THE SINE-COSINE FUNCTION METHOD FOR THE EXACT SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT
In this paper, we established a traveling wave solution by using Sine-Cosine function algorithm for nonlinear partial differential equations. The method is used to obtain the exact solutions for different types of nonlinear partial differential equations such as, the (2+1) - dimensional nonlinear Schrödinger equation, The Schrödinger-Hirota equation, Gardner equation, modified KdV equation, perturbed Burgers equation, and general Burger's-Fisher equation, which are the important Soliton equations.

Keywords: *Nonlinear PDEs, Exact Solutions, Nonlinear Waves, Schrödinger equation, Gardner equation, Sine-Cosine function method, modified KdV equation, perturbed Burgers equation, general Burger's-Fisher equation*.

1. INTRODUCTION

Nonlinear evolution equations have a major role in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. A variety of powerful methods, such as, tanh sech method {Malfliet [1], Khater et al. [2], and Wazwaz [3]}, extended tanh method {El-Wakil et al. [4], Fan [5], Wazwaz [6]}, hyperbolic function method {Xia and Zhang[7], and Yusufoglu and Bekir [8]}, Jacobi elliptic function expansion method {Inc and Ergut [9]}, F-expansion method {Zhang [10]}, and the First Integral method {Feng [11], Ding and Li [12]} .The sine-cosine method {Mitchell [13], Parkes [14], and Khater [2]} has been used to solve different types of nonlinear systems of PDEs.
The aim of this paper is to find new exact solutions by the sine-cosine method of the (2+1) - dimensional nonlinear Schrödinger equation, The Schrödinger-Hirota equation, Gardner equation , modified KdV equation, perturbed Burgers equation, and general Burger's-Fisher equation, which are the important Soliton equations.

2. THE SINE-COSINE FUNCTION METHOD

Consider the nonlinear partial differential equation in the form

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{yy}, \dots\dots\dots) = 0 \qquad (1)$$

where u(x, y, t) is a traveling wave solution of nonlinear partial differential equation Eq. (1). We use the transformations,

$$u(x, y, t) = f(\xi) \qquad (2)$$

where $\xi = x + y - \lambda t$ This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -\lambda \frac{d}{d\xi}(.) \ , \ \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.) \, , \frac{\partial}{\partial y}(.) = \frac{d}{d\xi}(.) \qquad (3)$$

Using Eq. (3) to transfer the nonlinear partial differential equation Eq. (1) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots\dots\dots\dots.) = 0 \qquad (4)$$

The ordinary differential equation (4) is then integrated as long as all terms contain derivatives, where we neglect the integration constants. The solutions of many nonlinear equations can be expressed in the form: {Ali et al [15], Wazwaz [16]-[17]}

$$f(\xi) = \alpha \, sin^{\beta}(\mu\xi) \ , \qquad |\xi| \le \frac{\pi}{2\mu}$$

or in the form (5)

$$f(\xi) = \alpha \, cos^{\beta}(\mu\xi) \ , \qquad |\xi| \le \frac{\pi}{2\mu}$$

Where α , μ, and β are parameters to be determined, μ and c are the wave number and the wave speed, respectively {Parks [14], Mahmoud et al [18]}. We use

$$f(\xi) = \alpha \, sin^{\beta}(\mu\xi)$$

$$f'(\xi) = \alpha \beta \mu \, \sin^{\beta - 1}(\mu\xi) \, \cos(\mu\xi) \qquad (6)$$

$f''(\xi) = \alpha\,\beta(\beta-1)\,\mu^2\ \sin^{\beta-2}(\mu\xi) - \alpha\,\beta^2\,\mu^2\ \sin^{\beta}(\mu\xi)$

and their derivative. Or use

$$f(\xi) = \alpha\cos^{\beta}(\mu\xi)$$
$$f'(\xi) = -\,\alpha\,\beta\,\mu\ \cos^{\beta-1}(\mu\xi)\ \sin(\mu\xi) \qquad (7)$$
$$f''(\xi) = \alpha\,\beta(\beta-1)\,\mu^2\ \cos^{\beta-2}(\mu\xi) - \alpha\,\beta^2\,\mu^2\ \cos^{\beta}(\mu\xi)$$
$$f'''(\xi) = -\,\alpha\,\beta(\beta-1)(\beta-2)\,\mu^3\ \cos^{\beta-3}(\mu\xi)\sin(\mu\xi) + \alpha\,\beta^3\,\mu^3\ \cos^{\beta-1}(\mu\xi)\ \sin(\mu\xi)$$

and so on. We substitute (6) or (7) into the reduced equation (4), balance the terms of the sine functions when (6) are used, or balance the terms of the cosine functions when (7) are used, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $sin^{k}(\mu\xi)$ or $cos^{k}(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's α, μ and β, and solve the subsequent system.

3. APPLICATIONS

3.1 Example 1. Let us first consider the (2+1)-dimensional nonlinear Schrödinger equation {Zhou et al. [19]} that reads:

$$i\,q_t + a\,q_{xx} - b\,q_{yy} + c\,|q|^2\ q = 0 \qquad (8)$$

where a, b and c are nonzero constants. Firstly, we introduce the transformations

$$q(x,y,t) = e^{i\,\theta}.u(\xi)\ , \qquad \theta = \alpha x + \beta y + \delta t\ ,\ \ \xi = k(x + ly - \lambda t) \qquad (9)$$

where $\alpha, \beta, \delta, k, l,$ and λ are real constants. Substituting (9) into Equation (8) we obtain the $\lambda = 2(a\alpha - b\beta l)$ and $u(\xi)$ satisfy into the ODE:

$$-(\delta + a\,\alpha^2 - b\beta^2)\,u(\xi) + (a - b\,l^2)k^2u''(\xi) + c(u(\xi))^3 = 0 \qquad (10)$$

Rewrite this second-order ordinary differential equation as follows:

$$u'' + k_1u^3 - k_2\,u = 0 \qquad (11)$$

Where

$$k_1 = \frac{c}{(a-b\,l^2)k^2}\ ,\ k_2 = \frac{(\delta + a\,\alpha^2 - b\beta^2)}{(a-b\,l^2)k^2} \qquad (12)$$

Seeking solutions of the form (7) we get:

$$\alpha\,\beta(\beta-1)\,\mu^2\cos^{\beta-2}(\mu\xi) - \alpha\,\beta^2\,\mu^2\cos^{\beta}(\mu\xi) + k_1\alpha^3\cos^{3\beta}(\mu\xi) - k_2\,\alpha\cos^{\beta}(\mu\xi) = 0 \qquad (13)$$

Equating the exponents and the coefficients of each pair of the cosine functions we find the following algebraic system:

$$\beta - 2 = 3\beta$$
$$\alpha\,\beta(\beta-1)\,\mu^2 + k_1\alpha^3 = 0$$
$$-\alpha\,\beta^2\,\mu^2 - k_2\,\alpha = 0 \qquad (14)$$

By solving the algebraic system (14), we get,

$$\beta = -1\ , \qquad \mu = \pm\, i\sqrt{k_2}\ \ ,\ \ \alpha = \pm\sqrt{\frac{2\,k_2}{k_1}} \qquad (15)$$

Then by substituting Eq. (15) into Eq. (7) then, the exact soliton solution of equation (8) can be written in the form:

$$u(\xi) = \pm\sqrt{\frac{2\,k_2}{k_1}}\ \sec\left(\pm\, i\sqrt{k_2}\xi\right) = \pm\sqrt{\frac{2\,k_2}{k_1}}\ \mathrm{sech}\left(\sqrt{k_2}\xi\right) \qquad (16)$$

Therefore

$$u(x,y,t) = \pm\sqrt{\frac{2(\delta + a\,\alpha^2 - b\beta^2)}{c}}\ \mathrm{sech}\left(\sqrt{\frac{(\delta + a\,\alpha^2 - b\beta^2)}{(a-b\,l^2)k^2}}\ \ k(x + ly - \lambda t)\right)\ e^{i\,(\alpha x + \beta y + \delta t\ )} \qquad (17)$$

3.2 Example 2. Let us consider the nonlinear The Schrödinger-Hirota equation which governs the propagation of optical solitons in a dispersive optical fiber :

$$i\,q_t + \frac{1}{2}\ q_{xx} + \ |q|^2\ q + i\,\lambda\,q_{xxx} = 0 \qquad (18)$$

This equation studied by {Biswas et al [20]} by the ansatz method for bright and dark 1-soliton solution. The power law nonlinearity was assumed. The equation was solved also by using the tanh method.

introduce the transformations

$$q(x,t) = e^{i\,\theta}.u(\xi)\ , \qquad \theta = \alpha x + \beta t + \epsilon_0\ ,\ \ \xi = k_0(x - 2\alpha t + \chi) \qquad (19)$$

where $\alpha, \beta, \epsilon_0, k_0,$ and χ are real constants. Substituting (19) into Equation (18) we obtain that $\alpha = \frac{-1}{3\lambda}$ and $u(\xi)$ satisfy into the ODE:

$-\left(\frac{5}{54\lambda^2}+\beta\right)\, u(\xi)+\frac{3}{2}{k_0}^2 u''(\xi)+(u(\xi))^3=0$ (20)

Then we can write the following equation:

$u''+k_1u^3-k_2\,u=0$ (21)

Where

$k_1=\frac{1}{\frac{3}{2}{k_0}^2}\ ,\ k_2=\frac{\left(\frac{5}{54\lambda^2}+\beta\right)}{\frac{3}{2}{k_0}^2}$ (22)

Seeking solutions of the form (6) we get:

$\alpha\,\beta(\beta-1)\,\mu^2\sin^{\beta-2}(\mu\xi)-\alpha\,\beta^2\,\mu^2\,\sin^{\beta}(\mu\xi)+k_1\alpha^3\sin^{3\beta}(\mu\xi)-k_2\,\alpha\,\sin^{\beta}(\mu\xi)=0$ (23)

Equating the exponents and the coefficients of each pair of the cosine functions we find the following algebraic system:

$\beta-2=3\beta$

$\alpha\,\beta(\beta-1)\,\mu^2+k_1\alpha^3=0$

$-\alpha\,\beta^2\,\mu^2-k_2\,\alpha=0$ (24)

By solving the algebraic system (24), we get,

$\beta=-1\ ,\quad \mu=\pm\, i\sqrt{k_2}\ \ ,\ \ \alpha=\pm\sqrt{\frac{2\,k_2}{k_1}}$ (25)

Then by substituting Eq. (25) into Eq. (6) , the exact soliton solution of equation (21) can be written in the form:

$u(\xi)=\pm\sqrt{\frac{5}{27\lambda^2}+2\beta}\,\csc\left(\pm\, i\sqrt{k_2}\xi\right)=\mp\sqrt{\frac{5}{27\lambda^2}+2\beta}\,\mathrm{csch}\left(\sqrt{k_2}\xi\right)$ (26)

Therefore

$u(x,y,t)=\ \pm\sqrt{\left(\frac{5}{27\lambda^2}+2\beta\right)}\ \mathrm{csch}\left(\sqrt{\frac{\left(\frac{5}{54\lambda^2}+\beta\right)}{\frac{3}{2}{k_0}^2}}\ \ k_0(x+\frac{2}{3\lambda}t+\chi)\right)\ e^{i\,(\frac{-1}{3\lambda}x+\beta t+\epsilon_0)}$ (27)

3.3 Example 3. Let us consider the Gardner equation {Wazwaz [21], and Biswas [22]}

$u_t-6\,(u\,+\,\varepsilon^2\,\,u^2)\,u_x+\,u_{xxx}=0$ (28)

This equation known as the mixed KdV-mKdV equation is very widely studied in various areas of Physics that includes Plasma Physics, Fluid Dynamics, Quantum Field Theory, Solid State Physics and others {Biswas [20]}. We introduce the transformation $\xi=k(x-\lambda t)$, where k, and λ are real constants. Equation (28) transforms to the ODE:

$-k\lambda\,u'-3k(u^2)'-2\varepsilon^2k(u^3)'+\,k^3\,u'''=0$ (29)

Integrating (29) once with zero constant to get the following ordinary differential equation:

$\lambda\,u+3u^2+2\varepsilon^2\,u^3-\,k^2\,u''=0$ (30)

Seeking the solution in (7)

$\lambda\,\alpha\cos^{\beta}(\mu\xi)+3\alpha^2\cos^{2\beta}(\mu\xi)\,+2\varepsilon^2\,\alpha^3\cos^{3\beta}(\mu\xi)-\ \ \alpha\,\beta(\beta-1)k^2\,\mu^2\,\cos^{\beta-2}(\mu\xi)+\alpha\,\beta^2\,\mu^2\,k^2\cos^{\beta}(\mu\xi)\ =0$ (31)

Equating the exponents and the coefficients of each pair of the cosine functions we find the following algebraic system:

$\beta(\beta-1)(\beta-2)\neq 0$

$3\beta=\beta-2\ \ \rightarrow\beta=-1$ (32)

Substituting Eq. (32) into Eq. (31) to get:

$\lambda\,\alpha\cos^{-1}(\mu\xi)+3\alpha^2\cos^{-2}(\mu\xi)\,+2\varepsilon^2\,\alpha^3\cos^{-3}(\mu\xi)-\ 2\,\alpha\ k^2\,\mu^2\,\cos^{-3}(\mu\xi)+\alpha\ \mu^2\,k^2\cos^{-1}(\mu\xi)\ =0$
(33)

Equating the exponents and the coefficients of each pair of the cosine function, we obtain a system of algebraic equations:

$\cos^{-3}(\mu\xi)\ \ :2\varepsilon^2\,\alpha^3\,-2\,\alpha\,k^2\,\mu^2\,=0$

$\cos^{-2}(\mu\xi)\ \ :\ 3\,\alpha^2=0$

$\cos^{-1}(\mu\xi)\ \ :\ \lambda\,\alpha+\alpha\,\mu^2\,k^2=0$ (34)

By solving the algebraic system (34), we get,

$\beta=-1\ ,\ \ \lambda\ =-\mu^2\,k^2\ ,\qquad \alpha\ =\ \mp\frac{k\ \mu}{\varepsilon}$ (35)

Then by substituting Eq. (35) into Eq. (7) , the exact soliton solution of equation and equating the exponents and the coefficients of each pair of the cosine function, we obtain a system of algebraic equations: (30) can be written in the form

$$u(x,t) = \mp \frac{k\ \mu}{\varepsilon}\ sec\,(\mu\, k(x+\mu^2\, k^2 t))\quad , \quad 0 < \mu\, k(x+\mu^2\, k^2 t) < \pi \qquad (36)$$

3.4 Example 4: The (1+1)-dimensional nonlinear dispersive equation

$$u_t - \delta\, u^2\, u_x + u_{xxx} = 0 \qquad (37)$$

where δ is a nonzero positive constant. This equation is called the modified KdV equation {Elsayed et al [23]}, which arises in the process of understanding the role of nonlinear dispersion and in the formation of structures like liquid drops, and it exhibits compaction solitons with compact support. To find the traveling wave solutions of Eq.(37), {He et al [24]} used the Exp-function method, and {Elsayed et al [23]} used G'/G expansion Method.
Let us now solve Eq.(37) by the proposed method. We introduce the transformation $\xi = k(x-\lambda t)$, where k, and λ are real constants. Equation (37) transforms to the ODE:

$$-k\lambda\, u' - \frac{\delta}{3} k(u^3)' + k^3\, u''' = 0 \qquad (38)$$

Integrating (38) once with zero constant to get the following ordinary differential equation:

$$\lambda\, u + \frac{\delta}{3} u^3 - k^2\, u'' == 0 \qquad (39)$$

Seeking the solution in (7)

$$\lambda\, \alpha \cos^{\beta}(\mu\xi) + \frac{\delta}{3}\alpha^3 \cos^{3\beta}(\mu\xi) - \alpha\,\beta(\beta-1)k^2\,\mu^2 \cos^{\beta-2}(\mu\xi) + \alpha\,\beta^2\,\mu^2\,k^2 \cos^{\beta}(\mu\xi) = 0 \qquad (40)$$

Equating the exponents and the coefficients of each pair of the cosine functions we find the following algebraic system:

$$3\beta = \beta - 2 \ \rightarrow \beta = -1$$
$$\cos^{-3}(\mu\xi)\ :\ \frac{\delta}{3}\alpha^3 - 2\,\alpha\, k^2\,\mu^2 = 0$$
$$\cos^{-1}(\mu\xi)\ :\ \lambda\,\alpha + \alpha\,\mu^2\, k^2 = 0 \qquad (41)$$

By solving the algebraic system (41), we get,

$$\beta = -1\,,\ \lambda = -\mu^2\, k^2\,,\ \alpha = \mp\sqrt{\frac{6}{\delta}}\ k\ \mu \qquad (42)$$

Then by substituting Eq. (42) into Eq. (7) , the exact soliton solution of equation (37) can be written in the form

$$u(x,t) = \mp\sqrt{\frac{6}{\delta}}\ k\ \mu\ \ sec\,(\mu\, k(x+\mu^2\, k^2 t))\,,\quad 0 < \mu\, k(x+\mu^2\, k^2 t) < \pi \qquad (43)$$

3.5. Example 5. Perturbed Burgers equation

In this section the study is going to be focused on the perturbed Burgers equation. The solitary wave ansatz method will be adopted to obtain the exact 1-soliton solution of the Burgers equation in (1+1) dimensions. The search is going to be for a topological 1-soliton solution. The perturbed Burgers equation that is given by the following form {Anwar et al [25]}:

$$u_t + a\, u\, u_x + b\, u_{xx} = c\, u^2\, u_x + \beta\, u\, u_{xx} + \gamma\,(u_x)^2 + \delta\, u_{xxx} \qquad (44)$$

Eq. (44) appears in the study of gas dynamics and also in free surface motion of waves in heated fluids. The perturbation terms are obtained from long-wave perturbation theory. Eq. (44) shows up in the long-wave small-amplitude limit of extended systems dominated by dissipation, where dispersion is also present at a higher order {Anwar et al [25]}.
To solve Eq.(44) by the proposed method. We introduce the transformation $\xi = k(x-\lambda t)$, where k, and λ are real constants. Equation (44) transforms to the ODE:

$$-\lambda k\, u' + a\, k\, u\, u' + b\, k^2\, u'' = c\, k\, u^2\, u' + d\ k^2\, u\, u'' + \gamma\, k^2\,(u')^2 + \delta\ k^3\, u''' \qquad (45)$$

Seeking the solution in (7)

$$\lambda\,\alpha\,\beta\,\mu\cos^{\beta-1}(\mu\xi)\sin(\mu\xi) - a\,\alpha^2\,\beta\,\mu\cos^{2\beta-1}(\mu\xi)\sin(\mu\xi) + b\,k\,\alpha\,\beta(\beta-1)\,\mu^2\, cos^{\beta-2}(\mu\xi) -$$
$$b\,k\,\alpha\,\beta^2\,\mu^2\, cos^{\beta}(\mu\xi) + c\,\alpha^3\,\beta\,\mu\cos^{3\beta-1}(\mu\xi)\sin(\mu\xi) - d\,k\,\alpha^2\,\beta(\beta-1)\mu^2\cos^{2\beta-2}(\mu\xi) +$$
$$\mathrm{d}k\,\alpha^2\,\beta^2\,\mu^2\cos^{2\beta}(\mu\xi) - \gamma\,k\,\alpha^2\,\beta^2\,\mu^2\cos^{2\beta-2}(\mu\xi) + \gamma\,k\,\alpha^2\,\beta^2\,\mu^2\cos^{2\beta}(\mu\xi) + \alpha\,\beta(\beta-1)(\beta-$$
$$2)\mu^3\,\delta\,k^2\cos^{\beta-3}(\mu\xi)\sin(\mu\xi) - \alpha\,\beta^3\,\mu^3\,\delta\,k^2\cos^{\beta-1}(\mu\xi)\sin(\mu\xi) = 0 \qquad (46)$$

From (46), equating exponents $2\beta-2$ and $3\beta-1$ yield

$$2\beta - 2 = 3\beta - 1 \qquad (47)$$

, so that

$$\beta = -1 \qquad (48)$$

It needs to be noted that the same value of β is obtained when the exponent pairs $-2 = 2\beta-1$, $2\beta-2 = \beta-3$ are equated , Thus setting their coefficients to zero yields:

$$\left.\begin{array}{l} -\mathrm{d}\,k\,\alpha^2\,\beta(\beta-1)\mu^2-\gamma\,k\,\alpha^2\,\beta^2\,\mu^2 \;+\;\alpha\,\beta(\beta-1)(\beta-2)\mu^3\,\delta\;k^2=0 \\ b\,k\,\alpha\,\beta(\beta-1)\,\mu^2 \;-\;a\,\alpha^2\,\beta\,\mu=0 \\ (\mathrm{d}k+\gamma\,k)\alpha\,\beta\,\mu+\;\lambda\;-\beta^2\,\mu^2\,\delta\;k^2\;=0 \end{array}\right\} \quad (49)$$

By solving the algebraic system (49), we get,

$$\delta=\frac{(2d\;+\gamma)\mathrm{b}}{3\,\mathrm{a}}\;,\;\alpha=-\frac{2\,b\,k}{a}\;\;\mu\;\;,\;\;\lambda=[4d\;-5\,\gamma\,]\frac{\mathrm{b}}{3\mathrm{a}}\;\;k^2\mu^2$$

Then by substituting Eq. (49) into Eq. (7) , the exact soliton solution of equation (44) can be written in the form

$$\mathrm{u}(\mathrm{x},\mathrm{t})=-\frac{2\,b\,k}{a}\;\;\mu\;\;\sec\,[\,\mu k(x-[4d\;-5\,\gamma\,]\frac{\mathrm{b}}{3\mathrm{a}}\;\;k^2\mu^2 t)] \quad (50)$$

3.6 Example 6. The general Burgers-Fisher equation

Let us consider the following general Burger's-Fisher equation {Javidi [26]}

$$u_t-a\,u^n\,u_x\;+b\,u_{xx}+c\,u\,(1-\;u^n)=0 \quad (51)$$

where a, b and c are nonzero constants. We introduce the transformation $\xi=k(x-\lambda t)$, where $k,$ and λ are real constants. The traveling wave variable ξ permits us converting Eq. (51) into the following ODE:

$$-\lambda k u'+a\,k\,u^n\,u'\;+b\,k^2u''+c\,u-c\,u^{n+1}=0 \quad (52)$$

Seeking the solution in (7)

$$\lambda k\;\;\alpha\,\beta\,\mu\;\cos^{\beta-1}(\mu\xi)\;\sin(\mu\xi)-a\,k\,\alpha^{\mathrm{n}+1}\,\beta\,\mu\;\cos^{(\mathrm{n}+1)\beta-1}(\mu\xi)\;\sin(\mu\xi)$$
$$+b\,k^2\alpha\,\beta(\beta-1)\,\mu^2\;\cos^{\beta-2}(\mu\xi)-[b\,k^2\alpha\,\beta^2\,\mu^2-c\,\alpha]\cos^{\beta}(\mu\xi)-c\;\;\alpha^{\mathrm{n}+1}\cos^{(n+1)\beta}(\mu\xi)=0 \quad (53)$$

From (53), equating exponents $(\mathrm{n}+1)\beta$ and $\beta-1$ yield

$$(\mathrm{n}+1)\beta=\beta-1 \quad (54)$$

, so that

$$\beta=\frac{-1}{\mathrm{n}} \quad (55)$$

when the exponent pair $(\mathrm{n}+1)\beta-1=\beta-2$, is equated gave the same value of $\beta=\frac{-1}{\mathrm{n}}$, Thus setting their coefficients to zero yields:

$$c\;\;\alpha^{\mathrm{n}+1}+\lambda k\;\;\alpha\,\beta\,\mu\;\;=0$$
$$b\,k^2\alpha\,\beta(\beta-1)\,\mu^2\;\;-a\,k\,\alpha^{\mathrm{n}+1}\,\beta\,\mu\;\;=0 \quad (56)$$

By solving the algebraic system (49), we get,

$$\lambda=\;-\;\frac{b\;\mathrm{c}\;(\mathrm{n}+1)}{a}\,,\,\alpha=(\frac{b\;\;(\mathrm{n}+1)}{a\,n}k\,\mu\,)^{\frac{1}{n}} \quad (57)$$

Then by substituting Eq. (57) into Eq. (7) , the exact soliton solution of equation (51) can be written in the form

$$u(x,t)=[\frac{b(\mathrm{n}+1)}{a\,n}\;k\,\mu\;\;\;sec\;\;(\mu k(x+\frac{bc\,(n+1)}{a}t))]^{\frac{1}{n}} \quad (58)$$

4. CONCLUSION

In this Letter, the sine-cosine function method has been successfully applied to find the solution for nonlinear partial differential equations. The method is used to find a new exact solution. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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6. REFERENCES

[1]. Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations, Am. J. Phys, Vol. 60, No. 7, pp. 650-654.

[2]. Khater, A.H., Malfliet, W., Callebaut, D.K. and Kamel, E.S. (2002). The tanh method, a simple transformation and exact analytic al solutions for nonlinear reaction–diffusion equations, Chaos Solitons Fractals, Vol. 14, No. 3, PP. 513-522.

[3]. Wazwaz, A.M. (2006). Two reliable methods for solving variants of the KdV equation with compact and noncompact structures, Chaos Solitons Fractals, Vol. 28, No. 2, pp. 454-462.

[4]. El-Wakil, S.A, Abdou, M.A. (2007). New exact travelling wave solutions using modified extended tanh-function method, Chaos Solitons Fractals, Vol. 31, No. 4, pp. 840-852.

[5]. Fan, E. (2000). Extended tanh-function method and its applications to nonlinear equations. Phys Lett A, Vol. 277, No.4, pp. 212-218.

[6]. Wazwaz, A.M. (2005). The tanh-function method: Solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos Solitons and Fractals, Vol. 25, No. 1, pp. 55-63.
[7]. Xia, T.C., Li, B. and Zhang, H.Q. (2001). New explicit and exact solutions for the Nizhnik- Novikov-Vesselov equation. Appl. Math. E-Notes, Vol. 1, pp. 139-142.
[8]. Yusufoglu, E., Bekir A. (2006). Solitons and periodic solutions of coupled nonlinear evolution equations by using Sine-Cosine method, Internat. J. Comput. Math, Vol. 83, No. 12, pp. 915-924.
[9]. Inc, M., Ergut, M. (2005). Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method, Appl. Math. E-Notes, Vol. 5, pp. 89-96.
[10]. Zhang, Sheng. (2006). The periodic wave solutions for the (2+1) -dimensional Konopelchenko Dubrovsky equations, Chaos Solitons Fractals, Vol. 30, pp. 1213-1220.
[11]. Feng, Z.S. (2002). The first integer method to study the Burgers-Korteweg-de Vries equation, J Phys. A. Math. Gen, Vol. 35, No. 2, pp. 343-349.
[12]. Ding, T.R., Li, C.Z. (1996). Ordinary differential equations. Peking University Press, Peking.
[13]. Mitchell A. R. and D. F. Griffiths(1980), The Finite Difference Method in Partial Differential Equations, John Wiley & Sons.
[14]. Parkes E. J. and B. R. Duffy(1998), An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations, Comput. Phys. Commun. 98 , 288-300.
[15]. Ali A.H.A. , A.A. Soliman , and K.R. Raslan,(2007), Soliton solution for nonlinear partial differential equations by cosine-function method, Physics Letters A 368 (2007) 299–304.
[16]. Wazwaz, A.M. (2004). A sine-cosine method for handling nonlinear wave equations, Math. Comput. Modelling, Vol. 40, No.5, pp. 499-508.
[17]. Wazwaz, A.M. (2004). The sine-cosine method for obtaining solutions with compact and non compact structures, Appl. Math. Comput, Vol. 159, No. 2, pp. 559-576.
[18]. Mahmoud M. El-Borai 1, Afaf A. Zaghrout 2 & Amal M. Elshaer, (2011), IJRRAS vol.9 issue (3)
[19]. Zhou, Y., Wang, M. and Miao, T. (2004). The periodic wave solutions and solitary for a class of nonlinear partial differential equations, Phys. Lett. A. Vol. 323, pp.77−88
[20]. Biswas Anjan, Anwar Ja'afar Muhammad Jawad, Wayne N. Manrakhan , Amarendra K. Sarma, Kaisar R. Khan,(2012), Optical Soliton and complexitons of the Schrödinger–Hirota equation, Optics & Laser Technology, Volume 44, Issue 7, Pages 2265–2269
[21]. Wazwaz, A.M. (2007). New solitons and kink solutions for the Gardner equation, Communications in Nonlinear Science and Numerical Simulation. Vol. 12, pp. 1395−1404.
[22]. Biswas, A. (2008). Soliton perturbation theory for the Gardner equation, Adv Studies Theor Phys. Vol. 16, No.2, pp. 787 – 794.
[23]. Elsayed.M.E. Zayed , Shorog Al-Joudi,(2009), The TravelingWave Solutions for Nonlinear Partial Differential Equations Using the (G'/G)-expansion Method, International Journal of Nonlinear Science, Vol.8, No.4,pp.435-447
[24]. He J. H, Wu X. H. ,(2006), Exp-function method for nonlinear wave equations. Chaos, Solitons & Fractals.30: 700-708.
[25]. Anwar Ja'afar Mohamad Jawad , Marko D. Petkovic, Anjan Biswas (2010), Soliton solutions of Burgers equations and perturbed Burgers equation, Applied Mathematics and Computation, vol.216 pp. 3370–3377.
[26]. Javidi. M. (2006), Spectral collocation method for the solution of the generalized Burgers-Fisher equation. Appl. Math. Comput, 174 ,345-352.