BAYESIAN AND NON BAYESIAN ESTIMATION OF ERLANG DISTRIBUTION UNDER PROGRESSIVE CENSORING

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ABSTRACT
Based on progressively Type-II censored samples, the maximum likelihood and Bayes estimators for the scale parameter, reliability and cumulative hazard functions are derived. The Bayes estimators are studied under symmetric (squared error) loss function and asymmetric (LINEX and general entropy) loss functions. Tow techniques are used for computing the Bayes estimates; standard Bayes and importance sampling methods. The performance of the estimates are compared by using the mean square error and the relative absolute bias through Monte Carlo simulation study.

Keywords: *Erlang distribution; Importance sampling technique; Monte Carlo Simulation; Progressive Type-II censoring; Symmetric and asymmetric loss functions.*

Mathematics Subject Classification: 62F10; 62F15; 62N02; 62N05.

1. INTRODUCTION

The progressive Type-II censoring scheme is played a vital role in life-testing. This type of censoring allows the experimenter to remove items from the experiment before its end, thus resulting in a saving in cost as well as experimental time. Also, this type of censoring has been attracting in many fields of application, for example, science, engineering and medicine (see, Balakrishnan and Aggarwala (2000)).

The progressive Type-II censoring can be described as follows: n units are placed on a life-testing experiment and only $m\,(<n)$ units are completely observed until failure. The censoring occurs progressively in m stages. These m stages offer failure times of the m completely observed units. At the time of the first failure (the first stage), r_1 of the $n-1$ surviving units are randomly withdrawn from the experiment. At the time of the second failure (the second stage), r_2 of the $n-2-r_1$ surviving units are withdrawn and so on. Finally, at the time of the m^{th} failure (the m^{th} stage), all the remaining $r_m = n-m-r_1-r_2-...-r_{m-1}$ surviving units are withdrawn. We will refer to this as progressive Type-II right censoring with scheme $(r_1, r_2, ..., r_m)$.

The Erlang variate is the sum of a number of exponential variates. It was developed as the distribution of waiting time and message length in telephone traffic. If the durations of individual calls are exponentially distributed, the duration of a succession of calls has an Erlang distribution. The Erlang variate is a gamma variate with an integer shape parameter. The probability density function (pdf) of an Erlang variate is given by

$$f\,(x;\kappa,b) = \frac{(x\,/b)^{\kappa-1} e^{-x/b}}{b\,(\kappa-1)!}, \qquad x \geq 0, b>0, \kappa>0, \tag{1}$$

Where b and κ are the scale and the shape parameters, respectively. Such that κ is an integer number. For more details about this distribution see, Evans et al. (2000).

Several authors interested in estimation of the scale parameter of gamma distribution with known shape parameter, among them Ghosh and Singh (1970) and Berger (1980). Also, Johnson et al. (1994) gave a good review about this distribution in the literature.

Some authors interested in gamma distribution with an integer shape parameter for example, Constantine et al. (1986) studied the estimation of $P(Y < X)$ in the gamma case with known integer-valued shape parameter.

Fang (2001) presented the study of the hyper-Erlang distribution model and its applications in wireless networks and mobile computing systems. Also, the moments of order statistics from nonidentically distributed Erlang variables computed by Abdelkader (2003). Willmot and Lin (2011) presented a review of analytical and computational properties of the mixed Erlang distribution in the context of risk analysis.

In this paper, we consider the Erlang distribution with shape parameter $(\kappa = 3)$ in (1), which has pdf and cumulative distribution function (cdf), respectively, as

$$f(x;b) = \frac{x^2}{2b^3} e^{-x/b}, \quad x \geq 0, \ b > 0, \tag{2}$$

$$F(x;b) = 1 - \frac{e^{-x/b}}{2}\left[\left(\frac{x}{b}+1\right)^2 + 1\right], \ \ x \geq 0, \ b > 0. \tag{3}$$

Also, the reliability and cumulative hazard functions of this distribution are given, respectively, by

$$R(x;b) = \frac{e^{-x/b}}{2}\left[\left(\frac{x}{b}+1\right)^2 + 1\right], \ \ x \geq 0, \ b > 0, \tag{4}$$

$$H(x;b) = -Log[R(x;b)], \ \ x \geq 0, \ b > 0. \tag{5}$$

We will denote the Erlang distribution in (2) by Er(*b*). Also, for simplicity, we will denote the scale parameter, reliability and cumulative hazard functions by, *b*, *R* and *H*, respectively.
Based on progressive Type-II censored samples, we consider the estimation of the scale parameter, reliability and cumulative hazard functions from Er(*b*). In Section 2, the maximum likelihood (ML) estimation for *b, R* and *H* are derived. Also, confidence interval (CI) based on the asymptotic distribution of the ML of *b* are obtained in this section. Bayesian estimation under squared error (SE), LINEX and general entropy (GE) loss functions are discussed in Section 3 by using tow techniques for computing the Bayes estimates. The performance of the estimates are compared by using the mean square error (MSE) and the relative absolute bias (RABias) through Monte Carlo simulation study based on different censoring schemes are investigated in Section 4. Finally, concluding remarks are presented in Section 5.

2. MAXIMUM LIKELIHOOD ESTIMATION

Suppose that $\underline{x} = (x_1, x_2, ..., x_m)$ is a progressive Type-II censored sample from a life test on n items whose lifetimes have an Erlang distribution, Er(*b*), with pdf given in (2), and $r_1, r_2, ..., r_m$ denote the corresponding numbers of units removed (withdrawn) from the test. The likelihood function based on the progressive Type-II censored sample (Balakrishnan and Aggarwala (2000)) is given by

$$l(b;\underline{x}) = A\prod_{i=1}^{m} f(x_i;b)[1-F(x_i;b)]^{r_i}, \tag{6}$$

where $A = n(n-1-r_1)(n-2-r_1-r_2)...(n-\sum_{i=1}^{m-1}(r_i+1)),$

$f(x)$ and $F(x)$ are given, respectively, by (2) and (3).
Substituting (2) and (3), into (6), then the likelihood function for Er(*b*) is

$$l(b;\underline{x}) = A\, 2^{-\sum_{i=1}^{m}(r_i+1)} b^{-3m} \prod_{i=1}^{m} x_i^2 e^{-b^{-1}(1+r_i)x_i}\left[1+\left(1+\frac{x_i}{b}\right)^2\right]^{r_i}. \tag{7}$$

And the natural logarithm of the likelihood function is given by

$$L(b;\underline{x}) = Log(A) - m\,Log(2b^3) + 2\sum_{i=1}^{m} Log(x_i) - b^{-1}\sum_{i=1}^{m}(r_i+1)x_i + \sum_{i=1}^{m} r_i\left\{Log\left[\left(1+\left(1+\frac{x_i}{b}\right)^2\right)/2\right]\right\}. \tag{8}$$

To derive the ML estimation of the unknown parameter *b*, say $\hat{b}_{ML}$, we differentiate (8) with respect to *b* and then solve the following non-linear equation numerically by using Newton-Raphson method

$$-3mb+\sum_{i=1}^{m}(r_i+1)x_i-2\sum_{i=1}^{m}r_i\left[1+\left(1+\frac{x_i}{b}\right)^2\right]^{-1}\left(x_i+\frac{x_i^2}{b}\right)=b^2. \qquad (9)$$

The ML of the reliability function, *R*, and the cumulative hazard function, H, are given by replacing $\hat{b}_{ML}$ in (4) and (5), respectively.

The observed asymptotic variance of ML estimation for the parameter *b* is given by dropping the expectation operator from the element of the inverse of the Fisher information matrix as follows

$$Var(\hat{b}_{ML})=\left(\frac{-\partial^2 l}{\partial b^2}\right)^{-1}$$
$$=\{-b^{-3}[3mb-2\sum_{i=1}^{m}[x_i+r_i\,x_i(1+(1+\delta_i^2)^{-1}$$
$$\times[2b^{-2}(x_i^2+b\,x_i)(\delta_i^2+1)^{-1}\delta_i-x_i\,b^{-1}-2\delta_i\,])]\}^{-1}, \qquad (10)$$

where $\delta_i=x_i\,b^{-1}+1.$

The asymptotic normality of the ML estimator can be used to compute the approximate confidence interval for the parameter *b*. Thus, $(1-\alpha)100\%$ confidence interval for *b* becomes

$$\hat{b}_{ML}\pm z_{\alpha/2}\sqrt{Var(\hat{b}_{ML})},$$

where $z_{\alpha/2}$ is a standard normal percentile.

3. BAYES ESTIMATION

In this section we studied Bayes estimators under three loss functions. One is symmetric (squared error) loss function and the others are asymmetric (LINEX and general entropy) loss functions. The squared error (quadratic) loss function associates equal importance to the losses due to overestimation and underestimation of equal magnitude. However, in real applications, the estimation of the parameters or function as reliability function an overestimation is more serious than the underestimate; thus, the use of a symmetrical loss function is inappropriate. (see, Canfield (1970) and Basu and Ebrahimi (1991)). In this case, an asymmetric loss functions must be considered. The LINEX loss function rises approximately exponentially on one side of zero and approximately linearly on the other side. This function was introduced by Varian (1975) and several authors interested in, among them Soliman (2000, 2002) and Bakoban (2010). The general entropy (GE) loss is also asymmetric loss function which is used in several papers, for example, Dey et al. (1987), Dey and Liu (1992) and Soliman (2005, 2006).
In our estimation of the parameters, we compute the estimators by using two techniques, standard Bayes and importance sampling methods. Importance sampling method is used for numerically approximating integrals. Also, it is viewed as a variance reduction technique. Many authors interested in this method among them, Kundu and Pradhan (2009) and Kundu and Howlader (2010) and Klakattawi et al. (2011). Furthermore, Yaun and Druzdzel (2006) and Tokdar and Kass (2010) gave an algorithm for the importance sampling method.
In Bayes study, we assume that the parameter b has an inverted gamma $(IG(c,\lambda))$ prior distribution with pdf

$$p(b)=\frac{e^{-\lambda/b}\lambda^c}{\Gamma(c)}\left(1/b\right)^{c+1},\quad b>0,c,\lambda>0, \qquad (11)$$

where c is the shape parameter, λ the scale parameter and the variate $1/b$ is a gamma variate with the same shape and scale parameters.
Combining (7) and (11), we obtain the posterior density of b as

$$\pi(b\,|\,\underline{x})=k\,b^{-3m-c-1}e^{-b^{-1}\phi(\lambda)+\zeta(x_i,b)}, \qquad (12)$$

where

$$\zeta(x_i,b)=\sum_{i=1}^{m}\left\{2\,Log(x_i)+r_i\,Log\left[\left(\frac{x_i}{b}+1\right)^2+1\right]\right\}, \qquad (13)$$

$$\phi(\lambda)=\lambda+\sum_{i=1}^{m}x_i\,(1+r_i), \tag{14}$$

$$k^{-1}=\int_0^\infty b^{-3m-c-1}e^{-b^{-1}\phi(\lambda)+\zeta(x_i,b)}db\,. \tag{15}$$

The Bayes estimators of a function of b, say $\varphi(b)$, will be derived under symmetric and asymmetric loss functions in the following subsections. For computing the Bayes estimators, we use the standard Bayes technique and the importance sampling technique. The importance sampling technique is designed to compute the Bayes estimates. The posterior density function (12) can be written as:

$$\pi(b|\underline{x})\propto\frac{[\phi(\lambda)]^{3m+c}}{\Gamma(3m+c)}b^{-3m-c-1}e^{-b^{-1}\phi(\lambda)+\zeta(x_i,b)},$$

$$\propto IG(b;3m+c,\phi(\lambda))\,g(b|\underline{x}), \tag{16}$$

$$\text{where}\quad g(b|\underline{x})=\exp[\zeta(x_i,b)]. \tag{17}$$

The right-hand side of (16), say $\pi_N(b|\underline{x})$, and $\pi(b|\underline{x})$ differ only by the proportionality constant. An approximate Bayes estimators can be computed using $\pi_N(b|\underline{x})$ as a posterior density function based on the importance sampling technique.

3.1 Symmetric Bayes estimates

The quadratic loss for Bayes estimate of a parameter, say $\varphi(b)$, is the posterior mean assuming that exists, say $\tilde{\varphi}_{BS}(b)$, which define as

$$\tilde{\varphi}_{BS}(b)=k\int_0^\infty\varphi(b)\,b^{-3m-c-1}e^{-b^{-1}\phi(\lambda)+\zeta(x_i,b)}db\,. \tag{18}$$

Equation (18) provided the standard Bayes estimate under the quadratic loss function.
According to the importance sampling technique, the approximate Bayes estimator under the quadratic loss function, say $\widehat{\varphi}_{SPBS}(b)$, can be computed by the following Algorithm:

Step 1. Generate $b\sim IG(b;3m+c,\phi(\lambda))$.

Step 2. Repeat Step 1 to obtain $b_1,b_2,...,b_{N_\tau}$.

Step 3. Compute the value

$$\widehat{\varphi}_{SPBS}(b)=\frac{\sum_{\tau=1}^{N_\tau}\varphi(b_\tau)\,g(b_\tau|\underline{x})}{\sum_{\tau=1}^{N_\tau}g(b_\tau|\underline{x})}, \tag{19}$$

$$\text{where}\quad g(b_\tau|\underline{x})=\exp[\zeta(x_i,b_\tau)]. \tag{20}$$

3.2 Asymmetric Bayes estimates

The LINEX loss function may be expressed as

$$\eta_1(\Delta)\propto e^{a\Delta}-a\Delta-1,\ a\neq 0, \tag{21}$$

where $\Delta=\hat{\beta}-\beta.$ The sign and magnitude of the shape parameter a reflects the direction and degree of asymmetry, respectively. (If $a>0,$ the overestimation is more serious than underestimation, and vice-versa). For a closed to zero, the LINEX loss is approximately squared error loss and therefore almost symmetric.
The posterior expectation of the LINEX loss function Equation (21) is

$$E_\beta[\eta_1(\hat{\beta}-\beta)]\propto\exp(a\hat{\beta})E_\beta[\exp(-a\beta)]-a(\hat{\beta}-E_\beta(\beta))-1, \tag{22}$$

where $E_{\beta}(.)$ denoting posterior expectation with respect to the posterior density of $\beta.$ By a result of Zellner (1986), the (unique) Bayes estimator of $\beta,$ denoted by $\hat{\beta}_{BL}$ under the LINEX loss is the value $\hat{\beta}$ which minimizes (22), is given by

$$\hat{\beta}_{BL} = -\frac{1}{a}\log\{E_{\beta}[\exp(-a\beta)]\}, \tag{23}$$

provided that the expectation $E_{\beta}[\exp(-a\beta)]$ exists and is finite [Calabria and Pulcini (1996)].

Next, under the assumption that the minimal loss occurs at $\tilde{u} = u,$ the general entropy for $u = u(b)$ (see, Soliman (2005)) is

$$\eta_2(\tilde{u}, u) \propto \left(\frac{\tilde{u}}{u}\right)^q - q\,Log\left(\frac{\tilde{u}}{u}\right) - 1. \tag{24}$$

When $q > 0,$ a positive error $(\tilde{u} > u)$ causes more serious consequences than a negative error. The Bayes estimate $\tilde{u}_{BG}$ of u under GE loss (24) is

$$\tilde{u}_{BG} = \left[E_u(u^{-q})\right]^{-1/q}, \tag{25}$$

provided that $E_u(u^{-q})$ exists and is finite.

3.2.1 Bayes estimates under LINEX loss function

According to (23), the standard Bayes estimators of $\varphi(b)$ under the LINEX loss function, say $\tilde{\varphi}_{BL}(b),$ is given by

$$\tilde{\varphi}_{BL}(b) = -\frac{1}{a}Log\left[k\int_0^{\infty} b^{-3m-c-1}e^{-[a\varphi(b)+b^{-1}\phi(\lambda)-\zeta(x_i,b)]}db\right], \tag{26}$$

where $\zeta(x_i, b), \phi(\lambda)$ and k^{-1} are given in (13), (14) and (15), respectively.

According to the importance sampling technique, the approximate Bayes estimator under LINEX loss function, say $\widehat{\varphi}_{SPBL}(b),$ can be computed by applying the steps 1 and 2 in the Algorithm that given in subsection (3.1), then the third step can be conducted from (23) as

Step 3. Compute the value

$$\widehat{\varphi}_{SPBL}(b) = -\frac{1}{a}Log\left[\frac{\sum_{\tau=1}^{N_\tau} e^{-a\varphi(b_\tau)}g(b_\tau|\underline{x})}{\sum_{\tau=1}^{N_\tau} g(b_\tau|\underline{x})}\right], \tag{27}$$

where $g(b_\tau|\underline{x})$ is defined in (20).

3.2.2 Bayes estimates under general entropy loss function

The standard Bayes estimate $\tilde{\varphi}_{BG}(b)$ of $\varphi(b)$ under GE loss (25) is given by

$$\tilde{\varphi}_{BG}(b) = \left[k\int_0^{\infty}[\varphi(b)]^{-q}\ b^{-3m-c-1}e^{-[b^{-1}\phi(\lambda)-\zeta(x_i,b)]}db\right]^{-1/q}, \tag{28}$$

where $\zeta(x_i, b), \phi(\lambda)$ and k^{-1} are given in (13), (14) and (15), respectively.

According to the importance sampling technique, the approximate Bayes estimator under GE loss function, say $\widehat{\varphi}_{SPBG}(b),$ can be computed by applying the steps 1 and 2 in the Algorithm that given in subsection (3.1), then the third step can be conducted from (25) as

Step 3. Compute the value

$$\widehat{\varphi}_{SPBG}(b) = \left[\frac{\sum_{\tau=1}^{N_\tau} [\varphi(b_\tau)]^{-q} \, g(b_\tau | \underline{x})}{\sum_{\tau=1}^{N_\tau} g(b_\tau | \underline{x})} \right]^{-1/q}, \qquad (29)$$

where $g(b_\tau | \underline{x})$ is defined in (20).

4. SIMULATION STUDY

In this section we discuss the numerical results of a simulation study testing the performance of the Bayes methods of estimation with the MLEs. Using the Algorithm presented in Balakrihnan and Sandhu (1995), we generate progressively Type-II censored sample of different sizes m from a random sample of different sizes n from Er(*b*) as follows

Step 1. Generate m independent Uniform (0, 1) observations $W_1, W_2, ..., W_m$.

Step 2. Determine the values of the censored scheme r_i, for $i = 1, 2, ..., m$.

Step 3. Set $E_i = 1/(i + \sum_{j=m-i+1}^{m} r_i)$ for $i = 1, 2, ..., m$.

Step 4. Set $V_i = W_i^{E_i}$ for $i = 1, 2, ..., m$.

Step 5. Set $U_i = 1 - \prod_{j=m-i+1}^{m} V_j$ for $i = 1, 2, ..., m$. Then $U_1, U_2, ..., U_m$ is progressive Type-II censored sample from the Uniform (0, 1) distribution.

Step 6. For given values of the prior parameters $(c = 3, \lambda = 2)$, generate a random value for *b* from the inverted gamma distribution whose density function given by Equation (11).

Step 7. Using $b = 1.1707$ obtained in step 6 and $U_1, U_2, ..., U_m$ from Step 5, we can obtain progressive Type-II censored sample $x_1, x_2, ..., x_m$ from Er(*b*) by solving the following equation numerically

$$U_i = 1 - \frac{e^{-x_i/b}}{2}\left[\left(\frac{x_i}{b} + 1\right)^2 + 1\right], \quad i = 1, 2, ..., m.$$

Step 8. Compute the estimates as the following:

a) Estimation of the scale parameter *b*:

Using $x_1, x_2, ..., x_m$ from step 7, the MLE of *b*, say $\hat{b}_{ML}$, were computed by solving Equation (9) numerically using Newton-Raphson method.

The Bayes estimates of *b*, are computed using the results that obtained in Section 3, by setting $\varphi(b) = b$ in the corresponding equations. Equations (18), (26) and (28) are used for standard Bayes estimates, say $\tilde{b}_{BS}, \tilde{b}_{BL}$ and $\tilde{b}_{BG}$, and Equations (19), (27) and (29) are used for approximate Bayes estimates say $\widehat{b}_{BS}, \widehat{b}_{BL}$ and $\widehat{b}_{BG}$, according to the importance sampling technique.

b) Estimation of the reliability function $R(t)$:

Substituting the ML of *b*, $\hat{b}_{ML}$, into (4), we obtain the ML of the reliability function, say $\hat{R}_{ML}$.

The Bayes estimates of *R*, are computed using the results that obtained in Section 3, by setting $\varphi(b) = R(t)$ in the corresponding equations. Equations (18), (26) and (28) are used for standard Bayes estimates, say $\tilde{R}_{BS}, \tilde{R}_{BL}$ and $\tilde{R}_{BG}$, and Equations (19), (27) and (29) are used for approximate Bayes estimates say $\widehat{R}_{BS}, \widehat{R}_{BL}$ and $\widehat{R}_{BG}$, according to the importance sampling technique. At $t_0 = 1$, we have $R(t_0) = 0.94447$ as a true value.

c) Estimation of the cumulative hazard function $H(t)$:

Substituting the ML of *b*, $\hat{b}_{ML}$, into (5), we obtain the ML of the cumulative hazard function, say $\hat{H}_{ML}$.

The Bayes estimates of *H*, are computed using the results that obtained in Section 3, by setting $\varphi(b) = H(t)$ in the corresponding equations. Equations (18), (26) and (28) are used for standard Bayes estimates, say $\tilde{H}_{BS}, \tilde{H}_{BL}$ and $\tilde{H}_{BG}$, and Equations (19), (27) and (29) are used for approximate Bayes estimates say $\widehat{H}_{BS}, \widehat{H}_{BL}$ and $\widehat{H}_{BG}$, according to the importance sampling technique. At $t_0 = 1$, we have $H(t_0) = 0.05713$ as a true value.

All above steps are repeated 1000 times to evaluate the mean square error (MSE) and the absolute relative bias (RABias) of the estimates, where

$$\text{MSE}(\hat{\alpha}) = E(\hat{\alpha} - \alpha)^2 \quad \text{and} \quad \text{RABias}(\hat{\alpha}) = \left|\frac{\hat{\alpha} - \alpha}{\alpha}\right|.$$

The computational results are presented in Tables (2-8). All results are obtained by using Mathematica 7.0.

Table 1 provides different censoring schemes for different sample sizes $n = 10, 20, 30, 40, 50$ and 100. For simplicity in notation, we denoted the censoring scheme (0, 0, 0, 0, 0, 0, 3) by $(6^{*0}, 3)$.

The MSE and RABias of the ML estimates of *b*, *R* and *H* for different censoring schemes are represented in Table 2. Tables 3, 4 and 5 contains the standard Bayes estimates of *b*, *R* and *H*, respectively. Also, Tables 6, 7 and 8 contains the approximate Bayes estimates of *b*, *R* and *H*, respectively, according to the importance sampling technique.

Table 1. Censoring schemes (C.S.) $r_i, i = 1, 2, ..., m$ for sample size *n* and observed sample size *m*.

n	***m***	***S**_i_*	**C.S.**	***n***	***m***	***S*_i_**	**C.S.**
10	7	S_1	$(6^{*0}, 3)$	40	30	S_7	$(1, 0, 2, 2^{*0}, 1, 0, 2, 3^{*0}, 1, 3^{*0}, 1, 2^{*0}, 1, 10^{*0}, 1)$
10	9	S_2	$(1, 8^{*0})$	40	10	S_8	$(9^{*0}, 30)$
20	5	S_3	$(15, 4^{*0})$	50	10	S_9	$(40, 9^{*0})$
20	10	S_4	$(9^{*0}, 10)$	50	20	S_{10}	$(15^{*2}, 5^{*0})$
30	15	S_5	$(4, 0, 2, 2^{*0}, 4, 3^{*0}, 2, 0, 1, 0, 1, 1$	100	20	S_{11}	$(80, 19^{*0})$
30	20	S_6	$(0, 1, 4^{*0}, 2, 3^{*0}, 2, 2^{*0}, 3, 2^{*0}, 1, 2^{*0}, 1)$	100	50	S_{12}	$(49^{*0}, 50)$

Table 2. MSEs and RABias (between parentheses) of ML estimates for the scale parameter, *R* and *H*.

n	***m***	***S*_i_**	$\hat{b}_{ML}$	$\hat{R}_{ML}$	$\hat{H}_{ML}$
10	7	S_1	0.06304 (0.00331)	0.00137 (0.01156)	0.00168 (0.21655)
10	9	S_2	0.04842 (0.00296)	0.00093 (0.00776)	0.00111 (0.14533)
20	5	S_3	0.07239 (0.00956)	0.00186 (0.01468)	0.00234 (0.27668)
20	10	S_4	0.03774 (0.00123)	0.00064 (0.0064)	0.00075 (0.11852)
30	15	S_5	0.02756 (0.00754)	0.00047 (0.0057)	0.00055 (0.10458)
30	20	S_6	0.02087 (0.00393)	0.00032 (0.0029)	0.00037 (0.05397)
40	10	S_7	0.03064 (0.01015)	0.00058 (0.0068)	0.00069 (0.125)
40	30	S_8	0.01304 (0.00255)	0.00019 (0.0025)	0.00022 (0.04569)
50	10	S_9	0.02856 (0.00465)	0.00051 (0.0056)	0.0006 (0.10322)
50	20	S_{10}	0.01876 (0.00133)	0.0003 (0.00337)	0.00035 (0.06207)
100	20	S_{11}	0.02052 (0.00100)	0.0003 (0.00318)	0.00034 (0.05872)
100	50	S_{12}	0.00688 (0.00079)	0.00009 (0.00123)	0.00011 (0.02245)

Table 3. MSEs and RABias (between parentheses) of Bayesian estimates for the scale parameter using standard Bayes method.

n	m	S_i	$\tilde{b}_{BS}$	$\tilde{b}_{BL}$, a=-1	$\tilde{b}_{BL}$, a=0.001	$\tilde{b}_{BL}$, a=1	$\tilde{b}_{BG}$, q=-3	$\tilde{b}_{BG}$, q=-1	$\tilde{b}_{BG}$, q=3
10	7	S_1	0.05411 (0.01269)	0.05806 (0.01192)	0.05410 (0.01271)	0.04633 (0.03478)	0.05734 (0.02646)	0.05411 (0.01269)	0.05157 (0.08255)
10	9	S_2	0.04358 (0.01502)	0.04897 (0.00833)	0.04358 (0.01504)	0.04145 (0.03561)	0.04849 (0.02146)	0.04358 (0.01502)	0.04614 (0.07869)
20	5	S_3	0.05897 (0.01886)	0.0662 (0.0185)	0.05896 (0.0189)	0.05888 (0.03886)	0.06478 (0.03759)	0.05897 (0.01886)	0.06682 (0.10077)
20	10	S_4	0.03430 (0.00616)	0.03432 (0.00183)	0.03429 (0.00617)	0.03237 (0.02046)	0.03406 (0.01171)	0.03430 (0.00616)	0.03445 (0.05269)
30	15	S_5	0.02575 (0.01131)	0.02625 (0.01089)	0.02575 (0.01132)	0.02562 (0.01502)	0.0263 (0.01819)	0.02575 (0.01131)	0.02671 (0.03925)
30	20	S_6	0.01969 (0.0003)	0.01883 (0.00586)	0.01969 (0.0003)	0.01715 (0.0149)	0.01885 (0.01168)	0.01969 (0.0003)	0.01807 (0.03424)
40	10	S_7	0.02844 (0.01155)	0.02884 (0.01046)	0.02844 (0.01156)	0.02445 (0.01926)	0.02888 (0.01884)	0.02844 (0.01155)	0.02616 (0.0466)
40	30	S_8	0.01255 (0.0052)	0.01383 (0.00176)	0.01255 (0.00521)	0.01265 (0.00761)	0.01381 (0.00594)	0.01255 (0.0052)	0.01301 (0.02162)
50	10	S_9	0.02661 (0.00542)	0.0398 (0.01943)	0.02661 (0.00543)	0.03912 (0.02348)	0.03985 (0.03062)	0.02661 (0.00542)	0.04176 (0.06023)
50	20	S_{10}	0.01784 (0.00374)	0.01857 (0.01246)	0.01784 (0.00375)	0.01857 (0.00911)	0.01868 (0.01782)	0.01784 (0.00374)	0.01909 (0.02706)
100	20	S_{11}	0.01932 (0.00275)	0.02169 (0.01013)	0.01932 (0.00276)	0.02122 (0.01911)	0.02176 (0.01632)	0.01932 (0.00275)	0.0224 (0.03963)
100	50	S_{12}	0.00675 (0.00180)	0.00674 (0.00280)	0.00675 (0.00181)	0.0068 (0.00749)	0.00672 (0.00067)	0.00675 (0.00180)	0.00696 (0.01463)

Table 4. MSEs and RABias of Bayesian estimates for the reliability function using standard Bayes method.

n	m	S_i	$\tilde{R}_{BS}$	$\tilde{R}_{BL}$, a=-1	$\tilde{R}_{BL}$, a=0.001	$\tilde{R}_{BL}$, a=1	$\tilde{R}_{BG}$, q=-3	$\tilde{R}_{BG}$, q=-1	$\tilde{R}_{BG}$, q=3
10	7	S_1	0.00165 (0.02088)	0.00161 (0.02016)	0.00165 (0.02088)	0.00145 (0.02022)	0.00153 (0.01938)	0.00165 (0.02088)	0.00167 (0.02256)
10	9	S_2	0.0013 (0.01817)	0.00125 (0.01769)	0.0013 (0.01817)	0.00132 (0.01896)	0.00119 (0.01705)	0.0013 (0.01817)	0.0015 (0.02097)
20	5	S_3	0.00222 (0.02658)	0.00208 (0.02517)	0.00222 (0.02658)	0.00233 (0.02634)	0.00195 (0.02401)	0.00222 (0.02658)	0.00286 (0.03019)
20	10	S_4	0.00076 (0.01245)	0.0008 (0.01327)	0.00076 (0.01245)	0.00077 (0.01264)	0.00077 (0.01286)	0.00076 (0.01245)	0.00084 (0.01383)
30	15	S_5	0.00055 (0.01026)	0.00047 (0.00816)	0.00055 (0.01026)	0.00056 (0.00959)	0.00046 (0.0079)	0.00055 (0.01026)	0.0006 (0.01039)
30	20	S_6	0.00036 (0.00653)	0.00034 (0.00661)	0.00036 (0.00653)	0.00033 (0.00733)	0.00033 (0.00642)	0.00036 (0.00653)	0.00035 (0.00791)
40	10	S_7	0.00067 (0.01154)	0.00058 (0.00965)	0.00067 (0.01155)	0.00059 (0.01053)	0.00057 (0.00934)	0.00067 (0.01154)	0.00063 (0.01145)
40	30	S_8	0.00021 (0.00515)	0.00022 (0.00506)	0.00021 (0.00515)	0.00022 (0.00479)	0.00022 (0.00493)	0.00021 (0.00515)	0.00023 (0.00517)
50	10	S_9	0.00058 (0.01000)	0.00083 (0.01231)	0.00058 (0.01000)	0.00109 (0.01538)	0.0008 (0.01185)	0.00058 (0.01000)	0.00121 (0.01691)
50	20	S_{10}	0.00034 (0.00662)	0.0003 (0.00515)	0.00034 (0.00662)	0.00034 (0.00651)	0.00029 (0.00498)	0.00034 (0.00662)	0.00036 (0.00703)
100	20	S_{11}	0.00035 (0.00708)	0.00037 (0.00668)	0.00035 (0.00708)	0.00042 (0.0088)	0.00037 (0.00647)	0.00035 (0.00708)	0.00044 (0.00945)
100	50	S_{12}	0.0001 (0.00251)	0.0001 (0.00303)	0.0001 (0.00251)	0.00011 (0.00297)	0.0001 (0.00297)	0.0001 (0.00251)	0.00011 (0.00315)

Table 5. MSEs and RABias of Bayesian estimates for the cumulative hazard function using standard Bayes method.

n	m	S_i	$\tilde{H}_{BS}$	$\tilde{H}_{BL}$, a=-1	$\tilde{H}_{BL}$, a=0.001	$\tilde{H}_{BL}$, a=1	$\tilde{H}_{BG}$, q=-3	$\tilde{H}_{BG}$, q=-1	$\tilde{H}_{BG}$, q=3
10	7	S_1	0.00213 (0.39653)	0.00229 (0.40953)	0.00213 (0.39652)	0.00171 (0.35754)	0.00373 (0.67328)	0.00213 (0.39653)	0.00067 (0.16240)
10	9	S_2	0.00169 (0.34299)	0.00169 (0.35509)	0.00169 (0.34298)	0.00155 (0.33551)	0.00276 (0.58753)	0.00169 (0.34299)	0.00064 (0.12357)
20	5	S_3	0.00297 (0.50997)	0.00314 (0.52356)	0.00297 (0.50994)	0.0028 (0.46795)	0.00567 (0.88618)	0.00297 (0.50997)	0.0009 (0.26563)
20	10	S_4	0.00092 (0.23244)	0.00103 (0.26071)	0.00092 (0.23243)	0.0009 (0.22296)	0.00156 (0.423)	0.00092 (0.23244)	0.00047 (0.09998)
30	15	S_5	0.00066 (0.18982)	0.00058 (0.16023)	0.00066 (0.18981)	0.00065 (0.16946)	0.00082 (0.27446)	0.00066 (0.18982)	0.0004 (0.06301)
30	20	S_6	0.00043 (0.12119)	0.00041 (0.12852)	0.00043 (0.12118)	0.00038 (0.1287)	0.00056 (0.21778)	0.00043 (0.12119)	0.00025 (0.0515)
40	10	S_7	0.00081 (0.21438)	0.00073 (0.19002)	0.00081 (0.21438)	0.00068 (0.18552)	0.00106 (0.32043)	0.00081 (0.21438)	0.00039 (0.08537)
40	30	S_8	0.00025 (0.09438)	0.00026 (0.09706)	0.00025 (0.09438)	0.00025 (0.08413)	0.00034 (0.16007)	0.00025 (0.09438)	0.00019 (0.04161)
50	10	S_9	0.0007 (0.1859)	0.00109 (0.24719)	0.0007 (0.18589)	0.00128 (0.27273)	0.00172 (0.43121)	0.0007 (0.1859)	0.00061 (0.10735)
50	20	S_{10}	0.0004 (0.12227)	0.00036 (0.10138)	0.0004 (0.12227)	0.00039 (0.11471)	0.00047 (0.18207)	0.0004 (0.12227)	0.00027 (0.05109)
100	20	S_{11}	0.00041 (0.13093)	0.00045 (0.13082)	0.00041 (0.13092)	0.00048 (0.15492)	0.00062 (0.226)	0.00041 (0.13093)	0.0003 (0.04066)
100	50	S_{12}	0.00011 (0.04586)	0.00012 (0.057)	0.00011 (0.04586)	0.00012 (0.05213)	0.00014 (0.0881)	0.00011 (0.04586)	0.0001 (0.0107)

Table 6. MSEs and RABias (between parentheses) of Bayesian estimates for the scale parameter using importance sampling method.

n	m	S_i	$\widehat{b}_{BS}$	$\widehat{b}_{BL}$, a=-1	$\widehat{b}_{BL}$, a=0.001	$\widehat{b}_{BL}$, a=1	$\widehat{b}_{BG}$, q=-3	$\widehat{b}_{BG}$, q=-1	$\widehat{b}_{BG}$, q=3
10	7	S_1	0.054582 (0.01242)	0.05844 (0.01262)	0.05457 (0.01244)	0.04616 (0.03432)	0.05772 (0.02711)	0.054582 (0.01242)	0.0513 (0.08172)
10	9	S_2	0.04361 (0.01459)	0.04909 (0.00855)	0.04361 (0.01461)	0.04151 (0.03578)	0.04861 (0.02168)	0.04361 (0.01459)	0.04617 (0.07886)
20	5	S_3	0.06589 (0.00317)	0.07263 (0.03487)	0.06589 (0.0032)	0.06546 (0.01648)	0.07117 (0.0516)	0.06589 (0.00317)	0.06958 (0.06357)
20	10	S_4	0.03757 (0.00487)	0.03764 (0.01044)	0.03757 (0.00486)	0.03635 (0.00626)	0.03729 (0.01918)	0.03757 (0.00487)	0.03742 (0.03104)
30	15	S_5	0.02824 (0.00238)	0.02862 (0.01801)	0.02824 (0.00239)	0.02746 (0.00408)	0.0286 (0.02447)	0.02824 (0.00238)	0.02804 (0.02309)
30	20	S_6	0.02033 (0.00217)	0.0192 (0.00689)	0.02033 (0.00217)	0.01771 (0.01408)	0.01919 (0.01257)	0.02033 (0.00217)	0.01877 (0.0324)
40	10	S_7	0.12116 (0.22708)	0.13235 (0.24223)	0.12116 (0.22708)	0.11979 (0.22827)	0.13296 (0.2435)	0.12116 (0.22708)	0.11642 (0.22331)
40	30	S_8	0.01265 (0.0046)	0.01395 (0.002)	0.01265 (0.00461)	0.01269 (0.00692)	0.01393 (0.00614)	0.01265 (0.0046)	0.01305 (0.02072)
50	10	S_9	0.25905 (0.38106)	0.06185 (0.08236)	0.25905 (0.38106)	0.05813 (0.0508)	0.06236 (0.08905)	0.25905 (0.38106)	0.05668 (0.03127)
50	20	S_{10}	0.02911 (0.06212)	0.03061 (0.07425)	0.02911 (0.06212)	0.02975 (0.05981)	0.03097 (0.07654)	0.02911 (0.06212)	0.0286 (0.05308)
100	20	S_{11}	0.03924 (0.07471)	0.04525 (0.08854)	0.03924 (0.07470)	0.04158 (0.06653)	0.04556 (0.09163)	0.03924 (0.07471)	0.04082 (0.05743)
100	50	S_{12}	0.03558 (0.13524)	0.03406 (0.13104)	0.03558 (0.13524)	0.03526 (0.1337)	0.03415 (0.13129)	0.03558 (0.13524)	0.03491 (0.1328)

Table 7. MSEs and RABias (between parentheses) of Bayesian estimates for the reliability function using importance sampling method.

n	m	S_i	$\hat{R}_{BS}$	$\hat{R}_{BL}$, a=-1	$\hat{R}_{BL}$, a=0.001	$\hat{R}_{BL}$, a=1	$\hat{R}_{BG}$, q=-3	$\hat{R}_{BG}$, q=-1	$\hat{R}_{BG}$, q=3
10	7	S_1	0.00166866 (0.02091)	0.00161 (0.02005)	0.00166872 (0.0209107)	0.00144 (0.02005)	0.00153 (0.01927)	0.00166866 (0.02091)	0.00165 (0.02233)
10	9	S_2	0.0013 (0.0181)	0.00125 (0.01766)	0.0013 (0.0181)	0.00133 (0.01901)	0.00119 (0.01702)	0.0013 (0.0181)	0.0015 (0.02102)
20	5	S_3	0.00211 (0.02258)	0.00192 (0.02049)	0.00211 (0.02258)	0.0023 (0.02176)	0.00182 (0.01964)	0.00211 (0.02258)	0.00269 (0.02426)
20	10	S_4	0.00076 (0.0102)	0.00082 (0.01128)	0.00076 (0.0102)	0.00075 (0.01009)	0.0008 (0.01097)	0.00076 (0.0102)	0.0008 (0.01089)
30	15	S_5	0.00056 (0.00857)	0.0005 (0.00665)	0.00056 (0.00857)	0.00055 (0.00761)	0.00049 (0.00646)	0.00056 (0.00857)	0.00058 (0.00818)
30	20	S_6	0.00037 (0.0062)	0.00034 (0.00636)	0.00037 (0.0062)	0.00034 (0.00723)	0.00034 (0.00618)	0.00037 (0.0062)	0.00036 (0.00777)
40	10	S_7	0.00057 (0.01871)	0.00058 (0.01962)	0.00057 (0.01871)	0.00057 (0.0192)	0.00058 (0.01963)	0.00057 (0.01871)	0.00057 (0.01915)
40	30	S_8	0.00022 (0.00505)	0.00022 (0.00501)	0.00022 (0.00505)	0.00022 (0.00469)	0.00022 (0.00488)	0.00022 (0.00505)	0.00023 (0.00506)
50	10	S_9	0.00089 (0.02922)	0.00065 (0.00111)	0.00089 (0.02922)	0.00083 (0.00346)	0.00064 (0.00094)	0.00089 (0.02922)	0.00087 (0.00404)
50	20	S_{10}	0.00028 (0.00367)	0.00026 (0.00492)	0.00028 (0.00367)	0.00029 (0.0037)	0.00026 (0.00496)	0.00028 (0.00367)	0.00029 (0.00358)
100	20	S_{11}	0.00035 (0.00387)	0.00036 (0.0047)	0.00035 (0.00387)	0.00038 (0.00296)	0.00036 (0.00478)	0.00035 (0.00387)	0.00039 (0.00274)
100	50	S_{12}	0.00025 (0.01442)	0.00024 (0.01395)	0.00025 (0.01442)	0.00025 (0.01428)	0.00024 (0.01395)	0.00025 (0.01442)	0.00025 (0.01427)

Table 8. MSEs and RABias (between parentheses) of Bayesian estimates for the cumulative hazard function using importance sampling method.

n	m	S_i	$\hat{H}_{BS}$	$\hat{H}_{BL}$, a=-1	$\hat{H}_{BL}$, a=0.001	$\hat{H}_{BL}$, a=1	$\hat{H}_{BG}$, q=-3	$\hat{H}_{BG}$, q=-1	$\hat{H}_{BG}$, q=3
10	7	S_1	0.00215 (0.39721)	0.00229 (0.40762)	0.00215 (0.39719)	0.00169 (0.35463)	0.00375 (0.66629)	0.00215 (0.39721)	0.00067 (0.16229)
10	9	S_2	0.0017 (0.34187)	0.00169 (0.35461)	0.0017 (0.34186)	0.00156 (0.33644)	0.00277 (0.58689)	0.0017 (0.34187)	0.00065 (0.11989)
20	5	S_3	0.00277 (0.43211)	0.00281 (0.42164)	0.00277 (0.43209)	0.00283 (0.39273)	0.00439 (0.65004)	0.00277 (0.43211)	0.00093 (0.27028)
20	10	S_4	0.00093 (0.19114)	0.00104 (0.22049)	0.00093 (0.19113)	0.00088 (0.17992)	0.00139 (0.32717)	0.00093 (0.19114)	0.00049 (0.10783)
30	15	S_5	0.00067 (0.15913)	0.00061 (0.13107)	0.00067 (0.15913)	0.00064 (0.13576)	0.00078 (0.21038)	0.00067 (0.15913)	0.00041 (0.07073)
30	20	S_6	0.00043 (0.11517)	0.00042 (0.12364)	0.00043 (0.11516)	0.00039 (0.12713)	0.00056 (0.20413)	0.00043 (0.11517)	0.00025 (0.05127)
40	10	S_7	0.00062 (0.3218)	0.00063 (0.33709)	0.00062 (0.3218)	0.00062 (0.33074)	0.00061 (0.32677)	0.00062 (0.3218)	0.0007 (0.3716)
40	30	S_8	0.00025 (0.09257)	0.00027 (0.09617)	0.00025 (0.09257)	0.00025 (0.08236)	0.00034 (0.15734)	0.00025 (0.09257)	0.00019 (0.04266)
50	10	S_9	0.00097 (0.50283)	0.00079 (0.03477)	0.00097 (0.50283)	0.00098 (0.06599)	0.00094 (0.10557)	0.00097 (0.50283)	0.00066 (0.1718)
50	20	S_{10}	0.00032 (0.06066)	0.00029 (0.08157)	0.00032 (0.06066)	0.00032 (0.06262)	0.00029 (0.06016)	0.00032 (0.06066)	0.00033 (0.13374)
100	20	S_{11}	0.0004 (0.06299)	0.00041 (0.07521)	0.0004 (0.06299)	0.00043 (0.04922)	0.00044 (0.04121)	0.0004 (0.06299)	0.00038 (0.14642)
100	50	S_{12}	0.00027 (0.24994)	0.00026 (0.24168)	0.00027 (0.24994)	0.00027 (0.24768)	0.00026 (0.2392)	0.00027 (0.24994)	0.00028 (0.25425)

5. CONCLUSIONS

In this paper we have presented Erlang distribution with shape parameter $\kappa = 3.$ Based on progressive Type-II censored samples drawn from Er(*b*), we have computed the ML and Bayes estimates of *b*, reliability, *R*, and cumulative hazard, *H*, functions. Under SE, LINEX and GE loss functions Bayesian estimates are computed by using tow techniques; standard Bayes and importance sampling. The performance of the estimates are conducted by using the MSE and RABias through Monte Carlo simulation study based on different censoring schemes.

From the results in Tables (2-8), we observe the following:

1. All of the obtained results can be specialized to both the complete sample case by taking ($m = n, r_i = 0, i = 1,2,3,...,m$) and the Type-II right censored sample for ($r_i = 0, i = 1,2,3,...,m-1, r_m = n-m$).
2. For fixed *n*, the MSEs of the estimates are decreasing as the observed sample proportion m/n is increasing.
3. The MSEs and RABiases of the Bayes estimates under LINEX loss function when the LINEX constant is close to zero, ($a = 0.001$), are very similar to their corresponding MSEs and RABiases under squared error loss function.
4. The MSEs and RABiases of the Bayes estimates under GE loss function when $q = -1$, are very similar to their corresponding MSEs and RABiases under squared error loss function.
5. For $S_1, S_2, ..., S_6,$ the results show that the standard Bayes and importance sampling techniques are given resulting estimates very close to each other.
6. The Bayesian estimates of *b* are similar to each other based on MSE. On the other hand, based on RABias, BS estimates have smaller values than the others, also, in some cases the standard Bayes method for $S_1, S_2, ..., S_8,$ BL, $a = -1$ have the smallest values. But ML estimates of *b* performs the best based on RABias for the two techniques.
7. By comparing the Bayesian estimates, BG estimates, $q = -3,$ of *R* have the minimum MSEs and RABiases. Furthermore, the ML estimates have the minimum MSEs and RABiases by comparing all estimates.
8. BG estimates, $q = 3,$ of *H* have the minimum RABiases for most cases.

From the previous discussion, we conclude that the importance sampling technique performs as well as standard Bayes technique. The BG estimates performs better than the Bayesian estimates for estimating *R* and *H*. Based on RABias, we recommend the ML estimator for estimating *b*. Also, we recommend ML estimators for estimating *R*. But we prefer to use BG estimators to estimate *H*.

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