

METABOLISM IS UNIVERSAL

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ABSTRACT

A mathematical theory attempted to predict the daily maintenance calorie intake (C) for a single male individual. It is a linear function of the weight, based on data gathered during two periods of strict water fasts with different fasting rates. These are somewhat unusual sets of data. F_A , the activity factor, is a measure of the amount of physical activity related to the lowest state of body metabolism, RMR. The height and weight of a normal adult male over 16 can be converted to the lean body mass (LBM), the sole predictor of RMR. So, an expression for the resting metabolism rate (RMR) can be derived and extended to Reference Man, and Levanzin, a man who underwent a famous prolonged fast, and the resulting linear formula agrees well with the Harris-Benedict equation for the basal metabolism rate for men. This proves that the RMR for normal adult men was given by extending the theory for the daily calorie intake for one man, so metabolism is universal. It was unknown what the calorie intake was on the non-fasting days, so in the theory it was assumed to be the maintenance calorie intake for the weight on that day. $C \equiv (RMR)(F_A)$ and for men $1.3 < F_A < 1.9$. The activity factor for U.S Reference Man is 1.6.

Keywords: *basal metabolism rate, calculus, Newton approximation, Reference Man, fasting.*

1. INTRODUCTION

The first object of this theoretical calculation was to get C, the daily calorie intake in cal/day, for a human subject, John H. Jennings, the author (JJ), on which there was weight data during 1985-1986 (see DATA FROM DOCTOR'S FILES). In the theory, it was assumed that C is linear in total body weight. The constants in the linear term were evaluated by solving a simple differential equation. The mathematics in this theory make it possible to apply to individuals without statistics and one of the novelties of this approach is to use a calculus technique called Newton approximation.

2. METHODS

There is a number in the literature, NAS 1980 [1], termed $C_{70} = 2700$ cal/day, which is the calories per pound to maintain the weight of a 70 kg man in the United States of height 177.8 cm and age between 23 and 50 years. There is another estimate for this same man in a book that uses the term Reference Man, ICRP [2], and the calorie requirement for that individual in the United States (age 22-35 years, 70 kg) is put at 2800 cal/day. Other sources give the value of 2700, NAS 1974 [3], or 2800, NAS 1968 [4], as the daily calorie intake for "U.S. Reference Man" but have his height as 69". Thus, in this paper the value of 2750 is used for U.S. Reference Man, USRM, and he is assumed to be 70" (177.8 cm) tall. Also, USRM's age is taken to be 30 years. 2700 cal/day is taken as a first estimate for JJ, who was 167.6 cm tall and 34 years old in 1985, from whom the data was collected. The 2700 figure was then revised. Since JJ is shorter than U.S. Reference Man, an initial value of 2700 is used for JJ and later an expression for the basal metabolism rate can be arrived at by making JJ's result agree with U.S. Reference Man, ICRP [2], and another well-studied man, Levanzin, the 40 year old male subject of Benedict [5]. The other constant Wishnofsky [6] used in this theory is $D = 3500$ cal/lb (or the equivalent value in cal/kg) and that is the number of calories needed to be eliminated for an individual to lose one pound of weight. However, a more accurate value for D appears in Kozusko [7] but 3500 is a nice round number. This is a simplified semi-empirical approach to get the RMR, or resting metabolic rate. Although the Harris-Benedict equations referred to the basal metabolism rate originally in Harris and Benedict [8], by methodology according to Frankenfield et al [9] these equations describe resting conditions, thus RMR is used here.

3. RESULTS

JJ underwent dieting periods using two different rates of fasting. The first was strict water fasting for three successive days/week every week during a 56-day period. The second, immediately following, was strict water fasting for one day/week over the rest of the time for 231 days, resulting in a weight loss of about 50 lb over a 9 ½ month period or 287 days. The data were weights taken of JJ on a doctor's scale (see Supplement) from April 12,

1985 to January 24, 1986 [10]. On the other days of the week for each fasting rate, in the theory the diet is assumed to be the normal maintenance diet for that day's weight. Using some results from human studies by Hume [11] one can make a general equation for adult men for the RMR and it can be made more exact by using the result for the calorie intake for JJ and introducing two unknowns (See Supplementary files 1, 2) thus making RMR agree with USRM and Levanzin, according to the Harris-Benedict equation for RMR [8] for a normal adult male of weight Ω (kg) and height ht(cm). It is Eq. X, presented here and is compared in Tables 1-3 with the Harris-Benedict equation [8] for men and Cunningham's formula [12], all three for the RMR for normal adult males.

$$\text{RMR(men)} = 13.89\Omega(\text{kg}) + 14.36\text{ht}(\text{cm}) - 1811 \quad \text{cal/day} \quad \text{Eq. X}$$

Table 1 RMR comparison for males - normal weight. All RMR in cal/day, age 30 years.

ht (cm)	Ω (kg)	Harris-Benedict	Cunningham	Jennings, Eq. X
150	50	1301	1317	1038
160	60	1489	1461	1320
170	75	1745	1641	1673
180	82	1891	1764	1914
190	100	2189	1965	2307

Table 2 RMR comparison for males - overweight.

ht (cm)	Ω (kg)	Harris-Benedict	Cunningham	Jennings, Eq. X
150	60	1439	1388	1177
160	72	1654	1547	1487
170	90	1951	1747	1881
180	105	2208	1923	2233
190	120	2464	2107	2585

Table 3 RMR comparison for males - underweight.

ht (cm)	Ω (kg)	Harris-Benedict	Cunningham	Jennings, Eq. X
150	45	1232	1282	969
160	50	1351	1391	1182
170	55	1470	1499	1395
180	67	1685	1658	1705
190	80	1914	1823	2029

4. DISCUSSION

The values for a and b in C, JJ's daily calorie intake (see Calculations in Supplement), are determined using the calculus. According to Cunningham [12] an error of 10% is to be expected in these kinds of calculations, but in this treatment b and B can be calculated with arbitrary accuracy. In some respects, the normal human body is like a biological machine, so a single man's data is enough to get the RMR for men. The figures in the tables show that the agreement (average deviation 9.4%) between Cunningham's formula for $190 \geq \text{ht} \geq 160$ cm (normal height for men) and Eq. X is best around the height of JJ, but that might be understandable because the whole theory is built around JJ's data of 1985-1986. (JJ's data are slightly unusual sets of data.) The agreement between Harris-Benedict and Eq. X is quite a bit better for the same normal height range (average deviation 5.5%) but that might be expected as USRM, Levanzin and the JJ theory were made to agree for Harris-Benedict. However, the mathematical treatment resulted in an equation of the same general form and it was important to incorporate Levanzin, a well-studied individual, and USRM, a standard, along with the theory. The Harris-Benedict equations and Cunningham's are both based on the Harris-Benedict data, but who is to say that Cunningham's recalculations reflect reality better? Data on a single man, using the theory presented here, predicts the same general result by Harris-Benedict. It appears that an individual can predict the result for all.

5. CONCLUSION

Provided here is a mathematical theory, which predicts the resting metabolism rate in adult men from one man. It shows fair agreement with two well-known formulas for RMR. All of this is an effort to better understand nutrition in humans, and may be able to be further tested with proper data on metabolism rates in men and a wider study.

6. ACKNOWLEDGMENTS

The support of Patrick J. Foley, Sr. encouraged me in writing this article and also I acknowledge YoungHee Lowrie for inspiration in starting this project.

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LEGEND

2750	U.S. Reference Man's daily calorie intake
2700	JJ's original estimate for calorie intake at 154 lb
177.8	U.S. Reference Man's height in cm
170.7	L. 1912's height in cm (Levanzin)
167.6	JJ's height in cm
154	U.S. Reference Man's weight in pounds
70	U.S. Reference Man's weight in kg
60.64	L. 1912's weight in kg (Levanzin)
40	L. 1912's age in years (Levanzin)
30	U.S. Reference Man's age in years for this study
a	0.297792 (for theory in pounds & inches)
b	0.00455979 lb ⁻¹ (for theory in pounds & inches)
A	0.297792 (for theory in kg & cm)
B	0.01005261 kg ⁻¹ (for theory in kg & cm)
c _i	actual calories consumed on day i
C	JJ's daily maintenance calorie intake = C ₇₀ (A + B Ω)
C	maintenance calorie intake = (RMR) (F _A)
C ₇₀	2700 cal / day
D	3500 cal / lb
Dk	7716.2 cal / kg
f	fraction of the week fasted = 1/7 or 3/7
f ₁	fasting one day per week = 1/7
f ₃	fasting three days per week = 3/7
F _A	activity factor, level of physical exertion
ht	height in centimeters
k	conversion factor = 2.204622476 lb/kg
w	weight in pounds

w_i weight in A.M. on day i
 Ω weight in kilograms
 Ω_o weight at beginning of weight loss
 RMR resting metabolism rate
 LBM lean body mass

SUPPLEMENTARY FILE 1 - *the calorie equation*

This section summarizes the original thinking concerning the derivation of RMR.

Given the expression for JJ's maintenance calorie intake as a function of weight, expressions can be derived for the basal metabolism rate (RMR) in terms of the lean body mass (LBM) and a general expression for all normal men giving their maintenance calorie intake, if one has their height and weight. The value for JJ's calorie intake is put at 2700 calories/day at age 34, but Reference Man is about 10 cm taller.

This is a result specific to a single man (JJ) from whom the weight data arose, but using Hume [11] there is an expression correlating the LBM with the weight w in kg, and height, ht in cm for all normal adult males over 16.

For normal men > age 16, Hume gives:

$$\text{LBM (kg)} = (0.32810) \Omega(\text{kg}) + (0.33929) \text{ ht}(\text{cm}) - 29.5336$$

Thus it is possible to convert between any two men's weights by the difference in their heights by converting through the LBM.

So, for two LBMs that are equal (one is JJ's), the constant drops out. Hume's expression allows for the fact that height increases the LBM at a given weight.

$$\begin{aligned} \text{LBM}_{\text{JJ}}(\text{kg}) &= \text{LBM}(\text{kg}) \quad (\text{JJ's height was } 167.6 \text{ cm.}) \\ (0.3281) \Omega_{\text{JJ}}(\text{kg}) + (0.33929) (\text{JJ's ht in cm}) &= \\ &= (0.3281) \Omega(\text{kg}) + (0.33929) \text{ ht}(\text{cm}) \\ \Omega_{\text{JJ}}(\text{kg}) &= \Omega(\text{kg}) + (0.33929)/(0.3281) (\text{ht}(\text{cm}) - \text{JJ's ht in cm}) \\ \Omega_{\text{JJ}}(\text{kg}) &= \Omega(\text{kg}) + (1.03411) (\text{ht}(\text{cm}) - 167.6 \text{ cm}) \end{aligned}$$

So, this is used to generalize the result for all adult men.

$C = (\text{RMR}) (F_A)$ and substituting for Ω_{JJ} makes $\text{RMR} = \text{RMR}_{\text{JJ}}$. Therefore $C / F_A = C_{\text{JJ}} / F_{\text{JJ}}$. The daily calorie intake for a man of weight Ω in kilograms and height ht in centimeters is here,

From (17.1) of the Calculations.

$$C = (2700) (0.297792 + 0.01005261 (\Omega + (1.03411) (\text{ht} - 167.6))) (F_A / 1.723243)$$

The equation for RMR is as follows, using only the result from the calculus:

$$\text{RMR} = C/F_A = (15.75) \Omega + (16.29) \text{ ht} - 2263$$

($F_A = 1.72343$ was based on a derivation with an invalid assumption.)

The above equation for RMR is useful when height (cm) and weight (kg) are known for males with age > 16 yrs, but RMR is extended to both USRM (ICRP, [2]) and Levanzin (Benedict, [5]) in Supplementary File 2, which follows.

SUPPLEMENTARY FILE 2 – *RMR for men*

The daily maintenance calorie intake and RMR, or resting metabolic rate for a man of weight Ω in kilograms and height ht in centimeters is shown here, Hume, as before, for men and also this semi-empirical theory. See Eqs. (14) - (17.2). C and RMR are in cal/day. (This uses 1.72343 as F_{JJ} , although it does not matter what value for F_{JJ} is chosen.) $\Delta ht = ht - 167.6$. (See SECTION 3 of CALCULATIONS, Eq. (19.1) for Harris-Benedict equation for men.)

$$\begin{aligned} C &= (2700) (0.297792 + 0.01005261(\Omega + 1.03411 (\Delta ht))) (F_A/1.723243) && \text{Eq. (17.1)} \\ \text{RMR} &= C/F_A \end{aligned}$$

Now, to generalize this expression for the RMR for men, one can make it true for USRM (R) and Levanzin (L) by adding two unknowns, E and N, including all significant figures, and using Harris-Benedict for RMR. (The divisor 1.723243 is irrelevant because of where the unknowns are added.)

For U.S. Reference Man:

$$\begin{aligned} \text{RMR}_R &= \\ &= (2700/E)(0.2977924503N + 0.01005261394(70 + 1.034105456(10.2)))/1.723243 \\ \text{RMR}_R &= \\ &= 66.5 + 13.75x70 + 5.003x177.8 - 6.775x30 = 1715.2834 \quad (\text{Harris-Benedict for men}) \end{aligned}$$

We proceed in the same way for Levanzin:

$$\begin{aligned} \text{RMR}_L &= \\ &= (2700/E)(0.2977924503N + 0.01005261394(60.64 + 1.034105456(3.1)))/1.723243 \\ \text{RMR}_L &= \\ &= 66.5 + 13.75x60.64 + 5.003x170.7 - 6.775x40 = 1483.3121 \quad (\text{Harris-Benedict for men}) \end{aligned}$$

Solving for E and N and substituting, we have RMR for men.

$$\text{RMR}(\text{men}) = 13.8887\Omega + 14.3624\text{ht} - 1810.5594 \quad \text{Eq. X}$$

In the text of the manuscript RMR is made to four significant figures. All significant figures for A and B were taken to avoid rounding errors. Remember the power of Newton approximation: A and B can be calculated to arbitrary accuracy.

CALCULATIONS

Section 1 – using the calculus

The following is the derivation of the weight-loss formula:

w_i = weight in A.M. on day i

c_i = actual calories consumed on day i

C = calories needed on day i to maintain the body's weight and

$C = C_{70} (a + b w_i)$ is the estimated daily maintenance calories for

JJ which allows for the fact that JJ's RDA calorie intake on day i depends on his bodyweight in a linear way and ($a + b w_i$) is the linear term. The object is to determine both a and b and their significance. Assuming C is a linear function of the weight is the main aim of this research project. Many of the formulas in nutrition that estimate the RMR are linear like this one.

For 154 lb or 70 kg U.S. Reference Man, $C_{70} = 2700$ cal /day.

When $C = 2700$ cal/day = $C_{70} (a + b(154))$. The reason 154 lb is used for the calculation is because the raw data were in pounds. The theoretical result is then converted to kg and cm, or SI units. There is a slight discrepancy between 154 lb and 70 kg, although they are close. Proceeding with the calculation, that means when $C = C_{70}$, $a + 154 b = 1$, exactly. The idea behind C_{70} is to give a first guess at JJ's calorie needs at weight 154 lb to maintain his bodyweight. That is why ($a + b w_i$) is chosen to collapse to one in the following expression when JJ's weight is 154 lb, but Reference Man's calorie intake is used as a reference point.

$$C = C_{70} (a + b w_i) \quad (\text{i})$$

JJ, from whom the data came from, was quite active during 1985-1986 when the data was gathered, and he was an avid backpacker from 1968-1983. The a and b apply to JJ in 1985-1986, who was height 167.6 cm, age 34 (birth date 2/19/1951), and 154 lb on or about December 25, 1985. Once this is solved in lb and inches, a and b become A and B and the units then used are kilograms and centimeters.

Notice that c_i may be more or less than the maintenance calorie intake C. The difference produces either a weight gain or loss, unless $c_i = C_{70} (a + b w_i)$ when $dw/dt = 0$.

$$(w_{i+1} - w_i) D / \Delta t = c_i - C_{70} (a + b w_i) \quad (\text{ii})$$

This is a finite difference equation where D is a constant to be found in the literature.

$\Delta t = t_{i+1} - t_i =$ one day, and one pound: $w_{i+1} - w_i = 1$. In other words, when the weight gain is one pound and the calorie difference between day i and day $i + 1$ is $c_n - C$, then $c_n - C_{70} (a + b w_n) = D$ cal/day. D is the calories to gain or lose one pound body weight on some day n . The precise conversion factor between pounds and kilograms is $k = 2.204622476$ lb/kg. $D = (3500 \text{ cal/lb}) (k) \approx 7716.2 \text{ cal / kg}$, Wishnofsky [6], and is used in this calculation. Another figure for D is given as 7770 cal/kg in Kozusko [7]. For a period of fasting, we assume

$$c_i = (1 - f) C \quad (\text{iii})$$

where f is the fraction of the week fasted. When $f = 0$, body weight is unchanged, but when fasting the daily weight loss is given by Eq. (20). During the periods of the fasts, f was constant and these were strict water fasts, three days in succession for DATA SET 1 and one day for DATA SET 2. Notice, that when $f = 0$, $c_i = C$. On days when the fasting rate is zero, the weight is exactly maintained at its present level. In other words, C is the daily maintenance calories and c_i is actual number of calories consumed per day, so dropping the subscripts, for JJ's data, $C = C_{70} (a + b w)$ when $dw/dt = 0$.

Gathering terms, we have the master equation:

$$(w_{i+1} - w_i) / \Delta t = -f C_{70} (a + b w_i) / D \text{ and} \quad (1)$$

taking the limit as i approaches $i + 1$ gives a differential equation:

$$d w / d t = -f C_{70} (a + b w) / D \quad (2)$$

Rearranging,

$$d w / (a + b w) = -(f C_{70} / D) d t \quad (3)$$

Integrating, we have two equations in two unknowns.

$$(1/b) \ln (a + b w) = -(f C_{70} / D) t + \text{constant} \quad (4)$$

This equation is more easily solved by using the raw data in pounds for the calculations and then converting the answer to kilograms later because of rounding errors. Newton approximation is sensitive enough to give the constants a and b to four significant figures by using a few cycles. In fact, the way this derivation works, it is only necessary to use the points that give the closest fit to a parabolic regression. The two common points drop out at the point where the curves cross. It is necessary to stay far away from 154 lb, where the equation behaves badly. (Consult Wikipedia - Newton approximation needs a well-behaved function.) The point used on both curves were the points where the slope of the parabolic regression was closest to the linear regression slope, namely at (168, 162.7) and (28, 186.3), as it seems that at those points the least-squares fit is best and most represents real-life. This could also be solved by a computer using the derived equation with the exponential.

Integrating, we have two equations in two unknowns, from the text:

$$(1/b) \ln (a + b w) = -(f C_{70} / D) t + C \quad (4)$$

From the data, the first equation, we have a sample calculation, from DATA SETS 1 & 2 and remember $a = 1 - 154 b$. It is necessary to use the raw data in these equations because of rounding errors in the conversion from pounds to kilograms. Also, it is easier to do it this way and convert to kilograms and centimeters later.

$$\ln (1 + (186.3 - 154) b) - \ln (1 + (173.5 - 154) b) = - (3/7) b (2700) (28 - 56) / 3500 \quad (5)$$

$$\text{So, } \ln (1 + 32.3 b) - \ln (1 + 19.5 b) = 9.257142857 (b) \quad (5.1)$$

and the second equation:

$$\ln (1 + (162.7 - 154) b) - \ln (1 + (173.5 - 154) b) = - (1/7) b (2700) (168 - 56) / 3500 \quad (6)$$

$$\text{Similarly, } \ln(1 + 8.7b) - \ln(1 + 19.5b) = -12.34285714(b) \quad (6.1)$$

Subtracting equation (6.1) from (5.1) and rearranging:

$$\ln(1 + 32.3b) - \ln(1 + 8.7b) - 21.6b = 0 \quad (7)$$

So, it is only necessary to use four points (t, w) in evaluating b. The numbers must actually be taken to 10 significant figures because the differences are small. The reason the 162.7 point was chosen was because it was near the linear regression curve and far from 154 where the function behaves badly. The points taken should be where the slope of the parabolic regression is closest to that of the linear regression because that is where the least-squares fit is best. Statistics seem to be unnecessary. Obviously, if one has a mainframe computer, an exponential curve-fit could be done on Eq. (18), but that is actually unnecessary, as Newton approximation does the job. In using a calculator, it must not create rounding errors. So, a calculator with sufficient significant figures is good and one must maintain 10 significant figures. It is very easy to get an inexpensive calculator where the entire calculation is retained and significant figures are not lost, as Newton approximation is arbitrarily precise and keeping the significant figures will make the calculation converge with fewer cycles.

How do you get b? To use Newton approximation, set the function equal to zero.

$$\ln(1 + 32.3b) - \ln(1 + 8.7b) - 21.6b = 0 \quad (8)$$

Taking Newton approximation, where $x_1 = x_0 - [f(x_0)/f'(x_0)]$, and starting with an estimate of $x_0 = 0.004$ (1st and 2nd order Taylor series approximations gave bounds of 0 and 0.0467 for the starting value)

it gets about 0.00455979 for b, actually an irrational number, so,

$$\text{to ten significant figures: } b = 0.004559789284 \quad (9.1)$$

One runs it through five cycles, that is $x_{n+1} = x_n - [f(x_n)/f'(x_n)]$

until the desired number of significant figures are obtained. Since the data are only about three significant figures, 0.004560 is close. Using this method

$$f(x) = \ln(1 + 32.3x) - \ln(1 + 8.7x) - 21.6x$$

and $f'(x_0)$ = the first derivative of $f(x)$ evaluated at $x = x_0$.

$$\text{So, } a = 1 - 154b = 1 - (154)(0.004559789284) = 0.2977924503 \text{ (exact)} \quad (9.2)$$

Note: This calculation was first done with the weight in pounds because the raw data are in lb. There is a bit of imprecision in analyzing this problem because 154 lb is not exactly 70 kg, the weight of U.S. Reference Man. Also, another value in the literature for D is 7770 cal/kg and that is not exactly equal to 3500 cal/lb. The only thing that could be done is to use the raw data as it stands and then convert to kg and cm later: the value for $a \rightarrow A$ and $b \rightarrow B$ in $C = C_{70}(a + bw) \rightarrow C = C_{70}(A + B\Omega)$, where Ω is weight in kilograms. The units of b are lb^{-1} and the units of B are kg^{-1} . This is because A and a are dimensionless. w is in pounds and Ω is the same weight in kg: $bw = B\Omega$. This conversion factor is used: $k = 2.204622476 \text{ lb/kg}$. In other words, $w/\Omega = k$.

$$B = b(w/\Omega) = (0.004559789284)(k) = 0.01005261394 \quad (10.1)$$

Then, when $w = 154 \text{ lb}$, $\Omega = 69.8532296 \text{ kg}$ (to preserve the exactness of calculating b).

Furthermore, when $\Omega = 69.8532296$, then $(A + B\Omega) = 1$. To calculate A, we have

$$A = 1 - B\Omega = 1 - (0.01005261394)(69.8532296) = 0.2977924503 = a \quad (10.2)$$

0.01005261 is taken for B and 0.297792 is taken for A. The more approximate values are 0.010053 and 0.2978 respectively for B and A. The raw data (in pounds) is only accurate to three significant figures, so the more approximate values suffice. However, because of rounding errors, the rounding is only done at the very end. Thus, these calculations give an equation in kilograms from the original raw data.

Section 2 – discussion on the resting metabolism rate

The formula for RMR, as Cunningham gives it, is $\text{RMR} = 501.6 + 21.6 \text{ LBM cal/day}$, where LBM is a function of weight in kilograms and height in centimeters, but it needs to be put to kilograms of JJ's body weight to compare with this theory. Cunningham notes that RMR is solely predicted by LBM. Using Hume's formula (19.2) for JJ's LBM, RMR_{JJ} is given as a function of kg bodyweight (height = 167.6 cm) in Eq. (11).

$$\text{RMR}_{\text{JJ}} = 1092.0 + 7.087 \Omega_{\text{JJ}} \quad \text{where } \Omega \text{ is in kg} \quad (11)$$

Eq. (12) is Cunningham's result for JJ.

$$C_{JJ} = (1092 + 7.087 \Omega_{JJ}) F_{JJ} \quad \text{cal/day} \quad (12)$$

Eq. (13) is JJ's calorie intake expressed in kilograms for the theory in this study, based on the estimate that his calorie intake at 154 lb. is 2700 cal/day, according to (i).

$$C_{JJ} = (2700) (0.2978 + 0.01005 \Omega_{JJ}) \quad \text{cal/day} \quad (13)$$

C is the required maintenance calorie intake as a function of body weight for JJ. The activity factor ($1.2 < F_A < 1.9$) adjusts the calorie intake multiplying the RMR according to the average level of daily physical exertion, according to the formula $C = (\text{RMR})(F_A)$. In the formula derived in this paper, JJ's activity factor is implicitly included in Eq. (13) above. When $(\Omega_{i+1} - \Omega_i) D / \Delta t = 0$, the resulting equation from (ii) gives (i), the maintenance cal/day, as there is no weight change. The calorie intake is

$C = C_{70} (A + B \Omega_i)$, which is (i) above expressed in kilograms. Eq. (16) enables the daily calorie intake C to be generalized because there is equality between the lean body mass of JJ and another man. Eq. (16) is substituted into Eq. (15). $C = (\text{RMR})(F_A)$ and $C_{JJ} = (\text{RMR}_{JJ})(F_{JJ})$. So, $C / F_A = C_{JJ} / F_{JJ}$ when the two individuals are near in height and weight, that is, when (14) is true. This is because the lean body masses are equal. Since U.S. Ref. Man has a daily calorie intake estimated variably as 2700-2800 cal/day from various sources, this was taken to be 2750 cal/day, as mentioned in the Introduction. Using 2750 and Eq. (19.1), a figure can be calculated for USRM's F_A and it is 1.603. Eq. (17.2) is C / F_A simplified and predicts RMR for men near JJ's height, but (17.2) was modified into Eq. X in Supplementary File 2.

$$\text{RMR} = \text{RMR}_{JJ} \quad \text{makes Eq. (16) true} \quad (14)$$

$$C_{JJ} = (2700) (0.2978 + 0.010053 \Omega_{JJ}) \quad \text{cal/day} \quad (15)$$

$$\Omega_{JJ} = \Omega + 1.0341 (ht - 167.6) \quad (16)$$

$$C = (2700) (0.2978 + 0.010053 (\Omega + 1.0341 (ht - 167.6))) (F_A / 1.7232) \quad \text{cal/day} \quad (17.1)$$

$$\text{RMR} = (15.75) \Omega + (16.29) ht - 2263 \quad \text{cal/day} \quad (17.2)$$

Eq. (17.1) was a provisional equation for adult males by using Hume's equation, Eq. (19.2). Equation (17.2) was the original prediction for the basal metabolism rate for men over 16 years of age by this theory. But, Eq. (17.2) was tweaked in SUPPLEMENTARY FILE 2 by adding correction factors and forcing it to equal the Harris-Benedict values for USRM and Levanzin values to get Eq. X for men.

Section 3 – the rate of weight loss

The differential equation that applies to this diet process is equation (2.1):

$$d\Omega / dt = -f C_{70} (A + B \Omega) / (D k) \quad (2.1)$$

The function that satisfies (2.1) is:

$$\Omega = (A / B + \Omega(0)) \exp(- (f B C_{70} t) / (D k)) - A / B \quad (18)$$

Here are the values for the unknowns.

A and B are 0.297792 and 0.01005261 according to Eqs. (10.2) and (10.1).

f = constant fasting rate for each of two data sets, at the end of this article.

$f_3 = 3/7$ and $f_1 = 1/7$: fasting at three days/week and one day/week.

The calorie intake on the non-fasting days was unknown and was assumed to be the normal maintenance calorie intake.

$\Omega(0)$ = weight in kg at time $t = 0$ for each fast, 3 days/week and 1 day/week:

91.172 kg and 78.698 kg, respectively.

$(f B C_{70}) / (D k) \approx 1/1000$

$C_{70} = 2700 \text{ cal/day}$ and $D = (3500 \text{ cal/lb}) (k) \approx 7716.2 \text{ cal/kg}$

The assumption was that the human body would tend to eat the right number of calories on the other 4 and 6 days of each week. Normal eating may not mean that $dw/dt = 0$, but it was taken to be so here. In the theory f, B, F_A and Ω are taken to be independent variables. (A value from a plot for JJ was $F_A = 1.7232$, but that was based on an erroneous assumption and is irrelevant, as the number drops out of the calculation for RMR.) It must be noted that

F_A goes from around 1.3 to 1.9 for men. The Harris-Benedict formula for men is included here as (19.1). Hume’s expression for the lean body mass is Eq. (19.2), valid for men over 16 years of age.

$$\begin{aligned} \text{RMR}(\text{men}) &= \\ &= 66.5 + 13.75 \Omega(\text{kg}) + 5.003 \text{ ht}(\text{cm}) - 6.775 \text{ age}(\text{years}) \quad \text{cal/day} \quad (19.1) \end{aligned}$$

$$\text{LBM}(\text{men}) = (0.32810) \Omega(\text{kg}) + (0.33929) \text{ ht}(\text{cm}) - 29.5336 \quad \text{kg} \quad (19.2)$$

Eq. (20) gives the initial rate of weight loss for the fraction of week fasted for JJ. It is a good approximation for $t < 20$ days: $f B C_{70} t / (D k) \approx 0.02 \exp(x) = 1 + x$ for small x in (18). It is pretty good for $t < 50$ days, as $50/1000 = 0.05$. For the limiting slope at $t = 0$, the following equation gives the weight loss rate. The solution to Eq. (2.1) is, of course, the exponential given in (18), but the exponent is very small and the slight curvature is not the concern of this study.

$$d \Omega / d t = - f (C_{70} (A + B \Omega_o) / (D k)) \quad \text{kg/day} \quad (20)$$

DATA FROM DOCTOR’S FILES

Here are personal fasting data from JJ, age 34, height 5’6” from April 12, 1985 to January 24, 1986. Raw data are in pounds from a doctor’s scale, Michael Lesser, M.D. 181 Vicente Road, Berkeley, CA 94705. These data were collected mostly while John H. Jennings (JJ) was at a stay in Bonita House, Berkeley, CA (USA). The two sets of raw data were originally collected in pounds and days. These data are published by Jennings and Lesser [10]. Only the starred data points are used for evaluating b in Eq. (4).

DATA SET 1			DATA SET 2		
Day	Weight (lb)	(kg)	Day	Weight (lb)	(kg)
Three successive days/week water only fast			One day/week water only fast		
t	w	Ω	t	w	Ω
0	201	91.17	56 (0)	173.5 *	78.70
28	186.3 *	84.50	77 (21)	172	78.02
56	173.5 *	78.70	109 (53)	169.3	76.79
			116 (60)	167.2	75.84
			151 (95)	163	73.94
			168 (112)	162.7 *	73.80
			196 (140)	160.5	72.80
			208 (152)	160.7	72.89
			224 (168)	158.3	71.80
			236 (180)	158.3	71.80
			243 (187)	155.7	70.62
			251 (195)	154.7	70.17
			287 (231)	150	68.04

Linear regression results for each data set: w is in pounds & Ω is in kilograms.

DATA SET 1 $f = 3/7$
 $w = - 0.4910714 t + 200.6833$ $dw/dt = - 0.4910714 \text{ lb/day}$
 $r = - 0.9992$ $d\Omega/dt = - 0.2227463 \text{ kg/day}$

DATA SET 2 $f = 1/7$
 $w = - 0.09535427 t + 173.6842$ $dw/dt = - 0.09535427 \text{ lb/day}$
 $r = - 0.989$ $d\Omega/dt = - 0.04325197 \text{ kg/day}$

Note: the points chosen to evaluate the integral were where the derivative of the parabolic regression was closest to the slope in the linear regression. The least-squares fit is best at those points and the common point, (56, 173.5), gives rise to like terms and they cancel when the two equations are added together. The starting point for each fast is, of course, taken to be (0, 201) for the three-day fasting period, and (56, 173.5) for the one-day fasting period. To solve for the number b , use Newton approximation.