MATHEMATICAL DEVELOPMENT OF THE OBTAINING OF THE MINIMAL AREA, OF A DETECTABLE UNHOMOGENEITY FOR THE EQUIPMENT OF MEASUREMENT OF CAPACITANCES, FOR A TWO-PHASE FLOW TO ANNUL IN A DUCT

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ABSTRACT

A mathematical model for the Electrical Capacitance Tomography (ECT) is proposed, of a two-phase flow regimes, dielectric annular in pipelines. An expression is obtained for the mutual capacitances, by means of the solution of a direct problem, which is in terms of the radiuses of regions, the error that the equipment commits on having measured and the permittivity of every phase of the fluid. The expressions of the mutual capacitance are used for to minimize a functional, to find the minimal area of a detectable unhomogeneity for the equipment of measurement, in the center of the duct. This result is very important since, will be used to raise a method of adapted discretization, to solve an inverse problem of the ECT. In addition the formula of the mutual capacitances, it will serve to calibrate the equipments of measurement nowadays existing; and the solution of the direct problem, it will serve to validate any method for solution of the inverse problem.

Keywords: *Electrical Capacitance Tomography, direct problem, inverse problem.*

1. INTRODUCTION

Electrical Capacitance Tomography (ECT) has recently been tested to obtain image cross-sections of several industrial problems that involve dielectric materials. We are interested in particular, in visualizing the cross section of a multiphase flow in pipelines, to be able to determine the permittivity distribution and hence the phase distribution in real time and without any a-priori information about the type of flow regime.

The relation between the capacitance C, the permittivity distribution defined on the two-dimensional cross section of the pipe line, $\varepsilon(x,y)$ and the generated potential V, can be expressed as.

$$C(\varepsilon) = -\frac{1}{V}\int_S \varepsilon(z)\frac{\partial V}{\partial n}\,ds, \qquad (1)$$

where S is the electrode surface and $z=(x,y)$. We can consider this relation as a functional relation between the mutual capacitance measured in the electrode array and the permittivity distribution at the cross section.

Using this relation it is possible to develop alternative methods to solve the inverse problem of determining the permittivity $\varepsilon(x,y)$ using measurements of the capacitances.

This problem has an answer theoretical proposed by Nachman in [1], he has put forward a number of ideas to construct $\varepsilon(x,y)$ by the knowledge of the Dirichlet-Neumann operator.

Nevertheless, up to the date only there exist two theoretically based results that show how to recover the function $\varepsilon(x,y)$ from the knowledge of infinite number of Cauchy´s data on the border. In [2] it shows an a numerical implementation of the reconstruction algorithm proposed by A. Nachman, nevertheless, the practical implementation of this algorithm is slightly feasible, since to obtain a good approximation of $\varepsilon(x,y)$ it requires a lot Cauchy´s data that correspond to a big number of measurements that not always are possible to realize in the practice. A more practical slope to solve this complex problem of identification when they are imposed "conditions in advance" to the function $\varepsilon(x,y)$ that one wants to recover. In [3], [4] and [5] it presents a theorem of uniqueness for the recovery of $\varepsilon(x,y)$ from the knowledge of Cauchy's alone couple, under the supposition in advance that $\varepsilon(x,y)$ it can take only two different values.

Provided that this problem of identification is ill-posed, is very important in advance decides the class where the solution is going to be looked and which in this class demonstrates a theorem of existence and uniqueness for the recovery of $\varepsilon(x,y)$ when it splits of information of Cauchy without error.

Generally this obvious in a lot of works where it is tried to solve this problem of a "practical" way and is for it that in a lot of works in which they propose algorithms without a solid theoretical justification there are obtained "good results" by the identification in some problems constructed of synthetic form and in others not and these algorithms cannot be reliable to be applied in real problems where "in advance" there is not known how there are distributed the values of the function $\varepsilon(x,y)$ that is wanted to identify.

In this work let's sense beforehand the exposition and the way of obtaining this information in advance; determining the "minimal size" of a detectable unhomogeneity in the flow for the equipment of measurement of capacitances, bearing in mind the error that the equipment commits on having measured. This information in advance is important due to the fact that problem of the ECT is an inverse problem, and up to the moment criteria are not had of discretization for this problem.

2. PROBLEM FORMULATION

The design of the cross section of a pipeline for multiphase flow is formed by three concentric circles which determine three regions. Within the inner most circle Ω_* (with radius R_*) we have the multiphase flow with unknown permittivity $\varepsilon_*(x,y)$, whereas in the other annular regions Ω_2 and Ω_3 with exterior radius R_2 and R_3, we have materials with known constant permittivity $\varepsilon_2(x,y)$ y $\varepsilon_3(x,y)$.

The situation that is described corresponds to what is known as a tomography of capacitance in which, from measurements related to the potential of the electrical field generated in the region $\Omega = \Omega_* \cup \Omega_2 \cup \Omega_3$, it is a question of recovering an electrical unknown characteristic of the way (permittivity) $\varepsilon_*(x,y)$, with the aim to construct an tomography that allows to describe the material composition of the region Ω.

The interior cylinder is composed of an insulating material where it supposes that a two-phase flow moves, composed for dielectric. Between the cylinders interior and intermediate, oil settles to offset the pressure produced by the two-phase flow in the walls of the interior cylinder. On the intermediate cylinder of radio R_2, is a circular arrangement of N electrodes of equal angular length. A small separation exists between two consecutive electrodes in which one finds an insulating material to diminish the capacitive effects between them. Between the intermediate cylinders and exterior one finds a dielectric material, of which also his electrical permittivity is known.

By means of a device the exterior cylinder is fixed to land so that the electrical potential on the above mentioned cylinder can be considered to be equal to zero. This device is connected to the arrangement of N electrodes arranged on the intermediate cylinder so that it produces an equal potential to 1 in one of the electrodes called of reference and zero in the remaining ones. In order that the model is considered to be stationary, it is necessary that the injected current has constant frequency, to support the distribution of potential needed in the arrangement of electrodes.

A stationary model serves us if the variation of the permittivity $\varepsilon_*(x,y)$, in the interval of time in which an tomography is effected by the above mentioned model, is despicable.

Then, the model that we consider later, is good when the measurements are effected to currents by very low frequencies and when the algorithm of solution of the problem of recovery of from the information of mutual capacitance, it is the sufficiently rapid thing.

When the injected current has a sufficiently small frequency (500 Hz) as in order that it could be considered to be a zero. In this case, it is possible to see that the length of corresponding wave is of the order of 30 meters, which is a very much major dimension that that of the electrodes, and for it, can consider to be the potential of constant reference in the electrode of reference.

In this article it will suppose that $\Omega_* = \Omega_0 \cup \Omega_1$, where Ω_0 and Ω_1 they are concentric circles of radiuses R_0 and R_1 respectively. We will denote by means of $V^{(i)}(\theta)$ with $i = 1,2,...,N$, to the potential produced in Ω, when an potential equal to 1 is fixed in the electrode i called of reference, and the remaining ones connect to land, and where $\theta = \arg z$. In addition we suppose that between the neighboring electrodes a separation exists of θ_0, in such a way that for when $|z| = R_2$ it is had:

$$V^{(i)}(\theta) = \begin{cases} 0, & \text{if} \quad \theta \le \theta_i^{(-)} - \theta_0; \\ \dfrac{\theta - \theta_i^{(-)}}{\theta_0} + 1, & \text{if} \quad \theta_i^{(-)} - \theta_0 \le \theta \le \theta_i^{(-)}; \\ 1 & \text{if} \quad \theta_i^{(-)} \le \theta \le \theta_i^{+}; \\ \dfrac{\theta_i^{(+)} - \theta}{\theta_0} + 1, & \text{if} \quad \theta_i^{(+)} \le \theta \le \theta_i^{(+)} + \theta_0; \\ 0, & \text{if} \quad \theta \ge \theta_i^{(+)} + \theta_0; \end{cases} \tag{4}$$

where $\theta_i^{(-)} = \dfrac{2\pi(i-1)}{N} + \dfrac{\theta_0}{2}$ and $\theta_i^{(+)} = \dfrac{2\pi i}{N} - \dfrac{\theta_0}{2}$;

It denotes by means of $V_j^{(i)}(z) = V^{(i)}(z)$ if $z \in \Omega_j$, $j = 0,1,2,3.$

In this case for $V_0^{(i)}(z), V_1^{(i)}(z), V_2^{(i)}(z), V_3^{(i)}(z),$ there is had that

$$\Delta V_0^{(i)}(z) = 0, \quad \text{in} \quad |z| < R_0; \tag{5}$$

$$\Delta V_1^{(i)}(z) = 0, \quad \text{in} \quad R_0 < |z| < R_1; \tag{6}$$

$$\Delta V_2^{(i)}(z) = 0, \quad \text{in} \quad R_1 < |z| < R_2; \tag{7}$$

$$\Delta V_3^{(i)}(z) = 0, \quad \text{in} \quad R_2 < |z| < R_3; \tag{8}$$

In addition there are fulfilled the following conditions of contour, of continuity of the potentials and of the normal currents

$$V_0^{(i)}(z) = V_1^{(i)}(z), \quad \text{in} \quad |z| = R_0; \tag{9}$$

$$V_1^{(i)}(z) = V_2^{(i)}(z), \quad \text{in} \quad |z| = R_1; \tag{10}$$

$$\varepsilon_0(z)\frac{\partial V_0^{(i)}}{\partial \vec{n}_0}(z) = \varepsilon_1(z)\frac{\partial V_1^{(i)}}{\partial \vec{n}_0}(z), \quad \text{in} \quad |z| = R_0; \tag{11}$$

$$\varepsilon_1(z)\frac{\partial V_1^{(i)}}{\partial \vec{n}_1}(z) = \varepsilon_2(z)\frac{\partial V_2^{(i)}}{\partial \vec{n}_1}(z), \quad \text{in} \quad |z| = R_1; \tag{12}$$

$$V_2^{(i)}(z) = V_3^{(i)}(z) = V^{(i)}(z), \quad \text{in} \quad |z| = R_2; \tag{13}$$

$$V_3^{(i)}(z)=0 \text{ in } |z|=R_3; \tag{14}$$

where $\vec{n}_0$ and $\vec{n}_1$ they are unitary normal exterior vectors to the circles of radiuses R_0 and R_1 respectively.

It supposes that the electrode i occupies the arch of circumference:
$S_i=\left\{z\in\mathbb{R}^2 : |z|=R_2,\ \frac{2\pi(i-1)}{N}+\frac{\theta_0}{2}\le \arg z\le \frac{2\pi i}{N}-\frac{\theta_0}{2}\right\}$; the inverse problem of the tomography of capacitances for the raised model is: on the knowledge of all the $N(N-1)/2$ mutual capacitances between the electrodes S_i and S_j, to determine of brought near form the value of $\varepsilon_*(x,y)$ using the model (5) - (14).
On the other hand, an ideal discretization for the problem of the ECT must think that the equipments of measurement commit a error at the moment to measure, therefore the mesh must have a minimal area limit in every point. To solve this problem, we will use the model (5) - (14) to obtain the explicit form of the mutual capacitances. With this formula, the capacitances are compared for the homogeneous case and for the not homogeneous case, and there appears an equation that will help to determine the minimal radius that must have an incorporation in order that it is detected by the equipment of measurement. With this information they will be able to determine the properties of the subregion of the section of flow, where the permittivity $\varepsilon(x,y)$ can reach one of his two constant values.
The determination of these properties of the function of permittivity will have to depend on the type of considered flow, which will correspond to the case in which a vertical duct is had, with a two-phase concentric mixture; where the interior component of the mixture corresponds to filaments or bubbles that are formed during the extraction or some process of separation.

3. DIRECT PROBLEM

The direct problem (5) - (14), is to find at the function $V_j^{(i)}(z)=V^{(i)}(z)$ if $z\in\Omega_j$, $j=0,1,2,3;$ it knows the functions $\varepsilon_*(x,y)$ and $V^{(i)}(z)$ in $|z|=R_2$.

3.1 Calculation of $V_3^{(i)}(z)$

Let's notice that $V_3^{(i)}(z)$ It is possible to disconnect of the model (5) - (14), Since it satisfies Dirichlet's problem:

$$\Delta V_3^{(i)}(z)=0,\quad \text{in } R_2<|z|<R_3;$$
$$V_3^{(i)}(z)=\begin{cases}V^{(i)}(z), \text{ in } |z|=R_2;\\ 0, \text{ in } |z|=R_3.\end{cases}$$

It is looked to the function $V_3^{(i)}(z)$ in the shape of series of Fourier, for it there is obtained first the series of Fourier of the function $V^{(i)}(z)$; which has the form:

$$V^{(i)}(z)=\frac{1}{N}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi\theta_0}\sin\frac{n\theta_0}{2}\left(\sin\frac{2\pi ni}{N}-\sin\frac{2\pi n(i-1)}{N}\right)\cos n\theta+ \sum_{n=1}^{\infty}\frac{2}{n^2\pi\theta_0}\sin\frac{n\theta_0}{2}\left(\cos\frac{2\pi n(i-1)}{N}-\cos\frac{2\pi ni}{N}\right)\sin n\theta. \tag{15}$$

Provided that it is looked to a harmonic function in a ring, there is obtained that $V_3^{(i)}(z)$ it has the form:

$$V_3^{(i)}(z)=\frac{\ln R_3}{N\ln\frac{R_3}{R_2}}+\frac{1}{N\ln\frac{R_2}{R_3}}\ln r+$$
$$\sum_{n=1}^{\infty}\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\sin\frac{2\pi ni}{N}-\sin\frac{2\pi n(i-1)}{N}\right)\left(\frac{R_2^nR_3^{2n}r^{-n}}{R_3^{2n}-R_2^{2n}}-\frac{R_2^nr^n}{R_3^{2n}-R_2^{2n}}\right)\cos n\theta+ \tag{16}$$
$$\sum_{n=1}^{\infty}\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\cos\frac{2\pi n(i-1)}{N}-\cos\frac{2\pi ni}{N}\right)\left(\frac{R_2^nR_3^{2n}r^{-n}}{R_3^{2n}-R_2^{2n}}-\frac{R_2^nr^n}{R_3^{2n}-R_2^{2n}}\right)\sin n\theta.$$

3.2 Calculation of $V_2^{(i)}(z)$

Using the model simplified on having disconnected $V_3^{(i)}(z)$, they are looked to the potentials $V_0^{(i)}(z)$, $V_1^{(i)}(z)$, y $V_2^{(i)}(z)$ in the shape of series of Fourier. Here only it appears to $V_2^{(i)}(z)$, Due to the fact that it is used to obtain the formula for the mutual capacitances. There is obtained that:

$$V_2^{(i)}(r,\theta)=\frac{1}{N}+\sum_{n=1}^{\infty}\left[\left(r^{-n}A_{n,2}^{(i)}+r^na_{n,2}^{(i)}\right)\cos n\theta+\left(r^{-n}B_{n,2}^{(i)}+r^nb_{n,2}^{(i)}\right)\sin n\theta\right];$$

where the coefficients are given for:

$$A_{n,2}^{(i)}=\frac{\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\sin\frac{2\pi ni}{N}-\sin\frac{2\pi n(i-1)}{N}\right)\left[R_1^{-2n}+R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)(\varepsilon_1+\varepsilon_2)}\right]}{\frac{(\varepsilon_2-\varepsilon_1)}{(\varepsilon_2+\varepsilon_1)}\left[R_0^{-2n}R_2^{-n}\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}+R_1^{-4n}R_2^n\right]+R_1^{-2n}\left[R_0^{-2n}R_2^n\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}+R_2^{-n}\right]}; \tag{17}$$

$$B_{n,2}^{(i)}=\frac{\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\cos\frac{2\pi n(i-1)}{N}-\cos\frac{2\pi ni}{N}\right)\left[R_1^{-2n}+R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)(\varepsilon_2-\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)(\varepsilon_1+\varepsilon_2)}\right]}{\frac{(\varepsilon_2-\varepsilon_1)}{(\varepsilon_2+\varepsilon_1)}\left[R_0^{-2n}R_2^{-n}\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}+R_1^{-4n}R_2^n\right]+R_1^{-2n}\left[R_0^{-2n}R_2^n\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}+R_2^{-n}\right]}; \tag{18}$$

$$a_{n,2}^{(i)}=\frac{\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\sin\frac{2\pi ni}{N}-\sin\frac{2\pi n(i-1)}{N}\right)\left[R_1^{-2n}+R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)(\varepsilon_2+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)(\varepsilon_2-\varepsilon_1)}\right]}{R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}\left[R_2^n\frac{(\varepsilon_2+\varepsilon_1)}{(\varepsilon_2-\varepsilon_1)}+R_1^{2n}R_2^{-n}\right]+R_1^{-2n}R_2^n+R_2^{-n}}; \tag{19}$$

$$b_{n,2}^{(i)} = \frac{\frac{2\sin\frac{n\theta_0}{2}}{n^2\pi\theta_0}\left(\cos\frac{2\pi n(i-1)}{N} - \cos\frac{2\pi ni}{N}\right)\left[R_1^{-2n} + R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)(\varepsilon_2+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)(\varepsilon_2-\varepsilon_1)}\right]}{R_0^{-2n}\frac{(\varepsilon_0+\varepsilon_1)}{(\varepsilon_1-\varepsilon_0)}\left[R_2^n\frac{(\varepsilon_2+\varepsilon_1)}{(\varepsilon_2-\varepsilon_1)} + R_1^{2n}R_2^{-n}\right] + R_1^{-2n}R_2^n + R_2^{-n}}. \quad (20)$$

3.2 The formula of the mutual capacitances

Later one presents the formula of the mutual capacitances, in terms of the potentials $V_3^{(i)}(z)$ and $V_2^{(i)}(z)$ for $i = 1, 2, ..., N$.

There is known that

$$C_{ij} = K\int_{S_j}\varepsilon(x,y)\frac{\partial V^{(i)}}{\partial n_2}ds; \quad (21)$$

where n_2 it is the unitary exterior vector to the circumference $|z| = R_2$ and K is a physical constant with dimensions of the inverse one of the potential [8].

Provided that C_{ij} , it is possible to express like

$$C_{ij} = K\left[\varepsilon_3\int_{S_j^+}\frac{\partial V_3^{(i)}}{\partial n_2}ds - \varepsilon_2\int_{S_j^-}\frac{\partial V_2^{(i)}}{\partial n_2}ds\right], \quad (22)$$

where by means of S_j^+ and S_j^- it is denoted for the same arch as part of $|z| = R_2$.

On having replaced the values of $\int_{S_j}\frac{\partial V_3^{(i)}}{\partial R_2}ds$ and $\int_{S_j}\frac{\partial V_2^{(i)}}{\partial R_2}ds$ in the relation (22), The formula is obtained for the mutual capacitances:

$$C_{ij} = K\frac{\varepsilon_3}{N}\left(\frac{2\pi}{N} - \theta_0\right)\frac{1}{\ln\frac{R_2}{R_3}} - \frac{8K}{\pi\theta_0}\sum_{n=1}^{\infty}\frac{M}{n^2}\sin\frac{n\theta_0}{2}\sin\frac{n\pi}{N}\sin n\left(\frac{\pi}{N} - \frac{\theta_0}{2}\right)\cos\frac{2n\pi(i-j)}{N} \quad (23)$$

where

$$M = -\varepsilon_3 \frac{R_3^{2n} + R_2^{2n}}{R_3^{2n} - R_2^{2n}} - \varepsilon_2 \frac{R_1^{-2n} + R_0^{-2n} \frac{(\varepsilon_0 + \varepsilon_1)}{(\varepsilon_1 - \varepsilon_0)} \left(\frac{(\varepsilon_2 + \varepsilon_1)}{(\varepsilon_2 - \varepsilon_1)} - R_1^{2n} R_2^{-2n} \right) - R_2^{-2n} \frac{(\varepsilon_2 + \varepsilon_1)}{(\varepsilon_2 - \varepsilon_1)}}{R_1^{-2n} + R_0^{-2n} \frac{(\varepsilon_0 + \varepsilon_1)}{(\varepsilon_1 - \varepsilon_0)} \left(\frac{(\varepsilon_2 + \varepsilon_1)}{(\varepsilon_2 - \varepsilon_1)} + R_1^{2n} R_2^{-2n} \right) + R_2^{-2n} \frac{(\varepsilon_2 + \varepsilon_1)}{(\varepsilon_2 - \varepsilon_1)}} \tag{24}$$

The series given in (24) is truncated so that if δ it is the error that commits in the measurement of the mutual capacitances C_{ij}, at the time the rest of the above mentioned series a major mistake does not introduce that δ. It is denoted for

$$C_{ij}(\delta) = K \frac{\varepsilon_3}{N} \left(\frac{2\pi}{N} - \theta_0 \right) \frac{1}{\ln \frac{R_2}{R_3}} - \frac{8K}{\pi \theta_0} \sum_{n=1}^{k(\delta)} \frac{M}{n^2} \sin \frac{n\theta_0}{2} \sin \frac{n\pi}{N} \sin n \left(\frac{\pi}{N} - \frac{\theta_0}{2} \right) \cos \frac{2n\pi (i-j)}{N}; \tag{25}$$

and

$$R_{k(\delta)} = \frac{8K}{\pi \theta_0} \sum_{n=k(\delta)+1}^{\infty} \frac{M}{n^2} \sin \frac{n\theta_0}{2} \sin \frac{n\pi}{N} \sin n \left(\frac{\pi}{N} - \frac{\theta_0}{2} \right) \cos \frac{2n\pi (i-j)}{N}; \tag{26}$$

where $k(\delta)$ it is chosen so that $\left| R_{k(\delta)} \right| \le \delta$.

It is possible to see that $|M| \le \varepsilon_2 + \varepsilon_3$ and $\sum_{n=k(\delta)+1}^{\infty} \frac{1}{n^2} \le \frac{1}{k(\delta)}$, so

$$\left| R_{k(\delta)} \right| \le \frac{8K(\varepsilon_2 + \varepsilon_3)}{\pi \theta_0} \sum_{n=k(\delta)+1}^{\infty} \frac{1}{n^2}; \tag{27}$$

for what it is necessary to choose

$$k(\delta) > \frac{8K(\varepsilon_2 + \varepsilon_3)}{\delta \pi \theta_0}; \tag{28}$$

this way we come to the formula of the mutual capacitances:

$$C_{ij}\left(\delta\right)=K\frac{\varepsilon_3}{N}\left(\frac{2\pi}{N}-\theta_0\right)\frac{1}{\ln\frac{R_2}{R_3}}-$$
$$\frac{8K}{\pi\theta_0}\sum_{n=1}^{\frac{8K(\varepsilon_2+\varepsilon_3)}{\delta\pi\theta_0}+1}\frac{M}{n^2}\sin\frac{n\theta_0}{2}\sin\frac{n\pi}{N}\sin n\left(\frac{\pi}{N}-\frac{\theta_0}{2}\right)\cos\frac{2n\pi\left(i-j\right)}{N}. \tag{29}$$

4. UNHOMOGENEITY 'S MINIMAL SIZE DETECTABLE FOR THE EQUIPMENT OF MEASUREMENT

A fundamental step to solve numerically the inverse problem of the Tomography Capacitances, is to solve the inverse problem for a discretised version of the model of the direct problem. For it a fundamental step is the best way finds of to discretise the region where one finds the flow. This problem is framed inside the theory of the problems of adaptive discretization to solve the contour problems. In addition it is studied enough in case of the direct problems, but it it is not in case of the inverse problems.

For to discretise it must be considered the region that the tomography of capacitances uses equipments of measurement to calculate the so called mutual capacitances, and these, on having effected the measurements they commit a error the one that is denoted for δ , then, very small regions exist that the change in the measurements is not detectable for the above mentioned equipments. On the other hand, it is known that the optimal criterion to define adaptive mesh in the solution of direct problems is to maximize the speed of convergence of the numerical algorithm. Nevertheless in the solution of inverse problems there is no still a optimal criterion.

It is known that the existing sensors, they have difficulty in the detection of the components in the center of the sensor [6], to confront this difficulty, proposes to compare the capacitances for the homogeneous case and for the not homogeneous case, and there appears an equation that will help to determine the minimal radius that must have an incorporation in the center of the sensor in order that it is detected by the equipment of measurement. For it, concentric dielectric calculates the formula of the mutual capacitances for the case of the section of flow, of a two-phase fluid. With the obtained formula, there appears an equation that helps to determine the minimal radius that must have an incorporation in order that it is detected by the equipment of measurement.

One proposes the analysis considering two cases. The first case corresponds to when dielectric has a single-phase flow with constant permittivity and another case corresponds to when a two-phase flow is had. In addition, when a single-phase flow is had, the formula of the mutual capacitances is denoted for

$$C_{ij}^0\left(\delta\right)=K\frac{\varepsilon_3}{N}\left(\frac{2\pi}{N}-\theta_0\right)\frac{1}{\ln\frac{R_2}{R_3}}-$$
$$\frac{8K}{\pi\theta_0}\sum_{n=1}^{\frac{8K(\varepsilon_2+\varepsilon_3)}{\delta\pi\theta_0}+1}\frac{M^0}{n^2}\sin\frac{n\theta_0}{2}\sin\frac{n\pi}{N}\sin n\left(\frac{\pi}{N}-\frac{\theta_0}{2}\right)\cos\frac{2n\pi\left(i-j\right)}{N}; \tag{30}$$

where

$$M^0=-\varepsilon_3\frac{R_3^{2n}+R_2^{2n}}{R_3^{2n}-R_2^{2n}}-\varepsilon_2\frac{\frac{\left(\varepsilon_2+\varepsilon_1\right)}{\left(\varepsilon_2-\varepsilon_1\right)}-R_1^{2n}R_2^{-2n}}{\frac{\left(\varepsilon_2+\varepsilon_1\right)}{\left(\varepsilon_2-\varepsilon_1\right)}+R_1^{2n}R_2^{-2n}}. \tag{31}$$

It is denoted for $C_{ij}^{*}(\delta)$, to the formula of the mutual capacitances given in (29) and that corresponds to a two-phase dielectric flow. To obtain the minimal size of an unhomogeneity in the center of the flow, the following analysis is done:

If $d\left(C_{ij}^{0}(\delta),C_{ij}^{*}(\delta)\right)<\delta$, It means that the equipment of measurement does not detect the incorporation since it δ is the error that the equipment commits on having measured. If $d\left(C_{ij}^{0}(\delta),C_{ij}^{*}(\delta)\right)>\delta$, it means that the equipment of measurement detects the incorporation and therefore if $d\left(C_{ij}^{0}(\delta),C_{ij}^{*}(\delta)\right)=\delta$, it means that the equipment detects the incorporation and in addition it is possible to find to the minimal size of an unhomogeneity.
So to find to the minimal size of the unhomogeneity it is necessary to resolve $d\left(C_{ij}^{0}(\delta),C_{ij}^{*}(\delta)\right)=\delta$ for R_0. Without loss of generality we take the Euclidean distance to obtain:

$$\left|\sum_{n=1}^{\frac{8K(\varepsilon_2+\varepsilon_3)}{\delta\pi\theta_0}+1}\frac{M-M^0}{n^2}\sin\frac{n\theta_0}{2}\sin\frac{n\pi}{N}\sin n\left(\frac{\pi}{N}-\frac{\theta_0}{2}\right)\cos\frac{2n\pi(i-j)}{N}\right|=\delta\,\frac{\pi\theta_0}{8K} \tag{32}$$

Finally, to find the minimum R_0 of the obtained radiuses using the previous formula.

5. NUMERICAL EXPERIMENTS

We present numerical experiments, in which there is known in advance the value of the radiuses and the permittivity. The given values correspond to information near the royal information, and later they are listed.

$$N=12,\ \ \theta_0=0.0246045\,rad,\ \ \delta=0.025\times10^{-15}\ \mathrm{F}d/m,\ \ K=8.841941283\times10^{-12}\ \mathrm{F}d/m,$$
$$R_1=0.045m,\ \ R_2=0.0509m,\ \ R_3=0.074m,\ \ \varepsilon_2=2.82K.$$

With this information there is obtained that $k(\delta)=1$, it is solved for R_0 the following equation:

$$\frac{8K}{\pi\theta_0}\left|\left|M_1-M_1^0\right|P\cos\frac{\pi(i-j)}{6}+\left|M_2-M_2^0\right|Q\cos\frac{\pi(i-j)}{3}\right|=\delta$$

where $\left|M_k-M_k^0\right|\ ;k=1,2$ are the expressions of $\left|M-M^0\right|$ for $n=1,2$ of the formulae (24) y (31), respectively, and

$$P=\sin\frac{\theta_0}{2}\sin\frac{\pi}{12}\sin\left(\frac{\pi}{12}-\frac{\theta_0}{2}\right),\ \ Q=\frac{\sin\theta_0}{4}\sin\frac{\pi}{6}\sin\left(\frac{\pi}{6}-\theta_0\right).$$

Later one presents a table with the results for R_0, when different two-phase flows are had. For example, the first line corresponds to the case in which there is had a flow consisted of oil with a circular incorporation of gas, for which obtained that the minimal area of the circular detectable unhomogeneity as the equipment of measurement has radio $R_0=0.00724393\ m$

Table 1. Minimal area of a circular detectable unhomogeneity for the equipment of measurement for a two-phase dielectric fluid.

ε_0	ε_1	$R_0\ m$	*Minimal area* $\left(m^2\right)$
K	$3K$	0.00724393	0.0001648535723
$3K$	K	0.0613517	0.011825052
$80K$	K	0.0474221	0.007064988133
K	$80K$	0.000397871	0.0000004973183037
$80K$	$3K$	0.0527696	0.0087481753
$3K$	$80K$	0.00040795	0.0000005228339104

5. OVERALL CONCLUSIONS

Between the most important conclusions on this work of investigation they are the following ones:

1. The analytical expression of the mutual capacitances given in (29), are used to calibrate the equipments of measurement nowadays available.
2. The formula given in (32), can be used to obtain minimal areas of circular detectable incorporations for the equipments of measurement, for other values of permittivity.
3. The information presented in the table 1, will be used to raise a method of disctretization for the problem of the tomography of capacitances, which gives a criterion optimal for the above mentioned inverse problem.
4. The solution of the direct problem, which he concludes with the expression of the mutual capacitances serves to validate any method of solution to the inverse problem of the ECT, with the conditions raised in this article.

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