytical formula for nucleon nucleus microscopic optical potential is constructed on the basis of the Glauber-Sitenko microscopic theory of scattering at high energies [15],[16]. The elastic scattering of proton on ^{12}C is studied and analyzed at proton incident energies 120 MeV, 160 MeV and 200 MeV, using the microscopic Eikonal phase of scattering in the so called optical limit of the multiple scattering theory of Glauber with the usual energy dependent scattering NN amplitude. An introduction for the elastic scattering of proton on ^{12}C at 120 MeV,160 MeV and 200 MeV incident energies is given in section1. In section2, we introduce the formulation of the used microscopic optical potentials. Numerical calculations and results are given in section3, while section4 is devoted for the discussion and conclusion.

2. THE MICROSCOPIC POTENTIAL

According to Glauber-Sitenko microscopic theory of scattering at high energies, the microscopic Eikonal phase of scattering on the bases of so called optical limit is given by:

$$\chi(b) = \int d^2 s \rho^0(s) F_{NN}(\bar{\zeta} = \bar{b} - s) \qquad (1)$$

where $\rho^o(s) = \int_{-\infty}^{\infty} \rho^o(\sqrt{s^2 + z^2})dz$ is the profile function of $\rho^o(r)$ and

$$F_{NN}(\zeta) = \frac{1}{(2\pi)^2} \int d^2 q e^{iq.\zeta} f_{NN}(q) \qquad (2)$$

where q and ζ are vectors quantities , the two body amplitude f_{NN}(q) is the form factor of the NN scattering amplitude usually taken to be of Gaussian form and is parameterized in the usual way [20],[21] and expressed as follows:

$$f_{NN}(q) = \frac{1}{2} \bar{\sigma}_{NN}(i + \bar{\alpha}_{NN}) e^{-q^2 B_{NN}/2} \qquad (3)$$

Here $\bar{\sigma}_{NN}$ is the total cross section of NN scattering amplitude , $\bar{\alpha}_{NN}$ is the ratio of real to imaginary part of the forward NN scattering amplitude, B_{NN} is the slope parameter which determine the fall of the angular distribution of the NN elastic scattering. All the three parameters are energy dependent and they are averaging on isotopic spins of colliding particles. The idea of obtaining a realistic microscopic optical potential is based on comparing the microscopic phase (1) [22],[23] with the phenomenological potential which is defined through the optical potential $U(r) = V(r) + iW(r)$ as follows:

$$\chi(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} U(\sqrt{b^2 + z^2})dz \qquad (4)$$

where ν is the relative velocity of the motion.

In deriving the microscopic realistic optical potential we have taken the potential corresponding to the microscopic phase (1) of the high energy approximation (HEA).Such potential can be obtained by using the inverse Fourier transform to (HEA) phase (1) [17] or independently in [18] ,by substituting the standard expression for simple folding potential [19] in the expression of phase (4).Then, according to the so called high energy approximation the central part of the microscopic optical potential is given by

$$U_{opt}(r) = V(r) + iW(r)$$

with
$$U_{opt}(r) = -\frac{2E}{(2\pi)^2 k} \bar{\sigma}_{NN}(i + \bar{\alpha}_{NN}) \int dq q^2 \rho^o(q) e^{-q^2 B_{NN}/2} j_o(qr) \qquad (5)$$

Where
$$V(r) = -\frac{2E}{(2\pi)^2 k} \bar{\sigma}_{NN} \bar{\alpha}_{NN} \int dq q^2 \rho^o(q) e^{-q^2 B_{NN}/2} j_o(qr)$$

and
$$W(r) = -\frac{2E}{(2\pi)^2 k} \bar{\sigma}_{NN} \int dq q^2 \rho^o(q) e^{-q^2 B_{NN}/2} j_o(qr)$$

where $j_o(qr)$ is the spherical Bessel function of order zero.

The density $\rho^o(q)$ is the Fourier transform of the density of the target nucleus

$$\rho^o(\bar{q}) = \int e^{iq.r} \rho(r) d\bar{r} \qquad (6)$$

According to Brink's model [14] , assuming zero orbital momentum ι = 0 in equation (11a) of Ref.[14], the spherical part of the density $\rho(r)$ of ^{12}C molecule in its ground state is given by

$$\rho(r) = \frac{1\sqrt{r}}{6b^3\sqrt{R_1}} [H I_{1/2}(\frac{2R_1 r}{b^2}) + F I_{1/2}(\frac{R_1 r}{b^2})] e^{-(r^2 + R_1^2)/b^2} \qquad (7)$$

Where R_1 is the distance between the center of mass of α particle to the center of nucleus and $I_{1/2}(x)$ is the modified Bessel function of the order $\frac{1}{2}$, and

$$H = 3N_1 + 12N_2 \qquad F = 6\sqrt{2}(N_1 - 2N_2)$$

$$N_1 = \frac{1}{3(1+2\eta)} \qquad N_2 = \frac{1}{6(1-\eta)} \qquad \eta = e^{-3R_1^2/4b^2}$$

Using Equations (6) and (7) into equation (5) for the microscopic nuclear optical potential and performing the integrals we obtain for the final expression of the optical limit microscopic potential the following analytic expression:

$$U_{opt}(r) = 2D'[He^{R_1^2/b^2} e^{-(r^2+R_1^2)/\lambda} I_{1/2}(\frac{2R_1 r}{\lambda}) + Fe^{R_1^2/4b^2} e^{-(r^2+4R_1^2)/4\lambda} I_{1/2}(\frac{R_1 r}{\lambda})] \qquad (8)$$

with

$$D' = \frac{\hbar\upsilon}{12R_1}\frac{\sigma_{NN}(1+i\alpha_{NN})}{\lambda} e^{-R_1^2/b^2} \qquad and \qquad \lambda = b^2 + 2B_{NN}$$

where, the parameters R_1 = 1.4 fm and b = 1.36 fm [14].

The total nuclear optical potential consists of the complex central part $U_{opt}(r)$ given by eqn.(5) and spin orbit part $U_{LS}(r)$. Therefore the complete nuclear microscopic optical potential is written as:

$$U_{OP}(r) = U_{opt}(r) + U_{LS}(r) \qquad (9)$$

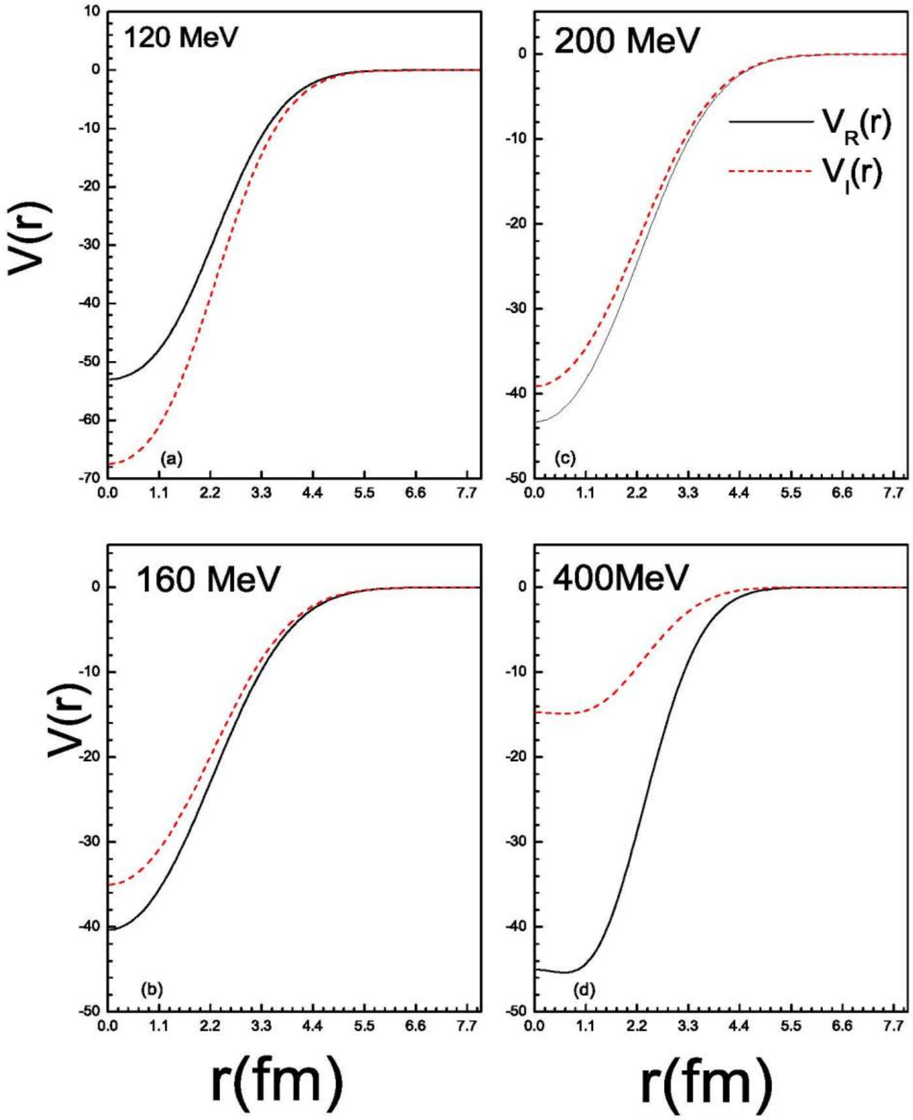
where $U_{LS}(r) = \lambda_\pi^2 (V_{SO} + iW_{SO}) g_{MSO}(r)(\overline{L}\,\overline{\sigma}),$ $\qquad g_{MSO}(r) = (1fm^3)\frac{1}{r}\frac{d}{dr}\rho'(r)$

and $\rho'(r) = 0.195[1+\exp(2.04r - 4.32)]^{-1} - 0.037[1+\exp(3.57r - 2.29)]^{-1}$ (10)

where ρ(r) is the nuclear point density for the ^{12}C nucleus [24],r is in fm , ρ(r) in fm^{-3} and $\lambda_\pi^2 = (\frac{\hbar}{m_\pi c})^2$.

3. NUMERICAL CALCULATIONS AND RESULTS

The experimental data for both the elastic differential cross section and analyzing power of protons scattering on ^{12}C nucleus at 120 MeV, 160 MeV,200 MeV and 400 MeV incident energies are taken from reference [24 , 25,26] .These data have been studied and analyzed using the nuclear microscopic optical potential in the optical limit on the basis of the Glauber Sitenko theory [15],[16]. On the basis of this theory we have derived the analytical expression (8) for the central part of the microscopic optical potential. The numerical calculations have been performed using the DWUCK4 [27] code. The nucleon-nucleon scattering amplitude f_{NN} given by equation (3) have been parameterized and calculated using the different parameters given in [21],[22] for the $\overline{\sigma}_{NN}$, $\overline{\alpha}_{NN}$,and $\overline{B}_{NN}$ scattering amplitudes. In [21] the scattering amplitudes parameters are given for the pp and pn collisions separately, because the proton and neutron densities in ^{12}C target nucleus are virtually the same to good accuracy. In this work we have used the mean values for the $\overline{\sigma}_{NN}$, $\overline{\alpha}_{NN}$,and $\overline{B}_{NN}$ scattering amplitudes given recently in [28,29,30,31]. In the present calculations ,the total microscopic optical potential given by equation (9) consists of the central part given by (8) as well as the spin orbit terms which are chosen [24] in the form of phenomenological Wood Saxon (WS) forms as given by equation (10). The best fit of the present theoretical calculations of both the differential cross sections and analyzing power with experimental data is obtained by readjusting the strengths of the real and imaginary parts of the spin orbit term (10).The different parameters for the spin orbit terms as well as the readjusted strengths of the real and imaginary parts at the different proton incident energies considered are presented in table (1). The radial behavior of the real and imaginary parts of the obtained microscopic optical potential and its variation with energy are displayed in figures (1a,1b,1c,and 1d) at the proton incident energies 120 MeV,160MeV,200MeV and 400 MeV respectively. Here we have introduced the behavior of the potential at 400 MeV just for comparison between intermediate energies and higher energies greater than 300 MeV.



Figure(1) *Radial distribution for the real and imaginary parts of the obtained theoretical microscopic optical potentials at the proton incident energies 120 MeV, 160 MeV , 200 MeV and 400 MeV (1a,1b, 1c and1d) respectively. The solid and dot curves, respectively, represents real and imaginary parts of the optical potentials.*

The theoretical values as well as the experimental data for both the differential cross sections and analyzing power are displayed in figures (2),(3) and (4) for different proton incident energies 120 MeV, 160MeV and 200MeV respectively.

Table (1) The best fit parameters of spin orbit term of the theoretical microscopic optical potential at different intermediate energies.

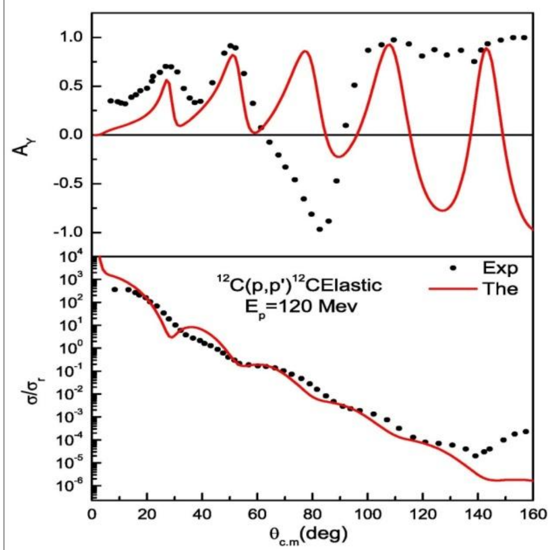
Energy / Parameters	120 MeV	160 MeV	200 MeV
V_{so} MeV	-6.217	-3.917	-5.184
r_{vso} (fm)	0.858	0.871	0.869
a_{vso}(fm)	0.483	0.546	0.558
W_{so} MeV	0.063	0.024	0.026
r_{wso}(fm)	0.810	0.815	0.872
a_{wso}(fm)	0.442	0.567	0.522
$\chi^2(\sigma)$	1569	2648	1518
$\chi^2(A_y)$	16372	7103	3715
σ_R mb	264	217	160
σ_{tot} mb	406	334	246

4. DISCUSSIONS AND CONCLUSIONS

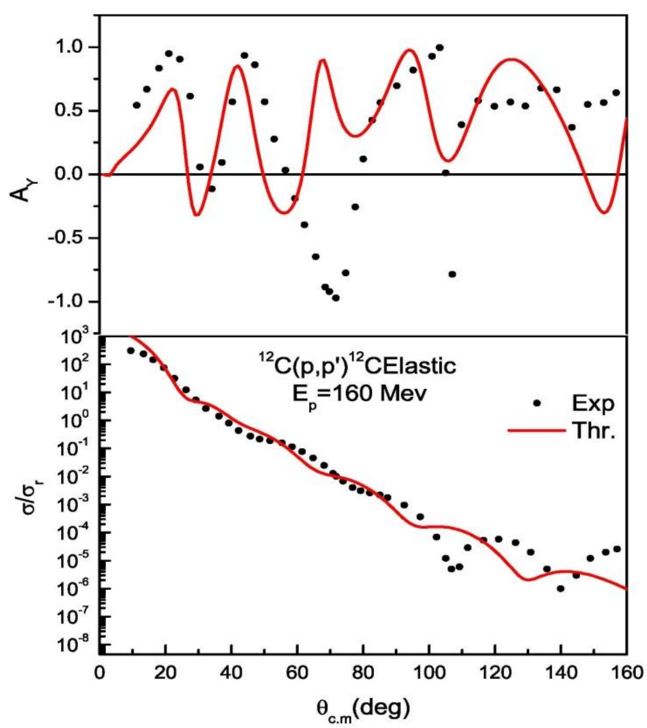
In the present calculations ,the total microscopic optical potential is obtained on the basis of Glauber Sitenko theory [15,16] in the optical limit at high energies. The radial behavior of the obtained microscopic potential is displayed in figure (1) for the different protons incident energies considered. It is clear from figure (1) that the strength of the real parts at small distances is slowly decreasing with increasing the energies of the incident proton , which is natural and expected for the behavior of the real parts of the optical potential at these intermediate energies. As for the imaginary parts of the microscopic optical potential it is shown from figures (1) at small distances that the strengths is strongly decreasing with increasing the incident proton energy where it ranges from -70 MeV (figure 1.a) up to -15 MeV (figure 1.d). This decrease in the strengths of imaginary part is expected with increasing the proton incident energy specially at higher energies greater than 300 MeV, where the strength of the imaginary part is small (-15 MeV at 400 MeV) compared with that one at lower energies (-70 MeV at 120 MeV). Thus the obtained theoretical microscopic optical potential is reliable and successful to describe the proton elastic scattering on ^{12}C target nucleus at the intermediate energies considered in the present calculations which is also supported by the calculated elastic cross section and analyzing power discussed below.

The best fit between the present theoretical calculations for both the differential cross section and analyzing power with the experimental data is obtained by readjusting strengths of the real and imaginary parts of the phenomenological spin orbit parts of the total microscopic optical potential given in table (1). The ratio of the elastic cross section to the Rutherford one σ/σ_R as well as the analyzing power A_Y are displayed in figures (2) ,(3) and (4) for the incident proton energies 120 MeV, 160 MeV and 200 MeV respectively. It is shown from the figures that there is a good agreement between the theoretical calculations for both the elastic cross sections and analyzing powers and experimental data [26] which is verified for the three intermediate incident proton energies considered. The obtained agreement between our theoretical calculations and experimental data indicates that our generated theoretical microscopic optical potential is powerful and reliable tool to describe the elastic scattering of protons on ^{12}C target nucleus at intermediate energies. It is shown form figures (2),(3) and (4) that the good fit between our calculations and experimental values exists for small angles as well as large angles (ranges from 0^o up to 160^o angles) which gives another support to our obtained microscopic optical potential based on the Glauber Sitenko theory [15],[16]. In a previous work [32] the scattering of protons on ^{16}O target nucleus at intermediate energies have been studied using single folding analysis with reasonable agreement with experimental data. In the next work the Glauber Sitenko theory [15],[16] will be used to study the elastic scattering of protons on ^{16}O target nucleus and other nuclei at intermediate and higher energies.

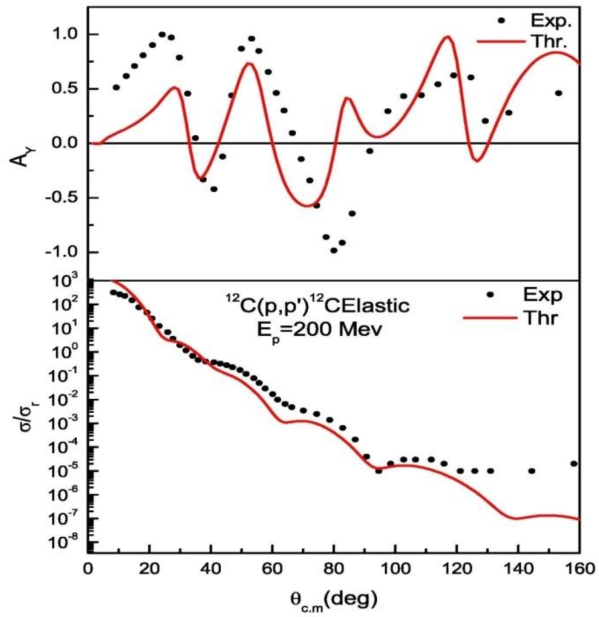
Finally from the above results and discussion, we reach to the following conclusions: The present microscopic optical potential model based on Glauber Sitenko theory on the optical limit at high energies gives a good agreement and fit with the experimental data for both differential cross section and analyzing power at 120 MeV 160 MeV and 200 MeV incident energy. The microscopic form of the central real, imaginary part and spin orbit part of the optical potential gives a good description for the experimental data of the considered reaction rather than the WS forms of all parts of the optical potential. In order to get confidence and generalization, the same model should be applied to greater range of energies and other types of target nuclei which is our next work in the very soon future .



Figure(2) *Ratio of the elastic scattering cross section to the Rutherford cross section σ/σ_R and analyzing power ,A_Y, are plotted vs the center of mass momentum scattering angle at 120 MeV*



Figure(3) *Ratio of the elastic scattering cross section to the Rutherford cross section σ/σ_R and analyzing power ,A_Y, are plotted vs the center of mass momentum scattering angle calculated at 160 MeV.*



Figure(4) *Ratio of the elastic scattering cross section to the Rutherford cross section* σ/σ_R *and analyzing power ,*A_Y*, are plotted vs the center of mass momentum scattering angle calculated at 200 MeV.*

In figures (2),(3) and (4) the solid curve represent the present theoretical calculations while the block circles represents experimental data taken from [25,26,27]

5. REFERENCES

[1]. L. Ray , G.W. Hoffman and W. R. Coker**,** Non relativistic and relativistic descriptions of proton-nucleus scattering. *Physics Reports* , C212, (1992), 223.

[2]. E.Bleszymski , M. Bleszymski and T. Jasasrewics, Optical potential approach in scattering of hadrons from light nuclei. *Phys. Rev*. ,C33, (1986), 1228.

[3]. C. Quigg and C. J. Joachain . Multiple scattering expansions in several particle dynamics.**,** *Rev.Mod.Phys*.,46, (1974), 279.

[4]. C. Bruge , In proceedings of the Seventh international conference on High Energy Physics and Nuclear Structure, edited by M.P. Socher, (Briskhouse,Basel and Stuttgart 1977 p.69).

[5]. R. Crespo, R.C. Johnson and J.A. Tostevin, Mean field calculations of nucleon-nucleus scattering. *Phys. Rev*. C53,(1996),3022.

[6]. K.M. Watson. Multiple Scattering by Quantum-Mechanical Systems , *Phys.Rev*.,105,(1975),1388 ; M.L.Goldberger and K.M.Watson," Collision theory" , John Wiley and Sons, Newyurk 1964.

[7]. H.O. Meyer, P. Schwamadi, G.L. Moake and P.P. Singh, Elastic scattering of 200 MeV polarized protons from $^{12,13}C$ and the optical potential . *Phys. Rev*. C23, (1981), 616.

[8]. L.R.B. Elton , Optical model analysis for protons at 180 MeV., *Nucl.Phys*.A89 ,(1966), 69.

[9]. P. E. Hodgson A survey of alpha-clustering. *Z.Phys*., A349, (1994),197.

[10]. D.M. Brink, H.Friedrich, A.Weigunay and W.Wang, Investigation of the alpha-particle model for light nuclei. *Phys.Lett*.,33B,(1970),143.

[11]. Y.Abgrall, P.Ga binski and J. Labarsouque, Electron scattering and α-cluster description of ^{12}C and ^{20}Ne., *Nucl.Phys*., A232, (1974), 235**.**

[12]. E.V. Inopin, V.S.K. Kunchakov, V.K. Sulycnov, and Y.U.S.Pol, Charge form factors of nuclei in the alpha cluster model., *Ann.. Phys.*118, (1979), 307.

[13]. H. Friedrich, A. Weiguny, Alpha clustering and angular momentum projection before and after variation. *Phys.Lett*., 35B, (1971), 105.

[14]. W. Wadia and S.Moharram., Structure of ^{4}He and ^{12}C nuclei . *Phys.Rev*.,C12,(1975),2050.

[15]. R.J. Glauber, Lectures in Theoretical Physics. (N.Y. Interscience,1959,p35).
[16]. A.G. Sitenko, *UKr. Fiz, Zhum*, 4, (1959), 152 (in Russian).
[17]. P. Shukla, Glauber model and the heavy ion reaction cross section. *Phys.Rev*.,C67, (2003), 054607.
[18]. V.K. Lukyanov, E.V. Zembyanaya and K.V. Lukyanov, P4-2004-115, JINR, Dubna, 2004, Nucleus–Nucleus Scattering in the High-Energy Approximation and Optical Folding Potential. *Phys. Atomic Nuclei* , 69, (2006), 240
[19]. G.R. Satchler ,and W.G.Love, Folding model potentials from realistic interactions for heavy-ion scattering. *Phys.Rep*.,55,(1979), 183.
[20]. L. Ray, Proton-nucleus total cross sections in the intermediate energy range. *Phys. Rev*. , C 20, (1979), 1857.
[21]. C.A. Bertulani, C.M. Campbell, and T. Glasmacher, A computer program for nuclear scattering at intermediate and high energies. *Comp. Phys.Comm*.,152, (2003), 317.
[22]. V.K. Lukyanov, E.V. Zemlyanaya, B.S lowi´nski, K.Hanna,, Glauber–Sitenko approach to scattering of nuclei with realistic density distribution and of restoring method optical potential. Izv.RAN .*Ser.Fiz*., 67, (1), (2003), 55.
[23]. K.M. Hanna,,K.V.Lukyanov,V.K. Lukyanov,B. Slowinski, and E.V. Zemlyanaya, Model for Restoration of Heavy-Ion Potentials at Intermediate Energies. *Izv. RAN.Ser.Fiz*., 69, (11), (2005), 1649.
[24]. H.O. Meyer, P. Schwandt, W.W. Jacobs, and J.R. Hall, Proton scattering from ^{12}C between 120 and 200 MeV and the optical potential . *Phys.Rev*., C27(3), (1983), 459.
[25]. L. Rikus, K. Nakano, and H.V. Von Geramb. Microscopic analysis of elastic and inelastic proton scattering from ^{12}C**.**, *Nucl. Phys.A* ,414, (1984), 413.
[26]. Zhen-Qiang Tan, Wen-Ying Ruan, Proton-nucleus optical potential in the alpha-particle model . *Nucl. Phys.*, 514, (1990), 295.
[27]. P.D.Kunz and E.Rost, in "Computational Nuclear Physics",V.2, P.88 (Ed Langanke K. et.al.Berlin:Springer-Verlag,1993); N.M.Clarke,"The Optical Model code OPTIM94,vs94.2", University of Birmingham.
[28]. W. Horiuchi, Y. Suzuki, B.Abu.Ibrahim and A. Kohama, Systematic analysis of reaction cross sections of carbon isotopes. *Phys. Rev. C*, 75, (2007), 044607.
[29]. B.Abu-Ibrahim, W. Horiuchi, A. Kohama, and Y.Suzuki, Reaction cross sections of carbon isotopes incident on a proton. *Phys. Rev. C*, 77, (2008), 034607.
[30]. Abu-Ibrahim B., Iwasaki S., Horiuchi W., Kohama A. and Suzuki Y., Elastic and total reaction cross sections of oxygen isotopes in Glauber theory. J. Phys. Soc.Jap. (2009) 78, 044201-1 -10 .
[31]. Ibrahim M.A. Tag El-Din, Samia S.A. Hassan, and and M. Fayez-Hassan., study of proton-nucleus total reaction cross section at 30 ≤ ep ≤ 2200 MeV using relativistic mean field densities *Jour. Nucl. Rad.Phys*., 5, (1), (2010), 35.
[32]. E.h. Esmael, and M.A. Allam, Single folding analysis of the elastic scattering of p-^{16}O. *Acta Physica Polonica B* , 39, (1), (2008), 159.