# PARAMETERS IDENTIFICATION AND PROJECTIVE DISLOCATED LAG SYNCHRONIZATION OF LIU CHAOTIC SYSTEM VIA ADAPTIVE CONTROL

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**ABSTRACT**
A new type of chaos synchronization, projective dislocated lag synchronization, is investigated of Liu chaotic system under the framework of response system with five uncertain parameters. Based on adaptive technique, the projective dislocated lag synchronization of Liu chaotic system is achieved by designing a novel nonlinear controller. Furthermore, the parameters identification is realized simultaneously. The conclusion is proved by Lyapunov stability theory and LaSalle's invariance principle. Finally, the numerical simulations are given to demonstrate the effectiveness and feasibility of the proposed method.

**Keywords:** *Projective dislocated lag synchronization, Adaptive control, Parameters identification, Liu chaotic system*

## 1. INTRODUCTION

Chaos synchronization has attracted great interest in recent years, largely because of its potential applications in many practical engineering fields, such as secure communication [1], signal processing [2], image encryption [3], and so on. In the past two decades, a wide variety of schemes have been proposed for chaos synchronization, including linear and nonlinear feedback approach [4,5], adaptive technique [5], backstepping method [6], impulsive control method [7], etc. At present, the researchers are concentrating on the following types of synchronization phenomena: complete synchronization [8], generalized synchronization [9], phase synchronization [10], lag synchronization [11], dislocated synchronization [12] and so on.

In 2004, Liu *et al.* [13] constructed a system of three-dimensional autonomous differential equations with only two quadratic terms. Recently, some literatures have been devoted to control and synchronization of Liu chaotic system. In Refs. [14-16]，the feedback control strategies for Liu chaotic system have been achieved. Hu *et al.* [17] has achieved the synchronization of Liu chaotic system with total parameters unknown by using a single nonlinear controller. In Ref. [18], a active backstepping design is used in controlling, synchronization and tracking Liu chaotic system. Chen and Jia [19] have investigated hybrid projective dislocated synchronization of Liu chaotic systems based on parameters identification. In this paper, we will discuss parameters identification and projective dislocated lag synchronization of Liu chaotic system.

The rest of this paper is organized as follows. Section 2 is problem formulation. In section 3, a general scheme for projective dislocated lag synchronization of Liu chaotic system and parameters identification is proved. Section 4 presents some numerical simulations to show the effectiveness of the proposed scheme. Finally, conclusions are shown.

## 2. PROBLEM FORMULATION

The nonlinear differential equations for describing Liu chaotic system [13] are

$$\begin{cases} \dot{x}_1(t) = a\left(x_2(t) - x_1(t)\right), \\ \dot{x}_2(t) = bx_1(t) - kx_1(t)x_3(t), \\ \dot{x}_3(t) = -cx_3(t) + hx_1^2(t), \end{cases} \tag{1}$$

having a chaotic attractor when $a = 10, b = 40, c = 2.5, h = 4, k = 1$. The phase portrait is shown in Figure 1.

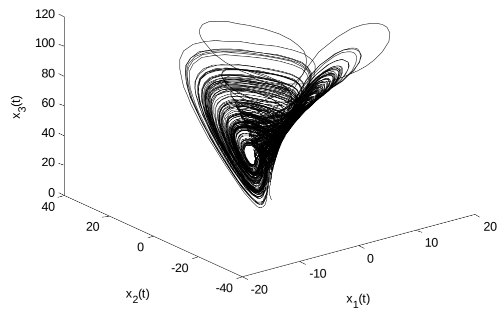



*Figure 1. The phase portrait of Liu chaotic system (1)*

Considering the drive system (1), the response system is controlled Liu chaotic system as following

$$\begin{cases} \dot{y}_1(t) = a_s\left(y_2(t) - y_1(t)\right) + u_1(t), \\ \dot{y}_2(t) = b_s y_1(t) - k_s y_1(t) y_3(t) + u_2(t), \\ \dot{y}_3(t) = -c_s y_3(t) + h_s y_1^2(t) + u_3(t), \end{cases} \tag{2}$$

where $a_s, b_s, c_s, h_s, k_s$ of (2) are unknown parameters which need to be identified in the response system, $U(t) = \left[u_1(t), u_2(t), u_3(t)\right]^T$ is the controller which should be designed such that two systems can be projective dislocated lag synchronized.

Let

$$\begin{cases} e_1(t) = y_1(t) - \lambda x_2(t-\tau), \\ e_2(t) = y_2(t) - \lambda x_3(t-\tau), \\ e_3(t) = y_3(t) - \lambda x_1(t-\tau). \end{cases} \tag{3}$$

Where $\lambda$ is the scaling factor and $\tau$ is the time delay for the errors dynamical system, which satisfy $\lambda \neq 0$ and $\tau > 0$.

Therefore, the goal of parameters identification and projective dislocated lag synchronization is to find an appropriate controller $U(t)$ and parameter adaptive laws of $a_s, b_s, c_s, h_s, k_s$, such that the synchronization errors

$$e_1(t) \to 0, e_2(t) \to 0, e_3(t) \to 0 \text{ as } t \to \infty \tag{4}$$

and the unknown parameters

$$\lim_{t\to\infty} a_s = a, \lim_{t\to\infty} b_s = b, \lim_{t\to\infty} c_s = c, \lim_{t\to\infty} h_s = h, \lim_{t\to\infty} k_s = k. \tag{5}$$

**Remark 1** When $\lambda = 1$, dislocated lag synchronization will appear, When $\lambda = -1$, dislocated lag anti-synchronization will appear.

**Remark 2** Here are another four types of hybrid projective dislocated synchronization errors

$$(\text{I})\begin{cases} e_1(t) = y_1(t) - \lambda x_1(t-\tau), \\ e_2(t) = y_2(t) - \lambda x_3(t-\tau), \\ e_3(t) = y_3(t) - \lambda x_2(t-\tau). \end{cases} (\text{II})\begin{cases} e_1(t) = y_1(t) - \lambda x_2(t-\tau), \\ e_2(t) = y_2(t) - \lambda x_1(t-\tau), \\ e_3(t) = y_3(t) - \lambda x_3(t-\tau). \end{cases} (\text{III})\begin{cases} e_1(t) = y_1(t) - \lambda x_3(t-\tau), \\ e_2(t) = y_2(t) - \lambda x_1(t-\tau), \\ e_3(t) = y_3(t) - \lambda x_2(t-\tau). \end{cases}$$

$$(\text{IV})\begin{cases} e_1(t) = y_1(t) - \lambda x_3(t-\tau), \\ e_2(t) = y_2(t) - \lambda x_2(t-\tau), \\ e_3(t) = y_3(t) - \lambda x_1(t-\tau). \end{cases}$$

For these cases, the discussions are similar to the method given in this paper.

## 3. PROJECTIVE DISLOCATED LAG SYNCHRONIZATION OF LIU CHAOTIC SYSTEM

In this section, based upon the nonlinear adaptive feedback control technique, a systematic design process of parameters identification and projective dislocated lag synchronization of Liu chaotic system under the situation of response system with unknown parameters is provided.
According to the systems (1) and (2), the errors dynamical system can be obtained as follows.

$$\begin{cases}\dot{e}_1(t)=-ae_1(t)-(a_s-a)y_1(t)+a_sy_2(t)+k\lambda x_1(t-\tau)x_3(t-\tau)-b\lambda x_1(t-\tau)-a\lambda x_2(t-\tau)+u_1(t),\\ \dot{e}_2(t)=-ce_2(t)+(b_s-b)y_1(t)-(k_s-k)y_1(t)y_3(t)-ky_1(t)y_3(t)+by_1+cy_2(t)-h\lambda x_1^2(t-\tau)+u_2(t),\\ \dot{e}_3(t)=-ae_3(t)-(c_s-c)y_3(t)+(h_s-h)y_1^2(t)+hy_1^2(t)-(c-a)y_3(t)-a\lambda x_2(t-\tau)+u_3(t).\end{cases}\tag{6}$$

Obviously, projective dislocated lag synchronization of systems (1) and (2) appears if the errors dynamical system (6) has an asymptotically stable equilibrium point $e(t)=0$, where $e(t)=\left[e_1(t),e_2(t),e_3(t)\right]^T$.
Then, we get the following theorem.

**Theorem** Assuming that the Liu chaotic system (1) drives the controlled Liu chaotic system (2), take

$$\begin{cases}u_1(t)=-a_sy_2(t)-k\lambda x_1(t-\tau)x_3(t-\tau)+b\lambda x_1(t-\tau)+a\lambda x_2(t-\tau),\\ u_2(t)=ky_1(t)y_3(t)-by_1(t)-cy_2(t)+h\lambda x_1^2(t-\tau),\\ u_3(t)=-hy_1^2(t)+(c-a)y_3(t)+a\lambda x_2(t-\tau),\end{cases}\tag{7}$$

and parameter adaptive laws

$$\begin{cases}\dot{a}_s=y_1(t)e_1(t),\\ \dot{b}_s=-y_1(t)e_2(t),\\ \dot{c}_s=y_3(t)e_3(t),\\ \dot{h}_s=-y_1^2(t)e_3(t),\\ \dot{k}_s=y_1(t)y_3(t)e_2(t).\end{cases}\tag{8}$$

Systems (1) and (2) can realize projective dislocated lag synchronization and the unknown parameters will be identified, i.e., Eqs. (4) and (5) will be achieved.

**Proof** Eq. (6) can be converted to the following form under the controller (7)

$$\begin{cases}\dot{e}_1(t)=-ae_1(t)-(a_s-a)y_1(t),\\ \dot{e}_2(t)=-ce_2(t)+(b_s-b)y_1(t)-(k_s-k)y_1(t)y_3(t),\\ \dot{e}_3(t)=-ae_3(t)-(c_s-c)y_3(t)+(h_s-h)y_1^2(t).\end{cases}\tag{9}$$

Consider the following Lyapunov function

$$V=\frac{1}{2}\left[e_1^2(t)+e_2^2(t)+e_3^2(t)+(a_s-a)^2+(b_s-b)^2+(c_s-c)^2+(h_s-h)^2+(k_s-k)^2\right],$$

Obviously, $V$ is a positive definite function. Taking its time derivative along with the trajectories of Eqs. (9) and (8) leads to

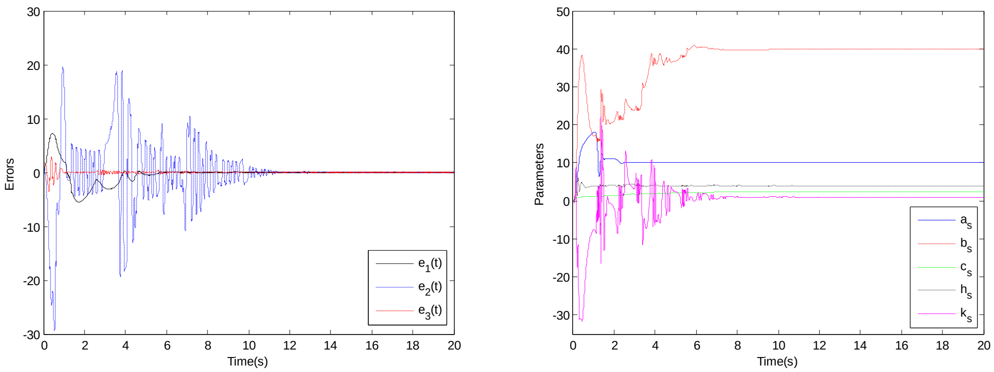
$$\begin{aligned}\dot{V}&=e_1(t)\dot{e}_1(t)+e_2(t)\dot{e}_2(t)+e_3(t)\dot{e}_3(t)+(a_s-a)\dot{a}_s+(b_s-b)\dot{b}_s+(c_s-c)\dot{c}_s+(k_s-k)\dot{k}_s+(h_s-h)h_s\\ &=-ae_1^2(t)-ce_2^2(t)-ae_3^2(t)=-e^TPe\le 0,\end{aligned}\tag{10}$$

where $P=diag\{a,c,a\}$.It is obvious that $\dot{V}=0$ if and only if $e_i(t)=0, i=1,2,3$, namely the set $M=\{e_1(t)=0,$ $e_2(t)=0, e_3(t)=0, a_s=a, b_s=b, c_s=c, h_s=h, k_s=k\}$ is the largest invariant set contained in $E=\{\dot{V}=0\}$ for Eq. (9). So according to the LaSalle's invariance principle [20], starting with arbitrary initial values of Eq. (9), the trajectory converges asymptotically to the set $M$, i.e., $e_1(t)\to 0, e_2(t)\to 0, e_3(t)\to 0, a_s\to a, b_s\to b,, c_s\to c,$ $h_s\to h$ and $k_s\to k$ as $t\to\infty$. This indicates that the projective dislocated lag synchronization of Liu chaotic system is achieved and the unknown parameters $a_s, b_s, c_s, h_s, k_s$ can be successfully identified by using controller (7) and parameter adaptive laws (8). Now the proof is completed.

## 4. NUMERICAL SIMULATIONS

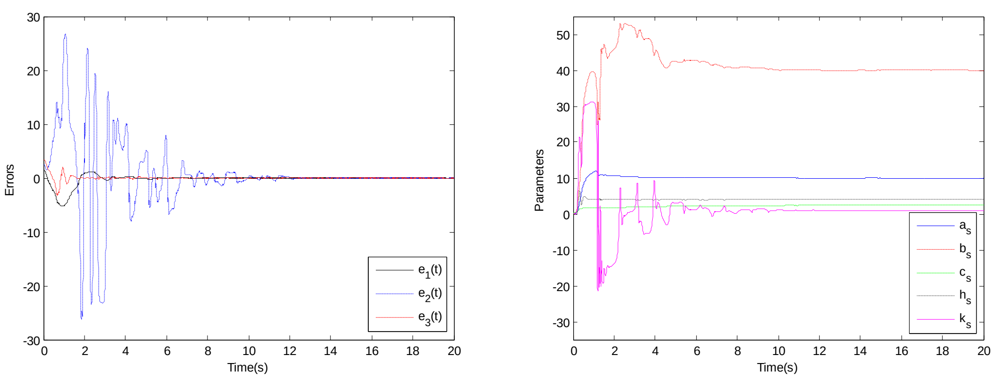
In order to verify the effectiveness and feasibility of the proposed method, we give some numerical simulations about the projective dislocated lag synchronization and parameters identification between systems (1) and (2). In the numerical simulations, all the differential equations are solved by using the fourth-order Runge-Kutta method.

For this numerical simulations, we assume that the initial states of drive system and response system are $x_1(0)=1$, $x_2(0)=1, x_3(0)=1$ and $y_1(0)=1, y_2(0)=2, y_3(0)=3$, the unknown parameters have zero initial condition, the time delay is chosen as $\tau=1$. The drive signals are from the Liu chaotic system (1) with system parameters $a=10$, $b=40, c=2.5, h=4, k=1$ so that it exhibits a chaotic attractor. The simulation results are shown in figures 2 and 3. Figures 2 and 3 display the evolution process of errors and unknown parameters when projective dislocated lag synchronization is realized between systems (1) and (2), respectively, with $\lambda=1$ and -0.5. From figures 2(a) and 3(a), we can see that the errors $e_i(t), i=1,2,3$ converge to zero less than 12 seconds. That is, the two systems achieve the projective dislocated lag synchronization quickly. At the same time, Figure 2(b) and 3(b) show that the unknown parameters $a_s, b_s, c_s, h_s$ and $k_s$, respectively, track $a=10, b=40, c=2.5, h=4$ and $k=1$ when $\lambda=1$ and -0.5. That is to say, the parameters identification is realized simultaneously.




*(a)* *(b)*

*Figure 2. The projective dislocated lag synchronization of the systems (1) and (2) with* $\lambda=1$.
*(a) Time evolution of synchronization errors, (b) Identification process of unknown parameters.*




*(a)* *(b)*

*Figure 3. The projective dislocated lag synchronization of the systems (1) and (2) with* $\lambda=-0.5$.
*(a) Time evolution of synchronization errors, (b) Identification process of unknown parameters.*

## 5. CONCLUSIONS

This paper investigates the adaptive projective dislocated lag synchronization for the Liu chaotic system with the response system parameters unknown. Based on Lyapunov stability theory and LaSalle's invariance principle, the controller and parameter adaptive laws are given to achieve projective dislocated lag synchronization and parameters identification simultaneously. Finally, numerical simulations are provided to demonstrate the effectiveness of the scheme proposed in this work.

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