# A BEAM ANALOGY PROCEDURE FOR STRENGTH OF INTERIOR SLAB-COLUMN CONNECTIONS OF UNBONDED POST-TENSIONED FLAT PLATES - PART A: DEVELOPMENT OF THE METHOD

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**ABSTRACT**

The development of a beam analogy procedure to assess the unbalanced bending moment strength of interior slab-column connections of unbonded post-tensioned flat plates without shear reinforcement subjected to pure gravity or combined shear and transfer of moment loading is outlined in the present study. The slab sections framing into each column face are idealized as beam sections capable of developing the ultimate shear, bending moment or torsional capacity appropriate for the given loading conditions. In contrast to other investigators, the enhancing effect due to compressive membrane action is taken into account in a more logical fashion by the introduction of suitable modification factors. The method however ignores any influence of cracking in the post-service range on the interaction between torsion and shear.

**Keywords:** *Slab, column, beam, shear, bending, torsion, punching, interaction.*

## 1. INTRODUCTION

Post-tensioned flat slabs have become very popular over the past several decades in the construction of apartment buildings, parking garages and warehouse complexes. Their main advantages are that at service loads such systems display relatively little cracking and minimal deflections compared with their reinforced concrete counterparts. For the post-tensioned flat plate variety that omits both drop panels and column capitals, relatively greater economy is possible because of the extremely simple formwork. However such structures are susceptible to punching failures which may occur suddenly and hence constitute a major disadvantage of these structures [1].

Punching failures can be triggered at an interior column of a flat plate structure by the application of a pure gravity loading over the complete floor area. The phenomenon is more critical however when the slab-column connection is subjected to a transfer of moment coupled with a shear loading. Such unbalanced moment transfer causes the shear stress distribution in the slab around the column to become non-uniform and effectively reduces the shear strength of the connection. This situation is prevalent in buildings subjected to horizontal wind loadings or earthquake effects [2].

Numerous tests have been carried out over the past several decades by investigators to shed light on the nature of punching failures in flat slabs. However in comparison with reinforced concrete flat slabs, studies on post-tensioned flat plates have been somewhat limited. Such investigations on prestressed flat plates [3–18] have dealt with many issues and aspects related to the punching problem. However several of the afore-mentioned test models, with the possible exception of those of [10], had span to depth ratios which were unrepresentative of the ones in prototype structures and did not adequately simulate the boundary conditions existing in such structures. Furthermore a number of such investigations were directed towards the post-punching behaviour of prestressed flat slabs. All of these shortcomings and anomalies have contributed to some extent to the slow development of general methods of dealing with the punching problem in post-tensioned flat plates.

As noted earlier, it is generally accepted that the occurrence of moment transfer loading imposes a more severe stress condition on the slab-column junction. There are a number of approaches in the literature dealing with the analysis or design of slab-column connections transferring unbalanced bending moment and shear. These procedures with the exception of the truss analogy method have earlier been largely reviewed by the Joint ASCE-ACI Committee 426 [19]. One method of analysis, as exemplified by the ACI 318-08 Code [20], is based on a linear variation of shear stress. The shear stresses on a critical perimeter close to the slab-column junction are assumed to vary linearly with distance from the centroidal axis of the perimeter. These shear stresses are induced by the shear force and a portion of the unbalanced bending moment; the remaining unbalanced bending moment is taken up by flexure in the slab. Another method of analysis based on elastic thin plate theory and incorporating finite elements has been employed by [21–23]. The method takes into account yielding and the substantial moment redistribution which may occur subsequently at the slab-column junction.

Yet another different approach is that of beam analogy where the slab adjacent to the column is modelled as acting as beams running in two directions at right angles framing into the column faces. Each beam is assumed to be capable of developing its ultimate bending moment, torsional moment and shear force at the critical sections close to the column faces, and allowance is also made for interaction effects. However the beam analogy procedure of Hawkins [24, 25] is relatively difficult to apply due to the large number of limiting combinations of bending, torsion and shear which must be considered. Park and Islam [26] developed a more simplified beam analogy method in which allowance was made for two-way slab action and the effect of compressive membrane forces by doubling the concrete shear and torsional strengths normally allowed for beams. The method however was primarily developed for reinforced concrete slabs. A more recent approach based on truss analogy is that due to Simmonds and Alexander [27, 28]. The model however, in respect of shear and unbalanced moment transfer, is principally applicable to edge slab-column connections.

The focus of the present study is the development of a beam analogy procedure to assess the unbalanced bending moment strength of unbonded post-tensioned flat plates at internal columns. As a starting point, the case of such connections without shear reinforcement is investigated. In order to arrive at a relatively simplified approach, the beam analogy procedure of [26] has been modified to deal with post-tensioned flat plates and to fully account for the enhancing effects of the well known compressive membrane action in two-way slabs.

## 2. METHODOLOGY

### 2.1 Strength of Interior Connections for Pure Vertical Loading

Figure 1(a) shows an interior slab-column connection transferring only shear. If $V_o$ is the ultimate shear to be transferred, then at the critical section surrounding the column the external shear is resisted by internal action within the slab as seen in Figure 1(b). Sections AB, BC, CD and DE are assumed to be the critical sections of the intersecting beams, and the equilibrium equation is

$$V_o = V_{AB} + V_{BC} + V_{CD} + V_{DA} \tag{1}$$

where $V_{AB}$, $V_{BC}$, $V_{CD}$ and $V_{DA}$ are the shear forces on the respective faces AB, BC, CD and DA. Following [26], the contributions of $V_{AB}$, $V_{BC}$, $V_{CD}$ and $V_{DA}$ are assumed to be in proportion to the tributary areas of the surrounding slab. In perfectly symmetrical cases it will be assumed that they make equal contributions, i.e. equal fractions of $V_o$ are transferred to the four faces. Based on the available test data, the maximum vertical shear stress on each face will be limited to

$$v = \left(2.682 + \frac{0.3 f_{pc}}{\sqrt{f_{cu}}}\right) \text{N/mm}^2 \tag{2}$$

where $f_{pc}$ is the average effective concrete prestress immediately after post-tensioning, in N/mm$^2$, and $f_{cu}$ is the cube crushing strength of concrete, in N/mm$^2$. If the critical section for shear in Figure 1(b) is assumed as customary to be at a distance 0.5d from the periphery of the column, where d is the effective depth of tension reinforcement, then the total punching shear to be transferred from Equation (1) is given by

$$V_o = \left(2.682 + \frac{0.3 f_{pc}}{\sqrt{f_{cu}}}\right) d\left[2(c_1 + d) + 2(c_2 + d)\right] \tag{3}$$

### 2.2 Strength of Interior Connections Transferring Shear and Unbalanced Bending Moments

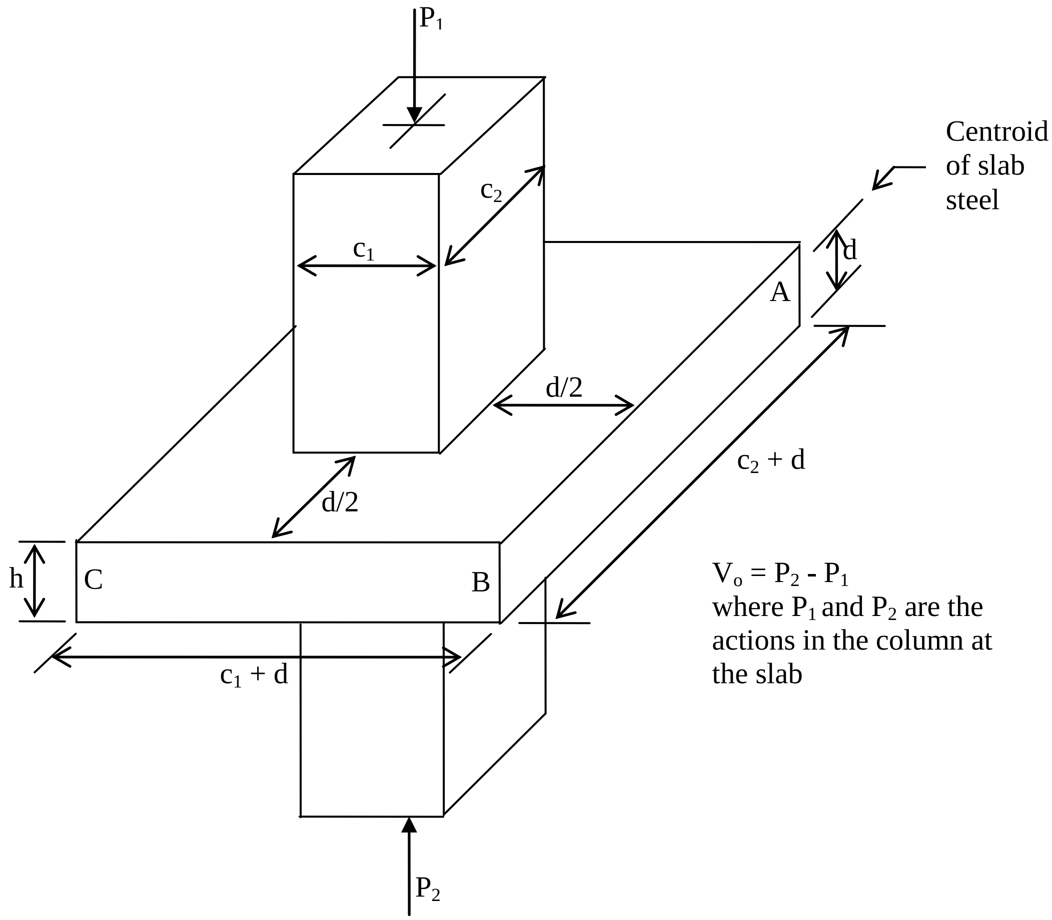
#### 2.2.1 Preliminaries

A typical connection transferring shear and unbalanced bending moment is shown in Figure 2(a).The ultimate internal shear and ultimate unbalanced bending moment to be transferred are denoted by $V_u$ and $M_u$ respectively. Assuming the critical section is taken as being at a distance 0.5d from the column periphery as before, then $V_u$ and $M_u$ are resisted by flexure, torsion and shear within the slab at the critical section as shown in Figure 2(b). The equilibrium equations hence become

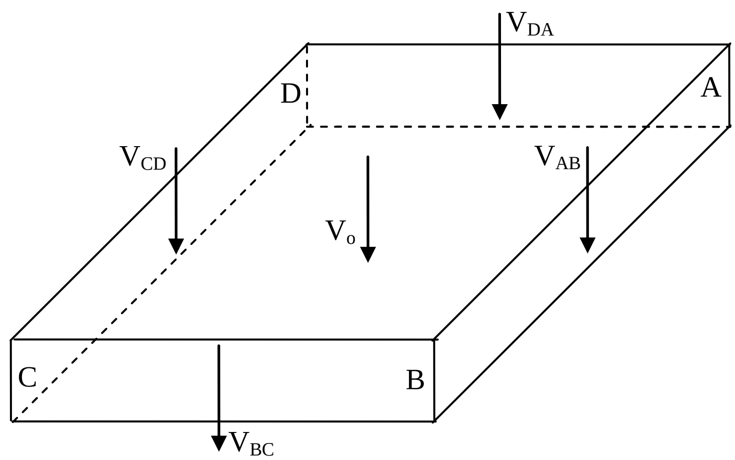
$$V_u = V_{AB} + V_{BC} + V_{CD} + V_{DA} \tag{4}$$

$$M_u = M_{AB} + M_{CD} + T_{BC} + T_{DA} + (V_{AB} - V_{CD})\left(\frac{c_1 + d}{2}\right) \tag{5}$$

where $V_{AB}$, $V_{BC}$, $V_{CD}$ and $V_{DA}$ are the shear forces acting on faces AB, BC, CD and DA respectively, $M_{AB}$ and $M_{CD}$ are the bending moments acting on faces AB and CD respectively, and $T_{BC}$ and $T_{DA}$ are the torsional moments acting on faces BC and DA respectively.

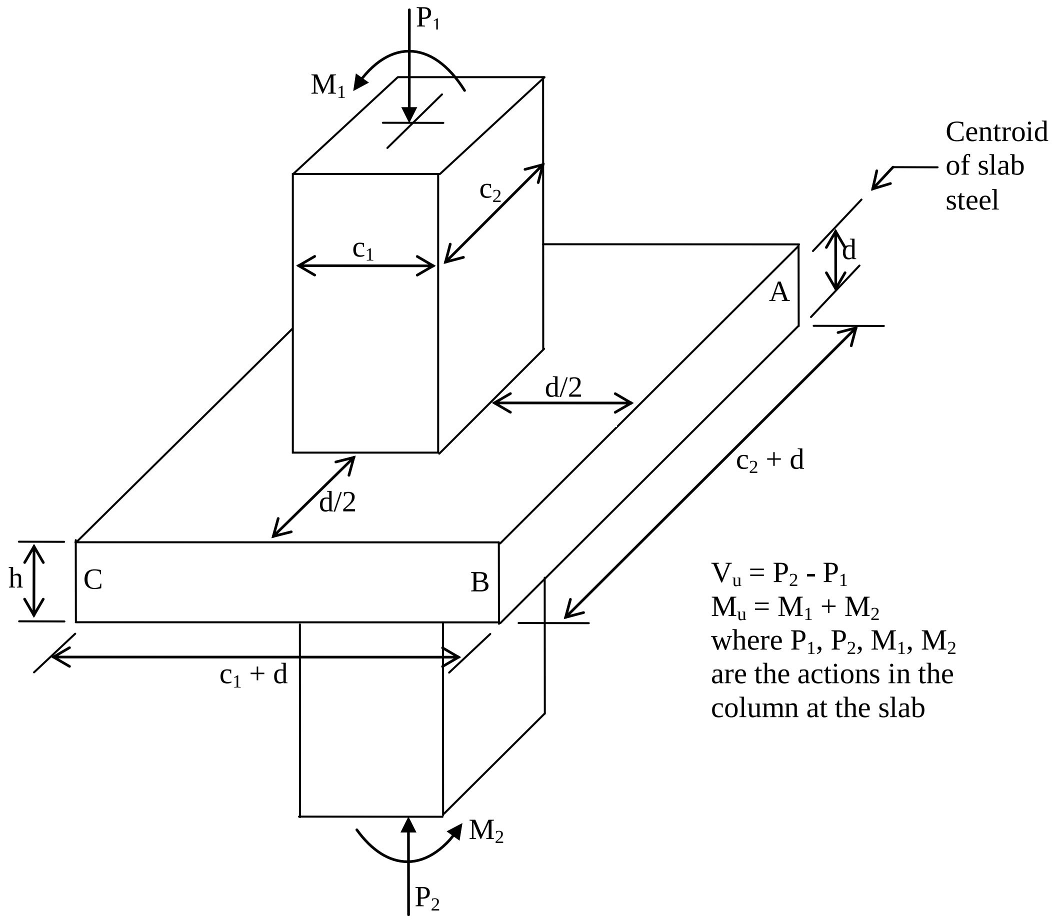



(a) External loads and the critical section at distance d/2 from column face

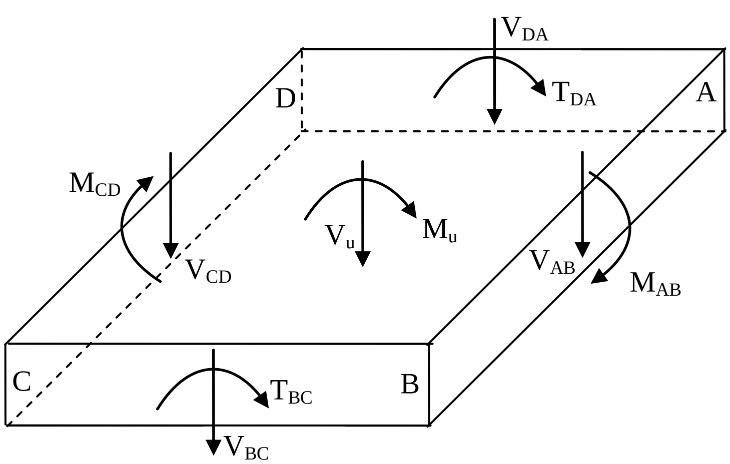



(a) Internal actions at the critical section

Figure 1. Slab-column connection with external loads and internal actions at critical section for vertical loading (after Park and Islam, [26] )




(a) External loads and the critical section at distance d/2 from column face




(a) Internal actions at the critical section

Figure 2. Slab-column connection with external loads and internal actions at critical section for combined loading (after Park and Islam, [26] )

In the present study the basic approach and assumptions made from this stage onwards are quite similar to that of Park and Islam [26] and Park and Gamble [2], and hence will not be repeated here. However the treatment of compressive membrane or in-plane forces which when present may considerably enhance the strength of the connection is markedly different. The approach adopted here is that proposed by Franklin et al [21] in which the calculated strength is multiplied by a factor $(1+H/2T_o)$, where H is the compressive membrane force per unit width and $T_o$ is the normal force in the reinforcement. The compressive membrane force H is given by

$$H = \frac{R^2 - (0.125L)^2}{R^2 + (0.125L)^2}\left(f_{sp} + f_{pc}\right)h \tag{6}$$

where $f_{sp}$ is the split cylinder tensile strength of the concrete, h is the overall slab depth and L is the slab span. For test slabs extending to the line of contraflexure (or simply supported slabs), R = 0.2L while for experimental slabs extending to mid-span between column centrelines, R = 0.5L. The normal force in the reinforcement $T_o$ is taken as

$$T_o = \rho_e f_y d \tag{7}$$

where $\rho_e$ is the equivalent reinforcement ratio and $f_y$ is the yield stress of ordinary bonded reinforcement in the post-tensioned slab.

Following Park and Islam [26], face AB in Figure 2(a) is the critical face for vertical shear since on this face the vertical shears due to the portion of $M_u$ transferred by shear stress and the fraction of $V_u$ transferred by the face are additive while in contrast, on face CD, these two shears act in opposite directions. Hence the limiting shear force on face AB taking into account the membrane effect will be

$$V_{AB} = \left(1 + \frac{H}{2T_o}\right)\left(2.682 + \frac{0.3f_{pc}}{\sqrt{f_{cu}}}\right)(c_2 + d)d \tag{8}$$

It will be assumed here that there will be no reduction in the ultimate shear capacity due to the development of the ultimate bending moment at the critical section.

Similar to the case of pure gravity loading highlighted earlier, the contributions of $V_{AB}$, $V_{BC}$, $V_{CD}$ and $V_{DA}$ in Equation (4) are assumed to be directly related to the tributary areas of the surrounding slab. However the values of $V_{AB}$ and $V_{CD}$ will also receive contributions from $M_u$ due to the shear induced by moment transfer. If the fraction of $V_u$ transferred by face AB is $K_{AB}V_u$, then the shear force induced on face AB by the moment transfer will be $(V_{AB} - K_{AB}V_u)$. From symmetry this is also the shear force induced on face CD by moment transfer, however these two moment induced shears act in opposite directions. Assuming the proportion of $V_u$ transferred by the face CD is $K_{CD}V_u$, the shear force $V_{CD}$ is given by

$$V_{CD} = K_{CD}V_u - \left(1 + \frac{H}{2T_o}\right)\left(2.682 + \frac{0.3f_{pc}}{\sqrt{f_{cu}}}\right)(c_2 + d)d - K_{AB}V_u \tag{9}$$

Under symmetrical conditions where equal fractions of $V_u$ are transferred to the four faces, $K_{AB} = K_{CD} = 0.25$.

In the case of combined gravity and transfer of moment loading, Franklin et al [21] relate the punching shear capacity for combined loading $V_u$ to the pure shear loading capacity $V_o$ by the expression

$$V_u = \left[\left(1 + \frac{H}{2T_o}\right)V_o\right] \Big/ \left(1 + K\frac{e}{L}\right) \tag{10}$$

where $K = k_1/k_2$. Here $k_1$ is a factor relating the vertical load to the average internal moment at the connection, $k_2$ relates the applied moment $V_u e$ to the corresponding internal slab moment and e is the eccentricity of column load. Franklin et al proposed a K value given by

$$K = 10 + 12.5\frac{e}{L} \tag{11}$$

The ultimate torsional capacity will be dealt with in a similar manner to the approach of Park and Islam [26], who adopted a circular interaction relationship between the torsional and vertical shear stress. However the ultimate torsional shear stress for the case of torsion in the absence of vertical shear will be assumed to be $(1+H/2T_o)$ times the value recommended by Hsu [29] for prestressed concrete beams without web reinforcement, to account for the enhancement due to the influence of compressive membrane forces. All that now remains is to proceed to develop the equation for the unbalanced moment strength of the slab-column connection.

### 2.2.2 Development of the basic equation

With reference to Figure 2 and Equation (5), the bending moments acting on faces AB and CD may be expressed as

$$M_{AB} + M_{CD} = \left(m_u + m_u'\right)\left(c_2 + d\right) \tag{12}$$

where $m_u$ and $m_u'$ are the ultimate positive and negative resisting moments per unit width at faces CD and AB respectively. In Equation (12) $m_u$ and $m_u'$ may be found from the expression of Franklin et al [21], that is

$$m_u = \left(1 + \frac{H}{2T_o}\right)\rho_e f_y d^2\left(1 - 0.60\rho_e \frac{f_y}{f_{cu}}\right) \tag{13a}$$

$$m_u' = \left(1 + \frac{H}{2T_o}\right)\rho_e' f_y d^2\left(1 - 0.60\rho_e' \frac{f_y}{f_{cu}}\right) \tag{13b}$$

where $\rho_e$ and $\rho_e'$ are the equivalent reinforcement ratios given respectively by

$$\rho_e = \rho_s + \rho_{ps}\left(\frac{f_{pb}}{f_y}\right)\left(\frac{d_{ps}}{d}\right) \tag{14a}$$

$$\rho_e' = \rho_s' + \rho_{ps}'\left(\frac{f_{pb}}{f_y}\right)\left(\frac{d_{ps}}{d}\right) \tag{14b}$$

in which $\rho s$ and $\rho_s'$ are the ratios of ordinary bonded bottom and top reinforcement respectively, $\rho_{ps}$ and $\rho_{ps}'$ are the ratios of prestressed bottom and top reinforcement respectively, $f_{pb}$ is the tensile stress in the prestressed tendon at slab failure, $d_{ps}$ is the effective depth to the actual tendon profile, d is the effective depth of tension reinforcement and $f_y$ is the yield stress of the ordinary bonded reinforcement.

In respect of the vertical shear forces acting on faces AB and CD of Figure 2, the contribution to the unbalanced moment transferred in Equation (5) as given by Equations (8) and (9) becomes after re-arranging

$$\left(V_{AB} - V_{CD}\right)\left(\frac{c_1 + d}{2}\right) = \left\{\left(1 + \frac{H}{2T_o}\right)\left(2.682 + \frac{0.3f_{pc}}{\sqrt{f_{cu}}}\right)\left(c_2 + d\right)d - 0.5V_u\left(K_{AB} + K_{CD}\right)\right\}\left(c_1 + d\right) \tag{15}$$

The torsional moments acting on faces BC and DA of Figure 2 required for Equation (5) may be written using the expression of Hsu [29] for the ultimate torque of a prestressed section. The calculations ignore bending–torsion interaction; however torsion–shear interaction is taken into account. The torsional stress of a rectangular section can be expressed as

$$\tau_u = \frac{T_o'}{\alpha x^2 y} \tag{16}$$

where x is the shorter side of the rectangular section, y is the longer side of the rectangular section, α is the torsion coefficient and $T_o'$ is the torsional moment. Assuming as stated earlier that the ultimate torsional shear stress is taken as $(1+H/2T_o)$ times the value recommended by Hsu [29] we have

$$\tau_u = \left(1+\frac{H}{2T_o}\right)0.5\sqrt{f_{cu}}\left(\sqrt{1+\frac{10f_{pc}}{f_{cu}}}\right) \tag{17}$$

where $f_{pc}$ is the average concrete prestress. Combining Equations (16) and (17) will yield

$$T_o{}' = \alpha x^2 y\left(1+\frac{H}{2T_o}\right)0.5\sqrt{f_{cu}}\left(\sqrt{1+\frac{10f_{pc}}{f_{cu}}}\right) \tag{18}$$

We elect in Equation (18) to adopt the value proposed by Zia and McGee [30] for the coefficient α, which is

$$\alpha = \frac{0.35}{\left(0.75+\frac{x}{y}\right)} \tag{19}$$

Equation (18) does not make any allowance for shear–torsion interaction. It is reasonable to assume that the simultaneous application of shear forces and torque will yield an interaction that will reduce the strength of the member compared with the cases when shear or torsion act alone. The interaction between torsion and shear for prestressed beams without web reinforcement may be reasonably represented by a circular curve [31]. Consequently we shall use the well established relationship

$$\left(\frac{V_u}{V_o}\right)^2 + \left(\frac{T_u}{T_o{}'}\right)^2 = 1 \tag{20}$$

where $V_u$ is the shear force at failure under combined loading, $T_u$ is the torsional moment at failure under combined loading, $V_o$ is the shear force of the member when subjected to flexural shear given in Equation (3) multiplied by the compressive membrane enhancement factor and $T_o{}'$ is the torsional moment of the member when subjected to torsion alone as given in Equation (18). Hence to allow for shear–torsion interaction, the maximum torsional strength of both faces BC and DA together in Figure 2 may be written as

$$\left(T_{BC}+T_{DA}\right) = 2\left(1+\frac{H}{2T_o}\right)\left(\alpha x^2 y\right)\left(0.5\sqrt{f_{cu}}\right)\left(\sqrt{1+\frac{10f_{pc}}{f_{cu}}}\right)\left(\sqrt{1-\left(\frac{V_u}{V_o}\right)^2}\right) \tag{21}$$

The unbalanced bending moment strength of the connection can now be obtained by summing up the contributions from the bending moment, vertical shear and torsional moment terms taking into account shear–torsion interaction. Substituting Equations (12), (15) and (21) into Equation (5) gives the unbalanced bending moment strength of the slab-column connection as follows:

$$\begin{aligned} M_u = {} & \left(m_u + m_u{}'\right)\left(c_2+d\right) + \left[\left(1+\frac{H}{2T_o}\right)\left(2.682+\frac{0.3f_{pc}}{\sqrt{f_{cu}}}\right)\left(c_2+d\right)d - 0.5V_u\left(K_{AB}+K_{CD}\right)\right]\left(c_1+d\right) \\ & + 2\left(1+\frac{H}{2T_o}\right)\left(\alpha x^2 y\right)\left(0.5\sqrt{f_{cu}}\right)\left(\sqrt{1+\frac{10f_{pc}}{f_{cu}}}\right)\left(\sqrt{1-\left(\frac{V_u}{V_o}\right)^2}\right) \end{aligned} \tag{22}$$

In Equation (22) $m_u$ and $m_u{}'$ are given by Equations 13(a, b) while $V_o$ is given by Equation (3) multiplied by the compressive membrane enhancement factor; $V_u$ is the ultimate shear force to be transferred as given by Equation (10).

## 3. RESULTS AND DISCUSSION

It is necessary to stress that for Equation (22), $V_u$ the ultimate shear force under combined shear and transfer of moment loading should not exceed $V_o$ the ultimate shear force in the absence of unbalanced moment. However when $V_u$ equals $V_o$, Equations (15) and (21) would imply that no unbalanced bending moment can be transferred by vertical shear or torsion, although an unbalanced moment may be resisted by the bending moment contributions of Equation (12) in Equation (22). This conclusion has also been noted by Park and Islam [26] in their investigations on reinforced concrete slab-column connections.

Although the approach in the present study is similar to that of Park and Islam, there are obvious differences. Firstly, the latter is only strictly applicable to reinforced concrete slabs while the beam analogy that has been developed is intended for unbonded post-tensioned slab-column connections. Secondly, the treatment of compressive membrane action as presented here appears to be more logical than that of Park and Islam who have neglected any influence of compressive membrane effects on the flexural strength; the enhancements for shear and torsion have been made by simply limiting the maximum vertical shear stress and the ultimate torsional shear stress values on the critical faces to twice the estimates normally recommended for beams. In contrast the enhancement factor $(1+H/2T_o)$ in the current study has been applied to the flexural strength, vertical shear and torsional moment contributions. It should be noted in passing that from the work of Franklin et al [21], the compressive membrane enhancement factor depends very largely on the extent of the slab boundaries beyond the line of contraflexure (approximately a distance 0.2L from the column centreline on either side of the column). This observation has also been confirmed by several other investigators, notably Cleland [32] and Rankin and Long [23]. There is very strong evidence from the latter study that the compressive membrane enhancement should be applied to the flexural strength.

One issue relating to the present study that should be mentioned at this stage is the influence of cracking on the torsion–shear interaction. The preceding work and discussion has not dwelt on this. However one approach would be to take the reduction in concrete torque upon cracking for a prestressed beam to be equal to that reduction upon cracking for an equivalent non-prestressed member, in the manner proposed by Zia and McGee [30]. This would almost certainly modify the torsional strength contribution in Equation (22) and lead to an even more complex expression. Hence for this latter reason, the problem has not been pursued any further here.

In summary the beam analogy that has been developed is along the lines of the earlier procedure of Park and Islam [26]. Consequently in contrast to that of Hawkins [24, 25] it is simpler to apply as the need to consider several possible limiting combinations of bending, shear and torsion is discounted. The proposed beam analogy assumes like that of Park and Islam that sufficient redistribution of internal actions can occur to allow the capacities in flexure, shear and torsion to be attained at the critical sections. The approach however is better suited to prestressed slabs than Park and Islam's method and moreover, its treatment of compressive membrane enhancing effects is markedly different and appears more rational.

## 4. CONCLUSIONS

A beam analogy procedure has been presented in this study to assess the unbalanced bending moment strength of interior slab-column connections of unbonded post-tensioned flat plates. The slab sections that frame into the column faces have been modelled as beam sections that are capable of attaining the ultimate shear, bending moment and torsional capacities relevant to the prescribed loading conditions. Although the procedure is similar to that of Park and Islam [26] it takes account of the well established compressive membrane action in a different and possibly more logical fashion. Unlike the beam analogy of Hawkins [24, 25] which utilizes a large number of equations on account of the various limiting cases, the procedure reported here is based on a single equation that includes all of the bending moment, shear and torsional strength contributions. The method however has not incorporated any effect of cracking in the post-service range on the shear–torsion interaction.

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