THE EXISTENCE OF POSITIVE SOLUTIONS OF SEMI-POSITONE 3×3 SYSTEMS

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ABSTRACT
In this paper, we consider the existence of positive solutions of the Semi-Positone Systems of the form.

$$\begin{aligned} &u''(t) = -\lambda f(u(t), v(t), w(t)), && t \in (0,1), \\ &v''(t) = -\lambda g(u(t), v(t), w(t)), && t \in (0,1), \\ &w''(t) = -\lambda h(u(t), v(t), w(t)), && t \in (0,1), \\ &u(0) = u(1) = 0,\ v(0) = v(1) = 0, && w(0) = w(1) = 0. \end{aligned}$$

where $\lambda > 0$ is a parameter, and *f, g, w* : $[0,\infty)\times[0,\infty)\times[0,\infty) \to \boldsymbol{R}$ are continuous such that $f(u,v,w) \ge -(M/3),\ g(u,v,w) \ge -(M/3),\ h(u,v,w) \ge -(M/3)$ for some $M > 0$. Our approach are based on the fixed point theory in a cone.

Key words: *existence, positive solutions, Semipositone, fixed point theory, cone.*
Mathematics Subject Classifications: *34B18, 37C25.*

1. INTRODUCTION

We study the existence of positive solutions to the system of three variable of the semipositone

$$\begin{aligned} &u''(t) = -\lambda f(u(t), v(t), w(t)), && t \in (0,1), \\ &v''(t) = -\lambda g(u(t), v(t), w(t)), && t \in (0,1), \\ &w''(t) = -\lambda h(u(t), v(t), w(t)), && t \in (0,1), \\ &u(0) = u(1) = 0,\ v(0) = v(1) = 0, && w(0) = w(1) = 0. \end{aligned} \tag{1.1}$$

where $\lambda > 0$ is a parameter. Existence of positive solutions for semi-positive problems have been studied by several authors, (see[1, 2, 5-9] for a single variable, and [3,4,10] for system of two variable) .In this paper we want to study system of three variable and establish results of positive solutions for the system (1.1). Our proofs are based on fixed point theory in a cone, and our result apply when $f(0,0,0)$ or $g(0,0,0)$ or $h(0,0,0)$ (or all of them) are negative (semi-poitone system).

For example, of single semi-positone equations, see [5, 6, 7], the authors studied the equation

$$\begin{cases} -\Delta u(x) = \lambda\ f(u(x)), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega, \end{cases} \tag{A}$$

and established multiple positive solutions, radial solutions, uniqueness and non-negative solutions for problem (A) .

In [3], Hai and Shivaji studied the existence of positive solutions for semilinear elliptic systems of two variable of the form

$$\begin{aligned} &(p(t)u')' = -\lambda f(u,v)p(t),\ t \in (a,b), \\ &(p(t)v')' = -\lambda g(u,v)p(t),\ t \in (a,b), \\ &u(a) = 0 = u(b),\ v(a) = 0 = v(b). \end{aligned}$$

Recently in [11] the 3×3 system

$$\begin{cases} -\Delta u = \lambda f(v, w), & x \in \Omega, \\ -\Delta v = \mu g(u, w), & x \in \Omega, \\ -\Delta w = \sigma h(v, w), & x \in \Omega, \\ u = v = w = 0, & x \in \partial\Omega, \end{cases}$$

Was discussed and nonexistence of positive solutions was established when two of the parameters λ, μ, σ are large . Our results extend existence results for single semi-positone and system of two variable of semi-positone equations to semi-positone system of three variable, we use the following assumptions to prove our results.

(A.1) $f,\ g,\ w\ :\ [0,\infty)\times[0,\infty)\times[0,\infty)\to \boldsymbol{R}$ are continuous and there exists $M > 0$ such that $f(u,v,w)\geq -(M/3),\ g(u,v,w)\geq -(M/3),\ h(u,v,w)\geq -(M/3)$ for every $(u,v,w)\in[0,\infty)\times[0,\infty)\times[0,\infty)$.

(A.2) $\lim_{v\to\infty,\, w\to\infty} f(u,v,w)=\infty$, $\lim_{u\to\infty,\, w\to\infty} g(u,v,w)=\infty$, $\lim_{u\to\infty,\, v\to\infty} h(u,v,w)=\infty$, where each limit is uniform with respect to the other variables , and $\lim_{n\to\infty}\left(h^*(n)/n\right)=\infty$,

where $h^*(n):=\inf_{u,\,v,\,w\geq n}\left(\min\left(f(u,v,w), g(u,v,w), h(u,v,w)\right)\right)$.

(A.3) $\lim_{v\to\infty,\, w\to\infty} f(u,v,w)=\infty$, $\lim_{u\to\infty,\, w\to\infty} g(u,v,w)=\infty$, $\lim_{u\to\infty,\, v\to\infty} h(u,v,w)=\infty$, where each limit is uniform with respect to the other variables , and $\lim_{n\to\infty}\left(\tilde{h}(n)/n\right)=0$,

where $\tilde{h}(n):=\sup_{0\leq u,\,v,\,w\leq n}\max\left(f(u,v,w), g(u,v,w), h(u,v,w)\right)$.

Our existence results for λ small.

Theorem 1.1. Let (A.1) and (A.2) hold. Then there exists $\lambda^*>0$ such that for $\lambda<\lambda^*$, the system (1.1) has a positive solution $(u_\lambda, v_\lambda, w_\lambda)$ with $|u_\lambda(t), v_\lambda(t), w_\lambda(t)|\to\infty$ as $\lambda\to 0$ uniformly for *t* in compact interval of (0,1).Here $|(u,v,w)|=|u|+|v|+|w|$.

Theorem 1.2. Let (A.2) and (A.3) hold. Then there exists $\tilde{\lambda}>0$ such that for $\lambda>\tilde{\lambda}$, the system (1.1) has a positive solution $(u_\lambda, v_\lambda, w_\lambda)$ with $\lambda^{-1}\max\ (u_\lambda(t), v_\lambda(t), w_\lambda(t))\to\infty$ as $\lambda\to\infty$ uniformly for t in compact interval of (0,1).

2. PROOF OF THEOREM 1.l

Let ξ be the solution of

$$\xi''=-\lambda M$$
$$\xi(0)=0=\xi(1)$$

Then there exists a $\tilde{K}>0$ such that

$$\xi(t)\leq\lambda\tilde{K}J(t) \tag{2.1}$$

where $J(t)=\min(t,1-t)$.

Next we note that (*u, v, w*) is a positive solution of (1.1) if and only if $\tilde{u}=u+\zeta,\ \tilde{v}=v+\zeta,\ \tilde{w}=w+\zeta$ is a solution of

$$\begin{aligned}
&\tilde{u}''=-\lambda\tilde{f}(\tilde{u}-\zeta,\tilde{v}-\zeta,\tilde{w}-\zeta), && t\in(0,1)\\
&\tilde{v}''=-\lambda\tilde{g}(\tilde{u}-\zeta,\tilde{v}-\zeta,\tilde{w}-\zeta), && t\in(0,1)\\
&\tilde{w}''=-\lambda\tilde{h}(\tilde{u}-\zeta,\tilde{v}-\zeta,\tilde{w}-\zeta), && t\in(0,1)\\
&\tilde{u}(0)=\tilde{u}(1)=0,\ \tilde{v}(0)=\tilde{v}(1)=0,\ \tilde{w}(0)=\tilde{w}(1)=0
\end{aligned} \tag{2.2}$$

with $\tilde{u}>\zeta, \tilde{v}>\zeta,\ \tilde{w}>\zeta,$ where

$$\tilde{f}(x,y,z)=f(\max(x,0,0),\max(0,y,0),\max(0,0,z))+M$$
$$\tilde{g}(x,y,z)=g(\max(x,0,0),\max(0,y,0),\max(0,0,z))+M$$

and

$$\tilde{h}(x,y,z)=h(\max(x,0,0),\max(0,y,0),\max(0,0,z))+M$$

Now for each $(\tilde{u},\tilde{v},\tilde{w})\in C[0,1]\times C[0,1]\times C[0,1]$, let $(u,v,w):=A(\bar{u},\bar{v},\bar{w})$ be the solution of

$$\begin{aligned}
&u''=-\lambda\tilde{f}(\bar{u}-\zeta,\bar{v}-\zeta,\bar{w}-\zeta), && t\in(0,1)\\
&v''=-\lambda\tilde{g}(\bar{u}-\zeta,\bar{v}-\zeta,\bar{w}-\zeta), && t\in(0,1)
\end{aligned} \tag{2.3}$$

$$w'' = -\lambda \tilde{h}(\bar{u}-\zeta, \bar{v}-\zeta, \bar{w}-\zeta), \qquad t \in (0, 1)$$
$$u(0) = u(1) = 0,\ v(0) = v(1) = 0,\ w(0) = w(1) = 0$$

Let K be a cone defined by

$$K = \{(u,v,w) \in C[0,1] \times C[0,1] \times C[0,1] : u(t) \ge |u|_0 J(t), v(t) \ge |v|_0 J(t), w(t) \ge |w|_0 J(t), t \in [0,1]\}$$

where $|\cdot|_0$ denotes the supremum norm .Then $A : K \to K$ and is completely continuous , see [2].

Theorem A: See[3] Let K be a cone in a Banach space E , and let $A : K \to K$ be a completely continuous operator , let $0 < r < R$ be such that

$$u \le Au \Rightarrow |u|_0 \ne r$$
$$u \ge Au \Rightarrow |u|_0 \ne R$$

Here $u \le v$ if $v - u \in K$ .Then A has a fixed point u with $r < |u|_0 < R$

Lemma 2.1: For $\lambda > 0$ small enough, there exists $A_\lambda > 0$ such that $(u, v, w) \le A(u, v, w)$ implies $|(u, v, w)|_0 \ne A_\lambda$: Further $A_\lambda \to \infty$ as $\lambda \to 0$.

Here $|(u, v, w)|_0 := \max |u|_0, |v|_0, |w|_0$.

Proof: Let $(u,v,w) \in K$ satisfy $(u,v,w) \le A(u,v,w)$; i.e

$$u(t) \le \lambda \int_0^1 K(t,s) \tilde{f}(u-\zeta, v-\zeta, w-\zeta) ds, \quad t \in [0,1]$$
$$v(t) \le \lambda \int_0^1 K(t,s) \tilde{g}(u-\zeta, v-\zeta, w-\zeta) ds, \quad t \in [0,1]$$
$$w(t) \le \lambda \int_0^1 K(t,s) \tilde{h}(u-\zeta, v-\zeta, w-\zeta) ds, \quad t \in [0,1]$$

where $K(t,s)$ is the Green function of $u'' = -N$ with Dirichlet boundary conditions. Then

$$u(t) \le \lambda C \sup \tilde{f}(r,s,t) : |(r,s,t)| \le |(u,v,w)|_0$$
$$v(t) \le \lambda C \sup \tilde{g}(r,s,t) : |(r,s,t)| \le |(u,v,w)|_0$$
$$w(t) \le \lambda C \sup \tilde{h}(r,s,t) : |(r,s,t)| \le |(u,v,w)|_0$$

where $C = |K|_0$ .This implies

$$|(u,v,w)|_0 \le \lambda C \sup_{0 \le r,s,t \le |(u,v,w)|_0} q(r,s,t) \equiv \lambda C H(|(u,v,w)|_0)$$

where $q(r,s,t) = \max(\tilde{f}(r,s,t), \tilde{g}(r,s,t), \tilde{h}(r,s,t))$ or

$$\frac{H(|(u,v,w)|_0)}{|(u,v,w)|_0} \ge \frac{1}{\lambda C} \tag{2.4}$$

Suppose that $\lambda < (1/(2CH(1)))$. Then $(H(1)/1) < (1/(2\lambda C))$ and since $\lim_{x\to\infty}(H(x)/x) = \infty$, see (A.2), there exists $A_\lambda > 1$ such that

$$\frac{H(A_\lambda)}{A_\lambda} = \frac{1}{2\lambda C} \tag{2.5}$$

From (2.4) and (2.5), we deduce that $|(u,v,w)|_0 \ne A_\lambda$. Since $H(A_\lambda) = (A_\lambda/(2\lambda C)) \ge (1/(2\lambda C)) \to \infty$ as $\lambda \to 0$, $A_\lambda \to \infty$ as $\lambda \to 0$ and therefore $(A_\lambda/\lambda) \to \infty$ as $\lambda \to 0$.

We next prove :

Lemma 2.2. There exists $R_\lambda > A_\lambda$ such that $(u,v,w) \ge A(u,v,w) \Rightarrow |(u,v,w)|_0 \ne R_\lambda$.

Proof. Let $(u,v,w)\in K$ satisfy $(u,v,w)\geq A(u,v,w)$,i.e

$$u(x)\geq \lambda\int_0^1 K(x,y)\tilde{f}(u-\zeta,v-\zeta,w-\zeta)dy, \quad x\in[0,1]$$

$$v(x)\geq \lambda\int_0^1 K(x,y)\tilde{g}(u-\zeta,v-\zeta,w-\zeta)dy, \quad x\in[0,1]$$

and

$$w(x)\geq \lambda\int_0^1 K(x,y)\tilde{h}(u-\zeta,v-\zeta,w-\zeta)dy, \quad x\in[0,1]$$

Suppose that $|(u,v,w)|_0 = |(u,v)|_0$ and let $[c,d]\subset(0,1)$. Then we have

$$(u-\zeta)(x)\geq |u|_0 J(x)-\lambda\tilde{K}J(x)\geq \left(|u|_0-\lambda\tilde{K}\right)\delta,\ x\in[c,d]$$
$$(v-\zeta)(x)\geq |v|_0 J(x)-\lambda\tilde{K}J(x)\geq \left(|v|_0-\lambda\tilde{K}\right)\delta,\ x\in[c,d]$$

where $\delta=\min_{[c,d]}J(x)$. Here, without loss on generality, we assume that $|u|_0>\lambda\tilde{K}$ and $|v|_0>\lambda\tilde{K}$. Hence

$$|w|_0\geq \lambda\tilde{c}\min\begin{cases}\inf\{\tilde{h}(r,s,t):(r,s)\geq\left(\left(|u|_0-\lambda\tilde{K}\right)\delta,\left(|v|_0-\lambda\tilde{K}\right)\delta\right),\ t\geq 0\}\\ \inf\{\tilde{g}(r,s,t):(r,t)\geq\left(\left(|v|_0-\lambda\tilde{K}\right)\delta,\left(|u|_0-\lambda\tilde{K}\right)\delta\right),\ s\geq 0\}\\ \inf\{\tilde{f}(r,s,t):(s,t)\geq\left(\left(|u|_0-\lambda\tilde{K}\right)\delta,\left(|v|_0-\lambda\tilde{K}\right)\delta\right),\ r\geq 0\}\end{cases}$$
$$\equiv \overline{B}\left(|(u,v)|_0\right)$$

where $\bar{c}=\{\min_{[c,d]\times[c,d]}K(x,y)\}(c-d)$. Further,

$$|(u,v)|_0\geq \lambda\tilde{c}\min\begin{cases}\inf\{\tilde{f}(r,s,t):(r,s)\geq\left(\left(|u|_0-\lambda\tilde{K}\right)\delta,\left(|v|_0-\lambda\tilde{K}\right)\delta\right),\ t\geq B\left(|(u,v)|_0\right)\}\\ \inf\{\tilde{g}(r,s,t):(r,t)\geq\left(\left(|u|_0-\lambda\tilde{K}\right)\delta,\left(|v|_0-\lambda\tilde{K}\right)\delta\right),\ s\geq B\left(|(u,v)|_0\right)\}\\ \inf\{\tilde{h}(r,s,t):(s,t)\geq\left(\left(|u|_0-\lambda\tilde{K}\right)\delta,\left(|v|_0-\lambda\tilde{K}\right)\delta\right),\ r\geq B\left(|(u,v)|_0\right)\}\end{cases}$$
$$\equiv \lambda\bar{c}\overline{A}\left(|(u,v)|_0\right)$$

where $B\left(|(u,v)|_0\right)=\left(\overline{B}\left(|(u,v)|_0\right)-\lambda\tilde{K}\right)\delta$ and hence

$$\frac{\overline{A}\left(|u,v|_0\right)}{|(u,v)|_0}\leq\frac{1}{\lambda\bar{c}}.$$

Once again, without loss of generality, we assume $|(u,v)|_0$ is large enough so that $\overline{B}\left(|(u,v)|_0\right)>\lambda\tilde{K}$.

Since $\lim_{x\to\infty}\left(\overline{A}(x)/x\right)=\infty$; see (A.2), there exists $R_\lambda>A_\lambda$ such that $\left(\overline{A}(R_\lambda)/R_\lambda\right)>\left(2/(\lambda\bar{c})\right)$. Consequently, $|(u,v)|_0\neq R_\lambda$. Similarly , $|(u,w)|_0\neq R_\lambda$ if $|(u,v,w)|_0=|(u,w)|_0$ and $|(v,w)|_0\neq R_\lambda$ if $|(u,v,w)|_0=|(v,w)|_0$

Now, using lemma 2.1 and 2.2 and Theorem A, we establish Theorem 1.1.

Proof of Theorem 1.1. From lemmas 2.1,2.2 and Theorem A, it follows that there exists $(\tilde{u},\tilde{v},\tilde{w})\in K$ with $|(\tilde{u},\tilde{v},\tilde{w})|_0\geq A_\lambda$ such that $(\tilde{u},\tilde{v},\tilde{w})=A(\tilde{u},\tilde{v},\tilde{w})$.Without loss of generality, we assume that $|(\tilde{u},\tilde{v},\tilde{w})|=|(\tilde{u},\tilde{v})|_0$.

Then

$$u_\lambda(x)=\tilde{u}_\lambda(x)-\zeta(x)$$
$$\geq\left(|\tilde{u}_\lambda|_0-\lambda\tilde{K}\right)J(x)$$

and

$$v_\lambda(x)=\tilde{v}_\lambda(x)-\zeta(x)$$
$$\geq\left(|\tilde{v}_\lambda|_0-\lambda\tilde{K}\right)J(x)$$

This implies that
$|(u_\lambda, v_\lambda)| = (|(u_\lambda, v_\lambda)|_0 - \lambda\tilde{K})J(x) \geq (A_\lambda - \lambda\tilde{K})J(x) > 0$ for λ small since $A_\lambda \to \infty$ as $\lambda \to 0$ Also ,we obtain

$$|\tilde{w}|_0 \geq \lambda\bar{c}\inf\{\tilde{h}(r,s,t): |(r,s)| \geq (A_\lambda - \lambda\tilde{K})\delta, t \geq 0\} \equiv \lambda\bar{c}\bar{A}_\lambda$$

Where $\bar{c} = \{\min_{[c,d]\times[c,d]} K(x,y)\}(c-d)$. Note that $A_\lambda \to \infty$ as $\lambda \to 0$ and so $\bar{A}_\lambda \to \infty$ as $\lambda \to 0$. Consequently $w_\lambda(x) \geq |\tilde{w}_\lambda|_0 \; J(x) - \lambda\tilde{K}J(x) \geq \lambda(\bar{c}\bar{A}_\lambda - \tilde{K})J(x) > 0$ for λ small .This completes the proof of theorem 1.1□

3. PROOF OF THEOREM 1.2

First we prove

Lemma 3.1: For $\lambda > 0$ large enough, there exists $B_\lambda > 0$ with $(B_\lambda/\lambda) \to \infty$ as $\lambda \to \infty$ such that $(u,v,w) \geq A(u,v,w) \Rightarrow |(u,v,w)|_0 \neq B_\lambda$.

Proof. Let $(u,v,w) \in K$ satisfy $(u,v,w) \geq A(u,v,w)$. Suppose that $|(u,v,w)|_0 = |(u,v)|_0$ .Then, as in lemma 2.2,see(2.6), we obtain

$$|(u,v)|_0 \geq \lambda\bar{c}\bar{A}(|(u,v)|_0). \tag{3.1}$$

Now suppose that $\lambda > (2/(\bar{c}M_1))$; where $M_1 = (M/3) > 0$. Then

$$\frac{\bar{A}(1)}{1} > M_1 > \frac{2}{\lambda\bar{c}}$$

and since $\lim_{z\to\infty}(\bar{A}(z)/z) = 0$; there exists $B_\lambda > 0$ such that

$$\frac{\bar{A}(B_\lambda)}{B_\lambda} = \frac{2}{\lambda\bar{c}}. \tag{3.2}$$

From (3.1), we have $\bar{A}(|(u,v)|_0)/|(u,v)|_0 \leq (1/(\lambda\bar{c})) < (2/(\lambda\bar{c}))$ and hence by (3.2) we deduce that $|(u,v)|_0 \neq B_\lambda$. In a similar way, we obtain $|(u,w)|_0 \neq B_\lambda$ if $|(u,v,w)|_0 = |(u,w)|_0$ and $|(v,w)|_0 \neq B_\lambda$ if $|(u,v,w)|_0 = |(v,w)|_0$. From (3.2), it follows that $B_\lambda \to \infty$ as $\lambda \to \infty$; and since $\lim_{z\to\infty}\bar{A}(z) = \infty$, we obtain $B_\lambda/\lambda \to \infty$ as $\lambda \to \infty$

Lemma 3.2: Let λ be as in lemma 3.1. Then there exists $R_\lambda > B_\lambda$ such that $(u,v,w) \leq A(u,v,w) \Rightarrow |(u,v,w)|_0 \neq R_\lambda$.

Proof. Let $(u,v,w) \in K$ satisfy $(u,v,w) \leq A(u,v,w)$. Then
$|(u,v,w)|_0 \leq \lambda c\sup\{h_1(r,s,t): |(r,s,t)| \leq |(u,v,w)|_0\} \equiv \lambda c\tilde{h}(|(u,v,w)|_0)$ where
$h_1(r,s,t) = \max(\tilde{f}(r,s,t), \tilde{g}(r,s,t), \tilde{h}(r,s,t))$ and $\tilde{h}(z) = \sup\{h_1(r,s,t): 0 \leq r,s,t \leq z\}$.Thus,

$$\frac{\tilde{h}(|(u,v,w)|_0)}{|(u,v,w)|_0} \geq \frac{1}{\lambda c} \text{ while } \lim_{z\to\infty}(\tilde{h}(z)/z) = 0$$

Hence it follows that there exists $R_\lambda > B_\lambda$ such that $|(u,v,w)|_0 \neq R_\lambda$. We now establish Theorem 1.2 by using lemmas 3.1,3.2 and theorem A.

Proof of Theorem 1.2.

It follows from lemmas 3.1,3.2 and theorem A that there exists $(\tilde{u},\tilde{v},\tilde{w}) \in K$ with $|(\tilde{u},\tilde{v},\tilde{w})|_0 \geq B_\lambda$ if λ is large enough .Without loss of generality we can assume that $|(\tilde{u},\tilde{v},\tilde{w})|_0 = |(\tilde{u},\tilde{v})|_0$. Then

$$\begin{aligned} u_\lambda(t) &= \tilde{u}_\lambda(t) - \zeta(t) \\ &\geq |\tilde{u}_\lambda|_0 J(t) - \lambda\tilde{K}J(t) \\ v_\lambda(t) &= \tilde{v}_\lambda(t) - \zeta(t) \\ &\geq |\tilde{v}_\lambda|_0 J(t) - \lambda\tilde{K}J(t) \end{aligned}$$

we get $\left|\, (u_\lambda, v_\lambda)\,\right| = \left(\left|\, (\tilde{u}_\lambda, \tilde{v}_\lambda)\,\right|_0 - \lambda\tilde{K}\right) J(t) \ge \lambda\left(\dfrac{B_\lambda}{\lambda} - \tilde{K}\right) J(t) > 0$

for λ large. Consequently, from $\tilde{w}(t) = \lambda\int_0^1 K(t,s)\tilde{h}(\tilde{u}-\zeta, \tilde{v}-\zeta, \tilde{w}-\zeta)\,ds$ we obtain

$$\left| w \right|_0 \ge \lambda\bar{c}\inf \tilde{h}(r,s,t) : (r,s) \ge \lambda\left[(B_\lambda/\lambda) - \tilde{K}\right]\delta, t \ge 0 = \lambda\bar{c}D_\lambda$$

and

$$\begin{aligned} w_\lambda(t) &= \tilde{w}_\lambda(t) - \zeta(t) \\ &\ge \left| \tilde{w}_\lambda \right|_0 J(t) - \lambda\tilde{K}J(t) \\ &\ge \lambda\{\bar{c}D_\lambda - \tilde{K}\}J(t), \quad t \in (0,1) \end{aligned}$$

Since $D_\lambda \to \infty$ as $\lambda \to \infty$, this completes the proof of theorem 1.2.

4. EXAMPLES

In this section we discuss examples satisfying the hypotheses of Theorems 1.1,1.2.

Example 4.1 . Let

$$f(u,v,w) = u^2v^2w^2 + uvw + u + v + w - 1$$
$$g(u,v,w) = u^3v^3w^3 + u^2v^2w^2 + uvw + u + v + w - 1$$
$$h(u,v,w) = u^4v^4w^4 + u^3v^3w^3 + u^2v^2w^2 + uvw + u + v + w - 1$$

Its clearly (A.1) is satisfied. Further, $f(u,v,w) \ge v + w - 1,\ g(u,v,w) \ge u + w - 1$ and $h(u,v,w) \ge u + v - 1$ and $h^*(n) = n^6 + n^3 + 3n - 1$ and thus (A,2) is satisfied , and hence all the hypotheses of Theorem 1.1 are satisfied.

Example 4.2 . Let

$$f(u,v,w) = u^{1/2}v^{1/3}w^{1/4} + u^{1/2} + v^{1/3} + w^{1/4} - 1$$
$$g(u,v,w) = u^{2/3}v^{3/7}w^{1/3} + u^{1/2} + v^{1/3} + w^{1/4} - 1$$
$$h(u,v,w) = u^{1/4}v^{1/8}w^{1/7} + u^{1/3} + v^{1/3} + w^{1/3} - 1$$

Its clearly (A.1) is satisfied .Further, $f(u,v,w) \ge v^{1/3} + w^{1/4} - 1,\ g(u,v,w) \ge u^{1/2} + w^{1/4} - 1$ and $h(u,v,w) \ge u^{1/3} + v^{1/3} - 1$ and $\tilde{h}(n) = n^{13/12} + n^{1/2} + n^{1/3} + n^{1/4} - 1$ and thus (A,3) is satisfied, and hence all the hypotheses of Theorem1.2 are satisfied.

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