RANDOM FIXED POINT THEOREMS OF RANDOM MULTIVALUED OPERATORS ON POLISH SPACE

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ABSTRACT
In the present paper, we prove the existence of a common random fixed point of two random multivalued generalized contractions by using functional expressions.

Keywords : *Polish space, Random fixed point, Random multivalued operator, Measurable Mapping.*
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1. INTRODUCTION

The Prague School of Probabilists started the study of random fixed point theorems [6, 13]. Common random fixed point theorems are stochastic generalization of classical common fixed point theorems. Itoh [8, 9] extended several well known fixed point theorems (that is, for contraction, non-expansive and condensing mapping) to the random case. Afterwards various stochastic aspects of Schauder's fixed point theorem have been studied by Sehgal and Singh [13], Papageorgious [12], Lin [10] and many authors. In a separable metric space, random fixed point theorems for contraction mappings were proved by Spacek [14], Hans [5, 6, 7], Mukherjee [11]. Afterwards Beg and Shahzad [3, 4], Badshah and Sayyed [2], Badshah and Gagrani [1] studied the structure of common random fixed points. A random coincidence points of pair of compatible random operators and proved the random fixed point theorems for contraction random operators in Polish space. This paper is in continuation of these investigations and proves the existence of a common random fixed point of the random multivalued generalized contractions with functional expressions.

2. PRELIMINARIES

Let (X, d) be a Polish space, that is a separable complete metric space and (Ω, X) be a measurable space. Let 2^X be a family of all subsets of X and CB(X) denote the family of all nonempty bounded closed subsets of X. A mapping $T : \Omega \to 2^X$, is called measurable, if for any open subset C of X, $T^{-1}(C) = \{ \omega \in \Omega \mid T(\omega) \cap C \neq \phi\} \in a$. A mapping $\xi : \Omega \to X$ is said to be a measurable if for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$. A mapping $f : \Omega \times X \to CB(X)$ is called a random multivalued operator, if for every $x \in X$, T(.,x) is measurable. A measurable mapping $\xi : \Omega \to X$ is called the random fixed point of a random multivalued operator $T : \Omega \times X \to CB(X)$ ($f : \Omega \times X \to X$), if for every $\omega \in \Omega$, $(f(\omega, \xi(\omega)) = \xi(\omega))$. Let $T : \Omega \times X \to CB(X)$ be a random operator and $\{ \xi_n\}$ be a sequence of measurable mapping $\xi_n : \Omega \to X$. The sequence $\{ \xi_n\}$ is said to be an asymptotically T-regular, if $d(\xi_n(\omega),T(\omega,\xi_n(\omega))) \to 0$.

Beg and Shahzad [4] proved the following result :

Theorem 2.1 [4] Let X be a Polish space and let S, T: $\Omega \times X \to CB(X)$ be two continuous multivalued random operators. If there exists a measurable mapping $\alpha : \Omega \to (0, 1)$ such that for all $x, y \in X$ and all $\omega \in \Omega$,

$$H(S(\omega, x), T(\omega, y)) \le \alpha(\omega) \max \left\{d(x, y), d(x, S(\omega, x)), d(y, T(\omega, y)), \frac{d(x,T(\omega,y)) + d(y,S(\omega,x))}{2}\right\}.$$

Then there exists a common random fixed point of S and T. Now we give the stochastic version of a result of Beg and Shahzad [4].

3. MAIN RESULT

Theorem 3.1. Let X be a Polish space. Let T, S : $\Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings $\alpha, \beta : \Omega \to (0, 1)$ such that

$$H(S(\omega, x), T(\omega, y)) \le \alpha(\omega) \frac{d(x,S(\omega,x))^3 + d(y,T(\omega,y))^3}{d(x,S(\omega,x))^2 + d(y,T(\omega,y))^2} + \beta(\omega)\, d(x, y)$$

for each $x, y \in X$, $\omega \in \Omega$ and $\alpha, \beta \in R^{+}$ with $2\alpha + \beta < 1$, there exists a common random fixed point of S and T (here H represents the Hausdorff metric on CB(X) induced by the metric d).

Proof. Let $\xi_0 : \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_1(\omega) \in S(\omega, \xi_0(\omega))$ for each $\omega \in \Omega$. Then for each $\omega \in \Omega$,

$$H(S(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \le \alpha(\omega)\frac{d(\xi_0(\omega), S(\omega,\xi_0(\omega)))^3 + d(\xi_1(\omega), T(\omega,\xi_1(\omega)))^3}{d(\xi_0(\omega), S(\omega,\xi_0(\omega)))^2 + d(\xi_1(\omega), T(\omega,\xi_1(\omega)))^2} + \beta(\omega)\, d(\xi_0(\omega), \xi_1(\omega))$$

Further, there exists a measurable mapping $\xi_2 : \Omega \to X$ such that for all $\omega \in \Omega$, $\xi_2(\omega) \in T(\omega, \xi_1(\omega))$ and

$$d(\xi_1(\omega)), \xi_2(\omega))$$

$$\le \alpha(\omega)\frac{d(\xi_0(\omega),\xi_1(\omega))^3 + d(\xi_1(\omega),\xi_2(\omega))^3}{d(\xi_0(\omega),\xi_1(\omega))^2 + d(\xi_1(\omega),\xi_2(\omega))^2} + \beta(\omega)\, d(\xi_0(\omega), \xi_1(\omega))$$

$$\le \frac{\alpha(\omega)}{d(\xi_0(\omega),\xi_1(\omega))^2 + d(\xi_1(\omega),\xi_2(\omega))^2} \times \Big\{d(\xi_0(\omega),\xi_1(\omega)) + d(\xi_1(\omega),\xi_2(\omega))(d(\xi_0(\omega),\xi_1(\omega))^2$$

$$- \quad d(\xi_0(\omega),\xi_1(\omega))d(\xi_1(\omega),\xi_2(\omega)) + d(\xi_1(\omega),\xi_2(\omega))^2\Big\}$$

$$+ \ \beta(\omega)d(\xi_0(\omega),\xi_1(\omega))$$

$$\le \alpha(\omega)\frac{d(\xi_0(\omega),\xi_1(\omega))^2 + d(\xi_1(\omega),\xi_2(\omega))^2}{d(\xi_0(\omega),\xi_1(\omega)) + d(\xi_1(\omega),\xi_2(\omega))} \quad + \ \beta(\omega)d(\xi_0(\omega),\xi_1(\omega))$$

$$\le \alpha(\omega)\frac{\{d(\xi_0(\omega),\xi_1(\omega)) + d(\xi_1(\omega),\xi_2(\omega))\}^2 - 2d(\xi_0(\omega),\xi_1(\omega))d(\xi_1(\omega),\xi_2(\omega))}{d(\xi_0(\omega),\xi_1(\omega)) + d(\xi_1(\omega),\xi_2(\omega))}$$

$$+ \ \beta(\omega)d(\xi_0(\omega),\xi_1(\omega))$$

$$\le \alpha(\omega)\ d(\xi_0(\omega),\xi_1(\omega)) + d(\xi_1(\omega),\xi_2(\omega)) \ + \ \beta(\omega)d(\xi_0(\omega),\xi_1(\omega))$$

so,

$(1 - \alpha(\omega))\, d(\xi_1(\omega), \xi_2(\omega)) \le (\alpha(\omega) + \beta(\omega))\, d(\xi_0(\omega), \xi_1(\omega))$.

Thus, $d(\xi_1(\omega), \xi_2(\omega)) \le k(\omega) d(\xi_0(\omega), \xi_1(\omega))$

where, $k = k(\omega) = \dfrac{\alpha(\omega) + \beta(\omega)}{1 - \alpha(\omega)} < 1.$

By Beg and Shazad [Lemma 2.3], we obtain a measurable mapping $\xi_3 : \Omega \to X$ such that for all $\omega \in \Omega$, $\xi_3(\omega) \in S(\omega, \xi_2(\omega))$ and

$$d(\xi_2(\omega)), \xi_3(\omega)) \le \alpha(\omega)\frac{d(\xi_1(\omega),\xi_2(\omega))^3 + d(\xi_2(\omega),\xi_3(\omega))^3}{d(\xi_1(\omega),\xi_2(\omega))^2 + d(\xi_2(\omega),\xi_3(\omega))^2} + \beta(\omega)\, d(\xi_1(\omega), \xi_2(\omega))$$

Hence, $d(\xi_2(\omega), \xi_3(\omega)) \le \alpha(\omega)\{d(\xi_1(\omega), \xi_2(\omega)) + d(\xi_2(\omega), \xi_3(\omega))$

$$+ \beta(\omega)\, d(\xi_1(\omega), \xi_2(\omega))$$

or $(1 - \alpha(\omega))\, d(\xi_2(\omega), \xi_3(\omega)) \le (\alpha(\omega) + \beta(\omega))\, d(\xi_1(\omega), \xi_2(\omega))$.

Thus, $d(\xi_2(\omega), \xi_3(\omega)) \le k^2 d(\xi_0(\omega), \xi_1(\omega))$.

Similarly, proceeding in the same way, by induction we get a sequence of measurable mappings $\xi_n : \Omega \to X$, such that for $n > 0$, for any $\omega \in \Omega$.

$$\xi_{2n+1}(\omega) \in S(\omega, \xi_{2n}(\omega)), \quad \xi_{2n+2}(\omega) \in T(\omega, \xi_{2n+1}(\omega)) \quad \text{and}$$

$$d(\xi_n(\omega), \xi_{n+1}(\omega)) \le kd(\xi_{n-1}(\omega), \xi_n(\omega)) \le \dots \le k^n d(\xi_0(\omega), \xi_1(\omega)).$$

Further, for m > n,

$$\begin{aligned} d(\xi_n(\omega), \xi_m(\omega)) &\le d(\xi_n(\omega), \xi_{n+1}(\omega)) + \dots + d(\xi_{m-1}(\omega), \xi_m(\omega)) \\ &\le (k^n + k^{n+1} + \dots + k^{m-1})\, d(\xi_0(\omega), \xi_1(\omega)) \\ &\le \left(\frac{k^n}{1-k}\right) d(\xi_0(\omega), \xi_1(\omega)) \end{aligned}$$

which tends to zero as $n \to \infty$. It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$.

It implies that $\xi_{2n+1}(\omega) \to \xi(\omega)$ and $\xi_{2n+2}(\omega) \to \xi(\omega)$.

Thus, we have for any $\omega \in \Omega$,

$$\begin{aligned} d(\xi(\omega), S(\omega, \xi(\omega)) &\le d(\xi(\omega), \xi_{2n+2}(\omega)) + d(\xi_{2n+2}(\omega), S(\omega, \xi(\omega))) \\ &\le d(\xi(\omega), \xi_{2n+2}(\omega)) + H(T(\omega, \xi_{2n+1}(\omega)), S(\omega, \xi(\omega))). \end{aligned}$$

Therefore,

$$\begin{aligned} d(\xi(\omega), S(\omega, \xi(\omega))) \le\ & d(\xi(\omega), \xi_{2n+2}(\omega)) \\ & + \alpha(\omega)\frac{d(\xi(\omega), S(\omega,\xi(\omega)))^3 + d(\xi_{2n+1}(\omega), T(\omega, \xi_{2n+1}(\omega)))^3}{d(\xi(\omega), S(\omega,\xi(\omega)))^2 + d(\xi_{2n+1}(\omega), T(\omega, \xi_{2n+1}(\omega)))^2} \\ & + \beta(\omega)\, d(\xi(\omega), \xi_{2n+1}(\omega)). \end{aligned}$$

Taking $n \to \infty$, we have

$$d(\xi(\omega), S(\omega, \xi(\omega)) \le \alpha(\omega)\, d(\xi(\omega), S(\omega, \xi(\omega))).$$

Hence $\xi(\omega) \in S(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Similarly, for any $\omega \in \Omega$,

$$\begin{aligned} d(\xi(\omega), T(\omega, \xi(\omega)) &\le \alpha(\omega)d(\xi(\omega), \xi_{2n+1}(\omega)) + H(S(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi(\omega))) \\ &\le \alpha(\omega)d(\xi(\omega), T(\omega, \xi(\omega))). \end{aligned}$$

Hence, $\xi(\omega) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

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