QUADRATIC HEDGING FOR CONTINGENT CLAIMS WITH DELTA CONSTRAINT

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ABSTRACT
In this paper, under constraint of delta-strategy and by importing another related risky asset to compose a hedging portfolio comprising the underlying asset and riskless asset(the Bond). Firstly, we excellently devise a dynamic hedging program for contingent claims; and then, according to Principle of Dynamic Programming and by taking advantage of backward recursion technique, at each rebalance moment before option's maturity date, the optimal hedging strategies are acquired to (1) eliminate the diffusion risk by imposing delta constraint; and (2) depress the jump risk using the hedging portfolio, which minimize the mean squared error between the terminal valuation of hedging portfolio and the payment obligation that the option issuer may be charged with; lastly, at the end of this paper, empirical analysis and numerical results indicate that our proposed hedging strategy is not only efficacious and feasible but also convenient and simple to manipulate, at the same time, it is referential to hedging practice.

Keywords: *Contingent Claims, Dynamic Hedging, Quadratic Criteria, Delta-Constraint*

1. INTRODUCTION

Since Black-Scholes and Merton proposed delta hedging strategy[1], it has become a hot topic in financial world and been widely adopted for option contracts. We all know that, in diffusion settings without jump and by continuous trading, the delta-hedging strategy under Black-Scholes and Merton model can eliminate financial risks. However, because of the existence of transaction fee and the asset price jump occurring for exterior unexpected incidents, the financial market generally become incomplete, and in this case, it is impossible to eliminate risk by delta-hedging strategy only with the underlying asset. About hedging in such a jump-diffusion model, Artur Sepp and Merrill Lynch[2] researched the distribution of delta-hedging errors and found that, even many hedging strategy rebalancing executed, the volatility of a delta-hedging strategy will not be reduced much; He C. et al[3] and Kennedy J.S. et al[4] also studied dynamic hedging for European options under jump diffusion settings, by holding extra instruments in the hedging portfolio that protect against a sudden movement in the stock price and repeatedly solving an optimization problem, they minimize the jump risk after having eliminated the diffusion risk by delta neutral strategy at each strategy rebalancing time. In fact, as to European style option, it will be settled only at the maturity date, so, the objective to optimize is the terminal deviation between valuation of hedging portfolio and contingent claims.

Enlightened by and refer to aforementioned literatures[2-4], in this paper, adopting the prevalent quadratic hedging criterion, which has been studied in literates[5-8], we look on the terminal deviation as mean squared error between the terminal valuation of hedging portfolio and the payment obligation that the option issuer may be charged with, then, under the constraint of delta-hedging strategy and importing another related risky asset to compose a hedging portfolio comprising the underlying asset, we excellently devise a dynamic hedging scheme for contingent claims, and by Principle of Dynamic Programming and by taking advantage of backward recursion technique, at each rebalance time during option's hedging horizon, the optimal hedging strategies are acquired to eliminate the diffusion risk by imposing delta constraint and to depress the jump risk by the hedging portfolio which minimize the terminal deviation. Namely, during the option maturity, we first acquire the optimal strategy φ_{T-1}^* by solving the optimization problem $\min\limits_{(\varphi_{T-1})} E[(H_T - V_T)^2 \mid F_{T-1}]$,and found that the expectation $E[(H_T - V_T)^2 \mid F_{T-1}]$ can be transformed into three parts where only $(H_{T-1} - V_{T-1})^2$ is decided by strategy φ_{T-2} at rebalancing time $T-2$, so, similar to acquiring φ_{T-1}^*,we can solve $\min\limits_{(\varphi_{T-2})} E[(H_{T-1} - V_{T-1})^2 \mid F_{T-2}]$ and acquire the optimal strategy φ_{T-2}^*, repeatedly till time 0, all strategies φ_t^* may be achieved. At the end of this paper, numerical example is given and empirical analysis results indicate that our proposed method is efficacious and feasible for hedging while convenient and simple to manipulate, not least, it is referential to hedging practice.

2. SOME PRELIMINARIES

Let (Ω, F, P) be a complete probability space with filtration $F=(F_t)_{t\in[0,T]}$,and the price of risky assets $S=(S_t)_{t\in[0,T]}$, $I=(I_t)_{t\in[0,T]}$ be nonnegative and adapted to F , respectively solving:

$$dS_t = S_t(\mu_S dt + \sigma_S dw_t + q_S dN_t) \qquad (1)$$

$$dI_t = I_t(\mu_I dt + \sigma_I dw_t + q_I dN_t) \qquad (2)$$

where $\mu_i, \sigma_i^2, i = S, I$ respectively denote instantaneous expected yield and instantaneous variability, w_t is a standard Brownian motion, N_t is a Poisson process with Poisson strength λ and independent with w_t, $q_i, i = S, I$ is the amplitude of price jump.

Let $B=(B_t)_{t\in[0,T]}$ be riskless asset price process, solving:

$$dB_t = rB_t dt, \quad r \text{ is riskless interest rate} \qquad (3)$$

Definition 1: Call a 3-demension stochastic process $\varphi_t = (\vartheta_t, \delta_t, B_t)_{t=0,\cdots,T-1}$ be an investment strategy and the corresponding valuation process is

$$V_t(\varphi) := \vartheta_t S_t + \delta_t I_t + B_t \in L^2(P), \quad t \in \{0,1,\cdots,T-1\} \qquad (4)$$

Furthermore, denoting $R := e^r$,we call it a self-financing strategy when

$$\begin{aligned} V_{t+1} &= \vartheta_{t+1} S_{t+1} + \delta_{t+1} I_{t+1} + B_{t+1} \\ &= \vartheta_t S_{t+1} + \delta_t I_{t+1} + B_t e^r \\ &= V_t + \vartheta_t \Delta S_{t+1} + \delta_t \Delta I_{t+1} + B_t(R-1) \quad t = 0,1,\cdots,T-1 \end{aligned} \qquad (5)$$

Supposing an investor has written a share of European Call Option $C = C(S,t)$ with exercising price K and T horizon, in order to minimizing the terminal quadratic expectation $E[(H_T - V_T)^2]$, by self-financing strategies and under constraint of delta strategy, he hedges the option by holding risky assets S (the underlying), I (the other related asset)and riskless asset B (the bond) with positions $\varphi_t = (\vartheta_t, \delta_t, B_t)$ at discrete time $\{0,1,\cdots,T-1\}$, thus, the hedging model is just as following

$$\begin{cases} \min\limits_{(\varphi_0,\cdots,\varphi_{T-1})} E[H_T - V_T(\varphi)]^2 \\ s.t.\ \vartheta_t S_{t+1} + \delta_t I_{t+1} + B_t R = \vartheta_{t+1} S_{t+1} + \delta_{t+1} I_{t+1} + B_{t+1} \\ \quad \vartheta_t = \dfrac{\partial C}{\partial S}\Big|_t = N(d_t; 0,1) \qquad t = 0,1,\cdots,T-1 \end{cases} \qquad (6)$$

Where $H_T := (S_T - K)^+$, $d_t = \dfrac{\ln(S_t/K) + (\mu_S + \sigma_S^2/2)(T-t)}{\sigma_S\sqrt{T-t}}$ and $N(d_t;0,1)$ denotes the cumulate probability distribution function.

3. SOLUTIONS

3.1Parameter-estimating of the price processes(MLE)

According to the expression (1) ,there is

$$y_t = \ln(S_t / S_{t-1}) = \mu_S - \frac{\sigma_S^2}{2} + \sigma_S \varepsilon_t + Z_t J_t \qquad (7)$$

where $\varepsilon_t = w_t - w_{t-1} \sim N(0,1)$, $Z_t = N_t - N_{t-1} \sim p(\lambda)$, $J_t := \ln(1+q_S) \sim N(\mu_J, \sigma_J^2)$, and ε_t is independent with Z_t .The independent increment quality of Brownian motion and Poisson process indicates that $\{y_1,\cdots,y_T\}$ is an independent and identical distribution sequence with conditional probability density function and likelihood function just as following

$$p(y_t \mid Z_t = k) = \frac{1}{\sqrt{2\pi(\sigma_S^2 + k\sigma_J^2)}} \exp\{-\frac{[y_t - (\mu_S - \frac{\sigma_S^2}{2} + k\mu_J)]^2}{2(\sigma_S + k\sigma_J^2)}\} \quad (8)$$

$$L(y_1,\cdots,y_T) = \prod_{t=1}^{T} p(y_t) = \prod_{t=1}^{T}\{\sum_{k=0}^{\infty}[\frac{\lambda^k e^{-\lambda}}{k!} p(y_t \mid Z_t = k)]\}$$
$$= \prod_{t=1}^{T}\sum_{k=0}^{\infty}[\frac{\lambda^k e^{-\lambda}}{k!}\frac{1}{\sqrt{2\pi(\sigma_S^2 + k\sigma_J^2)}} \exp\{-\frac{[y_t - (\mu_S - \frac{\sigma_S^2}{2} + k\mu_J)]^2}{2(\sigma_S + k\sigma_J^2)}\}] \quad (9)$$

As for Poisson jump, when $\Delta t \to 0$,there are $p(N(\Delta t)=1) = \lambda\Delta t + o(\Delta t)$ and $p(N(\Delta t)=0) = 1-\lambda\Delta t + o(\Delta t)$, $p(N(\Delta t)\geq 2) = o(\Delta t)$ .In this paper, we estimate parameters taking advantage of high frequency historic data with time interval $\Delta t = 5$ minutes, during which, it is reasonable to think there is not more than one price jump happen, i.e., $p(J_t = 1) = \lambda$, $p(J_t = 0) = 1-\lambda$,thus, the expressions (9) about likelihood function may be substituted with

$$L(y_1,\cdots,y_T)$$
$$= \prod_{t=1}^{T}\sum_{k=0}^{1}[\frac{\lambda^k e^{-\lambda}}{k!}\frac{1}{\sqrt{2\pi(\sigma_S^2 + k\sigma_J^2)}} \exp\{-\frac{[y_t - (\mu_S - \frac{\sigma_S^2}{2} + k\mu_J)]^2}{2(\sigma_S + k\sigma_J^2)}\}] \quad (10)$$
$$= \prod_{t=1}^{T}\frac{e^{-\lambda}}{\sqrt{2\pi}}[\frac{1}{\sqrt{\sigma_S^2}}\exp\{-\frac{(y_t - (\mu_S - \frac{\sigma_S^2}{2}))^2}{2\sigma_S^2}\} + \frac{\lambda}{\sqrt{(\sigma_S^2 + \sigma_J^2)}}\exp\{-\frac{[y_t - (\mu_S - \frac{\sigma_S^2}{2} + k\mu_J)]^2}{2(\sigma_S + k\sigma_J^2)}\}]$$

According to (10) and refer to literature [9],we can acquire all parameters' ML estimator.

3.2 Principle of hedging strategy decision

By using Bellman's optimality principle[10] and under the restriction of self-financing, the hedging model(6) can be denoted as below

$$\min_{(\varphi_0,\cdots,\varphi_{T-1})} E[H_T - V_T(\varphi)]^2 = \min_{(\varphi_0,\cdots,\varphi_{T-2})} E\{\min_{\varphi_{T-1}} E[(H_T - V_T(\varphi))^2 \mid F_{T-1}]\}$$
$$= \cdots = \min_{\varphi_0} E\{\min_{\varphi_1}\cdots E\{\min_{\varphi_{T-2}} E\{\min_{\varphi_{T-1}} E[(H_T - V_T(\varphi))^2 \mid F_{T-1}] \mid F_{T-2}\}\cdots\} \quad (11)$$

Thus, we can start from time $T-1$ and seek the optimal strategies $\varphi_t^* = (\vartheta_t^*, \delta_t^*, B_t^*)$ by backward induction method.

3.3 The Concrete steps

Firstly, under the constraint of delta strategy, by solving the following optimization problem(12)

$$\begin{cases} \min_{\varphi_{T-1}} E[(H_T - V_T)^2 \mid F_{T-1}] \\ s.t.\, V_T = \vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R \\ \quad \vartheta_{T-1} = \frac{\partial C}{\partial S}\mid_{T-1} = N(d_{T-1};0,1) \end{cases} \quad (12)$$

We can achieve the optimal position $(\vartheta_{T-1}^*, \delta_{T-1}^*, B_{T-1}^*)$ at time $T-1$ as following

$$\begin{cases} \dfrac{\partial}{\partial \delta_{T-1}} E[(\vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R - H_T)^2 \mid F_{T-1}] \\ \qquad = 2E[(\vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R - H_T)I_T \mid F_{T-1}] = 0 \\ \dfrac{\partial}{\partial B_{T-1}} E[(\vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R - H_T)^2 \mid F_{T-1}] \\ \qquad = 2E[(\vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R - H_T)R \mid F_{T-1}] = 0 \\ \vartheta_{T-1} = \dfrac{\partial C}{\partial S}\Big|_{T-1} = N(d_{T-1};0,1) \end{cases}$$

$$\Rightarrow \begin{cases} \vartheta_{T-1}^* = \dfrac{\partial C}{\partial S}\Big|_{T-1} = N(d_{T-1};0,1) \\ \delta_{T-1}^* = \dfrac{[E(H_T I_T \mid F_{T-1}) - E(H_T \mid F_{T-1})E(I_T \mid F_{T-1})]}{Var(I_T \mid F_{T-1})} \\ \qquad - \dfrac{\vartheta_{T-1}^*[E(S_T I_T \mid F_{T-1}) - E(S_T \mid F_{T-1})E(I_T \mid F_{T-1})]}{Var(I_T \mid F_{T-1})} \\ B_{T-1}^* = \dfrac{E(H_T \mid F_{T-1}) - \vartheta_{T-1}^* E(S_T \mid F_{T-1}) - \delta_{T-1}^* E(I_T \mid F_{T-1})}{R} \end{cases} \tag{13}$$

By self-financing, and denoting

$$H_{T-1} := E[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1)) \mid F_{T-1}]$$

Then,

$$\begin{aligned} &\min_{\varphi_{T-1}} E[(H_T - V_T(\varphi))^2 \mid F_{T-1}] \\ &= V_{T-1}^2 - 2V_{T-1}E[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1)) \mid F_{T-1}] \\ &\qquad + E[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1))^2 \mid F_{T-1}] \\ &= (H_{T-1} - V_{T-1})^2 + E[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1))^2 \mid F_{T-1}] \\ &\qquad - E^2[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1)) \mid F_{T-1}] \\ &= (H_{T-1} - \vartheta_{T-2}S_{T-1} - \delta_{T-2}I_{T-1} - B_{T-2}R)^2 \\ &\quad + E[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1))^2 \mid F_{T-1}] \\ &\quad - E^2[(H_T - \vartheta_{T-1}^*\Delta S_T - \delta_{T-1}^*\Delta I_T - B_{T-1}^*(R-1)) \mid F_{T-1}] \end{aligned} \tag{14}$$

Where $\Delta S_T = S_T - S_{T-1}, \Delta I_T = I_T - I_{T-1}$

Up to now, we can find that the last two parts in equation (14) are irrespective of the strategy $(\vartheta_{T-2}, \delta_{T-2}, B_{T-2})$,so, under the restriction of self-financing and delta strategy, by expressions (11), the optimal hedging strategy $(\vartheta_{T-2}^*, \delta_{T-2}^*, B_{T-2}^*)$ at time $T-2$ can be achieved by solving optimization problem (15) as following

$$\begin{cases} \min\limits_{\varphi_{T-2}} E[(H_{T-1} - V_{T-1})^2 \mid F_{T-2}] = E[(H_{T-1} - \vartheta_{T-2}S_{T-1} - \delta_{T-2}I_{T-1} - B_{T-2}R)^2 \mid F_{T-2}] \\ s.t \quad \vartheta_{T-2} = \dfrac{\partial C}{\partial S}\Big|_{T-2} = N(d_{T-2};0,1) \end{cases} \tag{15}$$

Compare the expressions (15) with (12), there is only difference in time subscript, so, we can solve problem (15) with the same method as solving problem (12) and the optimal strategy is similar to expressions (13).

Repeatedly, at any time $T-k$, by self-financing and with delta strategy constraint, we can solve the following optimization problem and get the optimal strategy as expressions (16).

$$\begin{cases} \min\limits_{\varphi_{T-k}} E[(H_{T-k+1}-V_{T-k+1})^2 \mid F_{T-k}] = E[(H_{T-k+1}-\vartheta_{T-k}S_{T-k+1}-\delta_{T-k}I_{T-k+1}-B_{T-k+1}R)^2 \mid F_{T-k}] \\ s.t \quad \vartheta_{T-k} = \frac{\partial C}{\partial S}\big|_{T-k} = N(d_{T-k};0,1) \end{cases}$$

$$\Rightarrow \begin{cases} \vartheta^*_{T-k} = \frac{\partial C}{\partial S}\big|_{T-k} = N(d_{T-k};0,1) \\ \delta^*_{T-k} = \frac{[E(H_{T-k+1}I_{T-k+1} \mid F_{T-k}) - E(H_{T-k+1} \mid F_{T-k})E(I_{T-k+1} \mid F_{T-k})]}{Var(I_{T-k+1} \mid F_{T-k})} \\ \qquad - \frac{\vartheta^*_{T-k}[E(S_{T-k+1}I_{T-k+1} \mid F_{T-k}) - E(S_{T-k+1} \mid F_{T-k})E(I_{T-k+1} \mid F_{T-k})]}{Var(I_{T-k+1} \mid F_{T-k})} \\ B^*_{T-k} = \frac{E(H_{T-k+1} \mid F_{T-k}) - \vartheta^*_{T-k}E(S_{T-k+1} \mid F_{T-k}) - \delta^*_{T-k}E(I_{T-k+1} \mid F_{T-k})}{R} \end{cases} \qquad (16)$$

Here, $\{H_{T-n}, n = 0,1,\cdots,T-1\}$ meet the recursive relation as follows

$$\begin{cases} H_{T-n} = \frac{E(H_{T-n+1} \mid F_{T-n})}{R} + N(d_{T-n};0,1)(S_{T-n} - \frac{E(S_{T-n+1} \mid F_{T-n})}{R}) \\ + \frac{Cov(H_{T-n+1}, I_{T-n+1} \mid F_{T-n}) - N(d_{T-n};0,1)Cov(S_{T-n+1}, I_{T-n+1} \mid F_{T-n})}{Var(I_{T-n+1} \mid F_{T-n})}(I_{T-n} - \frac{E(I_{T-n+1} \mid F_{T-n})}{R}) \\ H_T = (S_T - K)^+ \end{cases}$$

(17)

3.4 Total Hedging Costs

The total hedging cost consists of the charge to construct the original hedging position, the commission to adjust all hedging positions at any moment before expiration and the terminal profit and loss（ f :the commission rate）

$$\begin{aligned} TC &= (\vartheta_0 S_0 + \delta_0 I_0 + B_0)e^{rT} + (|\vartheta_0| S_0 + |\delta_0| I_0) f e^{rT} \\ &+ \sum_{t=1}^{T-1}[f(S_t|\vartheta_t - \vartheta_{t-1}| + I_t|\delta_t - \delta_{t-1}|)e^{r(T-t)}] \\ &+ (|\vartheta_{T-1}| S_T + |\delta_{T-1}| I_T) f + (S_T - K)^+ - (\vartheta_{T-1}S_T + \delta_{T-1}I_T + B_{T-1}R) \end{aligned} \qquad (18)$$

4. NUMERICAL EXAMPLE

4.1 Parameter decision of the price process

In this paper, we select high frequency historic price data of ICBC(601398) and BOC(601988) from January,2010 to May 2011 as in-sample data to estimate the jump-diffusion process's parameters, and with the above two stocks' close price data from June to August as out-sample data（as in Figure 1）, we give an empirical example. we set the riskless interest rate be 0.36% offered by the PBC. the parameter-estimating results are expressed in table 1.

Table 1 parameter-estimating results of the jump-diffusion process

parameters	μ ($\mu^{(I)}$)	σ ($\sigma^{(I)}$)	μ_J ($\mu_J^{(I)}$)	σ_J ($\sigma_J^{(I)}$)	λ ($\lambda^{(I)}$)	ρ
Value(S)	0.000312	0.016095	0.0543	0.0034	0.005	0.85435

Value(I)	0.000252	0.0175	-0.0535	0.0001	0.0249	

Data resource: RESSET DATABASE

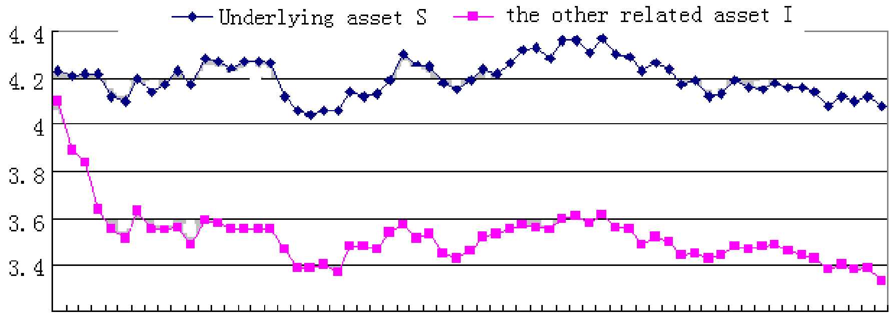

Figure 1 price tendency of the underlying asset and the other related asset

4.2 The analyzing results

We assume that an investor writes European call option, with 1(2,3,respectively)-month(s) maturity, based on ICBC(601398) at the 1st,June,2011, and the maturity date is the 1st,July,2011(1st,August,2011; 1st,September,2010, respectively), and she constructs hedging portfolio consisting of the underlying asset(ICBC stock),another related risky asset(BOC stock) and riskless Bond to hedge her written call option by minimizing the expectation of terminal quadratic error between the contingent claim and the portfolio value, i.e., $\min_{(\varphi_0,\cdots,\varphi_{T-1})} E[H_T - V_T(\varphi)]^2$ with restriction of self-financing and delta strategy. the strategy adjusting frequency is fixed to be 1-day(1-week, 2-week, respectively) and here we only discuss the hedging situation with exercising price $K = S_0$.

Firstly, European call option must be delivered only at maturity, so, the longer the remaining time to maturity date is, the stronger the price volatility risk may be, investors must spend more to attain some certain hedging goal. The following figure 2 denotes positions of the underlying at each rebalancing time of three different maturity call options with $K = S_0$ and 1-day rebalancing frequency. The three position dot lines generally incline rightward, i.e., higher positions correspond with more faraway to maturity date. Comparing three lines, we can find that the positions of underlying of 3-month expiry option are the highest, then those of 2-month expiry option, and the least positions are those of 1-month expiry option, which also indicates that the inverse relationship of risk and maturity time.

Secondly, as to hedging, positions of the underlying asset are closely related with underlying asset price, it is more propitious to hedging for investors to hold more underlying asset when its price is ascending and vice versa. For example, in figure 2, the later part of the position curve with 2-month maturity and the middle part of the longest curve with 3-month maturity are ascending, which really correspond to the price of the underlying asset ascending from its lowest location 4.06RMB to its highest location 4.37RMB. in addition, European call option's hedging position is tightly interrelated with the underlying asset's price, the following figure 3 denotes the relationship between the underlying asset positions and prices at rebalancing times $t = 5, 10, 15$, from which we can see that higher position relating to higher price.

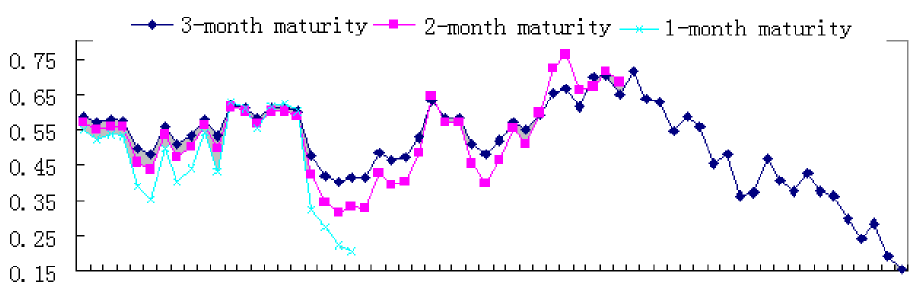

Figure 2 position tendency of the underlying asset

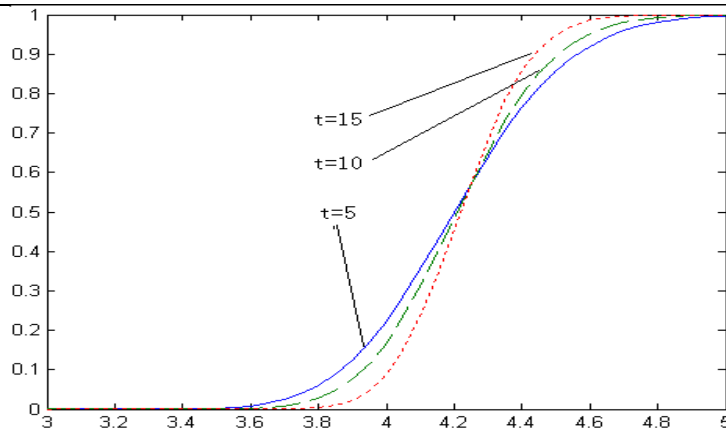

Figure 3 relationship between positions and prices of the underlying asset

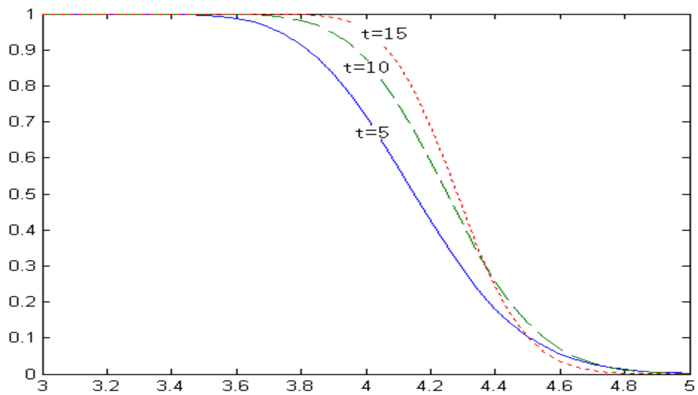

Figure 4 relationship between the striking price and the underlying asset price

Moreover, at the maturity date, the issuer of European call option will be charged $(S_T - K)^+$, which is obviously an decreasing function of the exercise price K, i.e., the probability of the issuer perform the payment obligation is smaller if the exercise price is higher, and vice versa. Figure 4, where $t = 5(10,15)$ denotes three different position rebalancing moments with 1-month expiration horizon, illustrates the reverse relationship between the underlying asset position and the striking price K. Besides, figure 4 also indicates that the underlying asset position tends towards 1 or 0 when the striking price tends enough lower or enough higher. In fact, 0-striking price presumes that the issuer of European call option will absolutely be charged with $(S_T - 0)^+$, so, in order to perfectly replicate the terminal contingent claim, he must hold a share of underlying asset; reversely, with enough high striking price, $(S_T - K)^+$ may always equal to 0, i.e., it is unnecessary to hedging the contingent claim which means no underlying asset will be held.

Fourthly, more violent price fluctuating indicates more formidable market risk, so, investors must spend more to hedge the possible risk and vice versa. (18) denotes that the total hedging profit and loss consists of the payment obligation at maturity, the valuation balance of the hedging portfolio at the beginning and end of hedging horizon, and the transaction fee for rebalancing positions, especially, after constructing the hedging positions, the terminal underlying asset price directly decides whether the option is in the money or not, which decides the hedging profit and loss. Table 2 denotes the total profit and loss and transaction fee of with different kinds of expiration horizons, position rebalancing frequencies and commission rates. From table 2, we know that there is profit for 1-month and 3-month maturity hedging situations while loss for 2-month maturity and 1-day rebalancing frequency, which is not a contradiction to the reality, because $K = S_0 = 4.23RMB$, and as for European call options with 1-month (3-month) maturity, there is $S_T = 4.06RMB$ (4.17RMB,respectively), so, to these two options, $(S_T - K)^+ = 0$, which denotes that no payment obligation occurs, thus, the total profit and loss is decided by the valuation balance of hedging portfolio at two ends of hedging term and the transaction fee; reversely, as to option with 2-month maturity, $S_T = 4.37$ leads to $(S_T - K)^+ = (4.37 - 4.23)^+ > 0$, i.e., different from the other two options, the issuer of this kind of 2-month maturity option will be charged with the terminal payment with the exception of transaction fee, especially, when the position rebalancing frequency is high enough, the overfull transaction fee may cause hedging loss, therefore, higher rebalancing frequency does not certainly benefit, investors should adjust strategy rebalancing frequency according to option maturity, the underlying asset price, and so on.

Table 2 Total profit and loss and transaction fee

fre / hor	Daily frequency	Weekly frequency
1-month maturity	0.1022（0.0177）* 0.0845（0.0354）** 0.0491（0.0708）***	0.2196（0.0127）* 0.2069（0.0254）** 0.1815（0.0507）***
2-month maturity	-0.0024（0.0315）* -0.0075（0.0630）** -0.0706（0.1260）***	0.1321（0.0196）* 0.1125（0.0391）** 0.0734（0.0783）***
3-month maturity	0.1318（0.0284）* 0.1034（0.0569）** 0.0465（0.1137）***	0.2575（0.0157）* 0.2418（0.0313）** 0.2105（0.0626）***

Notes: numbers in parentheses denote commission fee to adjusting portfolio positions; *：1‰ commission fee rate; **：2‰ commission fee rate;***：4‰ commission fee rate

Lastly, in figure 5, the left panel denotes the total profit and loss distribution of pure delta hedging, while the left part denotes that of our proposed hedging method. We can see from this figure that our hedging procedure dramatically reduces the exposure to risk, as also demonstrated by the results in table 3.

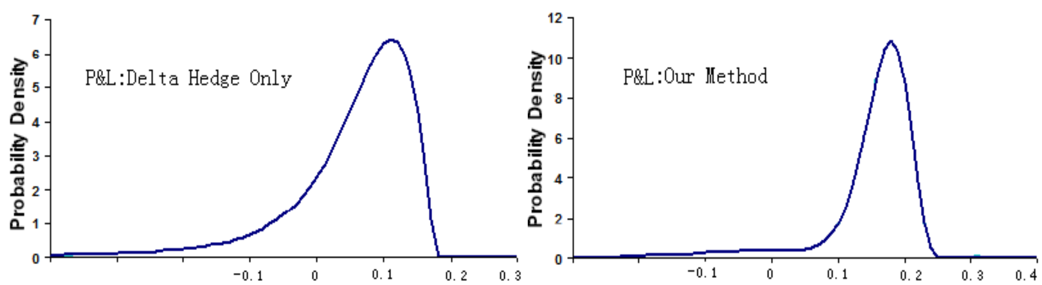

Figure 5 Total P&L distribution for pure Delta hedge (the left panel) and our hedging method(the right panel)

Table 3 Total P&L comparing

Hedging method	Mean	Std.Dev.	5%percentile	95%percentile
Pure Delta Hedging	0.0651	0.3957	-0.0427	0.1423
Our Hedging Procedure	0.1526	0.0126	0.0905	0.2001

5. CONCLUSIONS

It is well known that the goal of hedging is to decrease the risk arising from the price fluctuating, the core objective of hedging is to ascertain reasonable hedging strategies. Under the constraint of self-financing and delta strategy, this paper assumes that the underlying price obeys a jump-diffusion process and studies the quadratic hedging for European style contingent claims. By solving optimization problem (6), we have attained the strategy positions at each rebalancing moment, by which, it is convenient for investors to make their decision. Figures 2～4 and table 2 indicate our optimization model is reasonable and feasible, and the numerical example results indicate that our technology is valuable, while the results of figure 5 and table 3 demonstrate our hedging procedure is effective and helpful to hedging practice.

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