DISSIPATIVE ANALYSIS OF CONTINUOUS-TIME SYSTEMS WITH TWO ADDITIVE TIME-VARYING DELAYS

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ABSTRACT

This paper presents an application of dissipative concept for stability analysis of continuous-time system with two additive time-varying delays in the state. Our attention is focused on analysis of whether the continuous-time system with two additive time-varying delays in the state is asymptotically stable and dissipative. By exploiting Lyapunov-Krasovski functional and introducing free weighting matrix variables, the stability condition is derived by using linear matrix inequality (LMI) techniques.

Keywords: *Dissipative analysis, linear matrix inequality (LMI), time delay systems, networked control systems*

1. INTRODUCTION

The study of dissipative concept which is used to analyze and design of control systems was initially developed by Willems [1]. A system has the dissipative property if it always dissipates the energy. Many physical systems have input-output properties related to the conservation, dissipation and transport energy. Willems in [1] stated the following definition.

Definition 1. System $\dot{x} = f(x, w)$, $z = g(x, w)$, where x is the system state, w represents input to the system, z is the system output, is dissipative with respect to the supply function s(w, z), if there exists a storage function, $V(x) \ge 0$, such that the dissipation inequality

$$V\big(x(t_1)\big) \le V\big(x(t_0)\big) + \int_{t_0}^{t_1} s\big(w(t), z(t)\big)dt \tag{1}$$

holds along all possible trajectories of the system and for all $t_0 \le t_1$.

In the differential form, (1) is equivalent to the differential dissipation inequality,

$$\dot{V}(x) \le s\big(w(t), z(t)\big) \tag{2}$$

Equation (2) states that the rate of change of the stored energy is less than or equal to the input power, the difference being the rate of the energy dissipation. Willems in [2] showed that the notion of a dissipative system is a natural generalization of a Lyapunov functon. Analysis of dissipativity is a problem of finding a storage function (as a Lyapunov function candidate) which satisfy (2) with respect to a certain supply rate. In general, supply function is stated in quadratic function defined by

$$\begin{aligned} s(w(t), z(t)) &= \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} \\ &= z^T(t)Qz(t) + w^T(t)S^T z(t) + z^T(t)Sw(t) + w^T(t)Rw(t) \end{aligned} \tag{3}$$

Here, Q, S and R are real matrices with appropriate dimension and Q and R are symmetric matrices. The dissipative inequality (2) with supply function (3) is quite general, covering bounded real lemma and positive real (or passivity-based) as special cases.

Remark 1. Dissipativity analysis with quadratic supply function is quite general which includes H_∞ and passivity as special cases.

(1) H_∞ performance analysis can be covered by supply function (3) when $Q = -I$, $S = 0$ and $R = \gamma^2 I$.

(2) Passive systems are dissipative with respect to supply function (3) with $R = Q = 0$ and $S = I$.

Without loss of generality, we shall make the following assumption.

Assumption 1. $Q \le 0$

The H∞ approach builds on the small gain theorem whereas the positive real approach relies on the positivity theorem. In H∞ control, the small gain theorem is used to ensure robust stability by requiring that the loop gain is less than one at all frequencies. In this scheme, phase information is not used in guaranteeing stability. Phase information is considered in positivity theory which is widely used in the analysis of passive control systems. In the positivity theorem, a positive real system has its phase less than 90º so that the loop transfer function of the negative feedback connection of two positive real systems has a phase-lag less than 180º. This guarantees stability irrespective of the loop gain. Both the small-gain theorem and positivity theorem deal with gain and phase performances separately, and therefore may lead to conservative results in application. Dissipativeness provides an appropriate framework for a less conservative robust controller design, especially in application where both gain and phase performances are considered [3].

Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect. When information or energy is physically transmitted from one place to another, there is a delay associated with the transmission [4]. It is well known that the presence of time-delay is a source of instability [4,6]. Analysis and synthesis of time delay systems has been studied in a number of works. Robust H∞ control for system with single input/output delay was addressed in [4]. Some basic theories of stability and stabilization of systems with time-delay was presented in [7]. Stability analysis, robust H∞ control design and H∞ filter design of time-delay systems were presented in [8], and references therein. Dissipative controller for the system with delay in the state was developed in [3]. However, this work was restricted to system represented by a single delay. In [9], new model for time delay systems was proposed, in the form $\dot{x}(t) = Ax(t) + A_d x(t - \tau_1(t) - \tau_2(t))$, preliminary result on its stability analysis was addressed. The new model is motivated by practical situation in Networked Control Systems (NCSs), where τ1(t) is the time-delay from sensor to the controller and τ2(t) is the time-delay from controller to the actuator. Based on such a model, a new stability condition and H∞ performance were investigated in [10]. Stability analysis for uncertain systems based on such a model were presented in [11]. Subsequently, [12] derived a new and improved delay-dependent stability condition and providing less conservative delay upper bound compared to those of [5,9,10].

It is worth noting that in [12], only stability is analyzed, while the H∞ performance and dissipativity of the system have not been investigated. Following the work of [12], it is our focus in this paper to investigate the stability and dissipativity of continuous-time system with two time-varying delays in the state. By exploiting Lyapunov-Krasovski functional from [12] and introducing free weighting matrix variables, the stability condition for the system is derived by using linear matrix inequality (LMI) techniques.

Notation. The notation *X > 0* denotes a symmetric positive definite, asterisk (*) represents the elements of symmetric term in the symmetric block matrix. The superscripts "*T*" and "*-1*" represent the transpose and inverse matrix, respectively. $L_2[0,\infty)$ is the space of square integrable functions on [0,∞).

2. PROBLEM FORMULATION

Consider the following continuous-time dynamical system with two additive time varying delays in the state [10],

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x\big(t - \tau_1(t) - \tau_2(t)\big) + Ew(t), \\ z(t) &= Cx(t) + C_d x\big(t - \tau_1(t) - \tau_2(t)\big) + Fw(t), \\ x(t) &= \eta(t), \quad t \in [-\bar{\tau}, 0] \end{aligned} \tag{4}$$

where $x(t) \in \Re^n$ is the state vector; $z(t) \in \Re^p$ is the output vector; $\tau_1(t)$ and $\tau_2(t)$ represents two delays in the state; $\eta(t)$ is the initial condition on the segment $[-\bar{\tau}, 0]$; $w(t) \in \Re^l$ is the disturbance input which belongs to $L_2[0,\infty)$; A, A_d, E, C, C_d, and F are known system matrices with appropriate dimension. For dynamical system in (4), it is assumed that,

$$\begin{aligned} &0 \le \tau_1(t) \le \bar{\tau}_1 < \infty, \quad \dot{\tau}_1 \le d_1 < \infty, \quad 0 \le \tau_2(t) \le \bar{\tau}_2 < \infty, \quad \dot{\tau}_2 \le d_2 < \infty, \\ &\text{and } \bar{\tau} = \bar{\tau}_1 + \bar{\tau}_2, \; d = d_1 + d_2 \end{aligned} \tag{5}$$

Our objective is to investigate whether the continuous system with two time-varying delays is asymptotically stable and dissipative. The following lemmas will be used in the proof of the main results of the present paper.

Lemma 1 [13]. For any $z, y \in \Re^n$ and for any symmetric positive definite matrix $X \in \Re^{n \times n}$,

$$-2z^T y \le z^T X^{-1} z + y^T X y \tag{6}$$

3. MAIN RESULT

Main result of the present paper is stated in the following theorem.

Theorem 1. *Given real matrices Q, S, R where Q and R are symmetric, and consider system (4) subject to assumption 1. Then the system (4) satisfying (5) is asymptotically stable and (Q, S, R)-dissipative if there exist matrices* $P = P^T > 0$, $Q_1 = Q_1^T > 0$, $Q_2 = Q_2^T > 0$, $Q_3 = Q_3^T > 0$, $R_1 = R_1^T > 0$, $R_2 = R_2^T > 0$, $R_3 = R_3^T > 0$ *and* G_i, L_i, M_i, N_i, $i = 1,...,4$ *are free matrices with* $Q_2 \geq Q_3$ *satisfying,*

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \varphi_1 & C^T(-Q)^{1/2} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \varphi_2 & 0 \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & \varphi_3 & C_d^T(-Q)^{1/2} \\ * & * & * & \phi_{44} & \phi_{45} & \varphi_4 & 0 \\ * & * & * & * & \phi_{55} & \varphi_5 & F^T(-Q)^{1/2} \\ * & * & * & * & * & \varphi_6 & 0 \\ * & * & * & * & * & 0 & -I \end{bmatrix} < 0 \tag{7}$$

where,

$\phi_{11} = Q_1 + Q_2 + G_1A + A^TG_1^T + L_1 + L_1^T + M_1 + M_1^T$, $\phi_{12} = A^TG_2^T + L_2^T - M_1 + M_2^T + N_1$,
$\phi_{13} = G_1A_d + A^TG_3^T - L_1 + L_3^T + M_3^T - N_1$, $\phi_{14} = P - G_1 + A^TG_4^T + L_4^T + M_4^T$, $\phi_{15} = G_1E - C^TS$,
$\phi_{22} = -(1-d_1)(Q_2 - Q_3) - M_2 - M_2^T + N_2 + N_2^T$, $\phi_{23} = G_2A_d - L_2 - M_3^T - N_2 + N_3^T$, $\phi_{24} = -G_2 - M_4^T + N_4^T$,
$\phi_{25} = G_2E$, $\phi_{33} = -(1-d_1-d_2)(Q_1+Q_3) + G_3A_d + A_d^TG_3^T - L_3 - L_3^T - N_3 - N_3^T$,
$\phi_{34} = -G_3 + A_d^TG_4^T - L_4^T - N_4^T$, $\phi_{35} = G_3E - C_d^TS$, $\phi_{44} = \bar{\tau}R_1 + \bar{\tau}_1R_2 + \bar{\tau}_2R_3 - G_4 - G_4^T$, $\phi_{45} = G_4E$,
$\phi_{55} = -(S^TF + F^TS + R)$, $\varphi_5 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, $\varphi_6 = \text{diag}\{-\bar{\tau}^{-1}R_1, -\bar{\tau}_1^{-1}R_2, -\bar{\tau}_2^{-1}R_3\}$, $\varphi_i = \begin{bmatrix} L_i & M_i & N_i \end{bmatrix}$ for i = 1 to 4.

Proof.

Define a Lyapunov-Krasovski functional as in [12],

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

$$V_1(t) = x^T(t)Px(t) \tag{8}$$

$$V_2(t) = \int_{t-\tau_1(t)-\tau_2(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau_1(t)}^{t} x^T(s)Q_2x(s)ds + \int_{t-\tau_1(t)-\tau_2(t)}^{t-\tau_1(t)} x^T(s)Q_3x(s)ds \tag{9}$$

$$V_3(t) = \int_{t-\bar{\tau}_1-\bar{\tau}_2}^{t}\int_{\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{t-\bar{\tau}_1}^{t}\int_{\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta + \int_{t-\bar{\tau}_1-\bar{\tau}_2}^{t-\bar{\tau}_1}\int_{\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta \tag{10}$$

The time derivative of $V(t)$ satisfying condition (5) is given by (as done in [12])

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t), \tag{11}$$

$$\begin{aligned} \dot{V}_2(t) \leq\ & x^T(t)(Q_1+Q_2)x(t) - (1-d_1)x^T(t-\tau_1(t))(Q_2-Q_3)x(t-\tau_1(t)) \\ & -(1-d_1-d_2)x^T(t-\tau(t))(Q_1+Q_3)x(t-\tau(t)) \end{aligned} \tag{12}$$

$$\dot{V}_3(t) \leq \dot{x}^T(t)(\bar{\tau}R_1 + \bar{\tau}_1R_2 + \bar{\tau}_2R_3)\dot{x}(t) - \int_{t-\tau(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\tau_1(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-\tau(t)}^{t-\tau_1(t)} \dot{x}^T(s)R_3\dot{x}(s)ds \tag{13}$$

Now, introducing any free matrices G_i, i = 1, 2, 3, 4, one may write

$$\begin{aligned} \Sigma &= 2\left[x^T(t)G_1 + x^T(t-\tau_1(t))G_2 + x^T(t-\tau(t))G_3 + \dot{x}^T(t)G_4\right] \\ &\times\left[-\dot{x}(t) + Ax(t) + A_dx(t-\tau(t)) + Ew(t)\right] \\ &= 0 \end{aligned} \tag{14}$$

Simplifying (14), we get

$$\Sigma = \bar{\xi}^T(t)\begin{bmatrix} G_1A + A^TG_1^T & A^TG_2^T & G_1A_d + A^TG_3^T & -G_1 + A^TG_4^T & G_1E \\ * & 0 & G_2A_d & -G_2 & G_2E \\ * & * & G_3A_d + A_d^TG_3^T & -G_3 + A_d^TG_4^T & G_3E \\ * & * & * & -G_4 - G_4^T & G_4E \\ * & * & * & * & 0 \end{bmatrix}\bar{\xi}(t) = 0 \quad (15)$$

where $\bar{\xi}(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_1(t)) & x^T(t-\tau(t)) & \dot{x}^T(t) & w(t) \end{bmatrix}^T$.

By the Newton-Leibniz formula, and introducing free matrices L_i, i = 1, 2, 3, 4, we get

$$\begin{aligned} &2\left[x^T(t)L_1 + x^T(t-\tau_1(t))L_2 + x^T(t-\tau(t))L_3 + \dot{x}^T(t)L_4\right] \\ &\times\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{x}(s)ds\right] = 0 \end{aligned} \quad (16)$$

Simplifying (16), we obtain

$$\bar{\xi}^T(t)\begin{bmatrix} L_1 + L_1^T & L_2^T & -L_1 + L_3^T & L_4^T & 0 \\ * & 0 & -L_2 & 0 & 0 \\ * & * & -L_3 - L_3^T & -L_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}\bar{\xi}(t) + \int_{t-\tau(t)}^{t} -2\bar{\xi}^T(t)\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}\dot{x}(s)ds = 0 \quad (17)$$

Applying Lemma 1 on the last term of (17), we get

$$\int_{t-\tau(t)}^{t} -2\bar{\xi}^T(t)\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}\dot{x}(s)ds \le \tau(t)\bar{\xi}^T(t)\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}R_1^{-1}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}^T\bar{\xi}(t) + \int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \quad (18)$$

Substituting (18) in the last term of (17), we get

$$-\int_{t-\tau(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \le \bar{\xi}^T(t)\left\{\begin{bmatrix} L_1 + L_1^T & L_2^T & -L_1 + L_3^T & L_4^T & 0 \\ * & 0 & -L_2 & 0 & 0 \\ * & * & -L_3 - L_3^T & -L_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} + \bar{\tau}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}R_1^{-1}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}^T\right\}\bar{\xi}(t) \quad (19)$$

We can remove the last two terms of (13) (integral terms) using similar way as done for (16) – (19). We obtain,

$$-\int_{t-\tau_1(t)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds \le \bar{\xi}^T(t)\left\{\begin{bmatrix} M_1 + M_1^T & -M_1 + M_2^T & M_3^T & M_4^T & 0 \\ * & -M_2 - M_2^T & -M_3^T & -M_4^T & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} + \bar{\tau}_1\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix}R_2^{-1}\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix}^T\right\}\bar{\xi}(t) \quad (20)$$

$$-\int_{t-\tau(t)}^{t-\tau_1(t)}\dot{x}^T(s)R_3\dot{x}(s)ds \le \bar{\xi}^T(t)\left\{\begin{bmatrix} 0 & N_1 & -N_1 & 0 & 0 \\ * & N_2 + N_2^T & -N_2 + N_3^T & N_4^T & 0 \\ * & * & -N_3 - N_3^T & -N_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} + \bar{\tau}_2\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix}R_3^{-1}\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix}^T\right\}\bar{\xi}(t) \quad (21)$$

where M_i, i = 1, 2, 3, 4 and N_i, i = 1, 2, 3, 4 are free matrices.

Substituting (19), (20) and (21) in (13), we get

$$\dot{V}_3(t) \le \dot{x}^T(t)\left(\bar{\tau}R_1 + \bar{\tau}_1R_2 + \bar{\tau}_2R_3\right)\dot{x}(t) + \bar{\xi}^T(t)\left\{\dot{V}_{31} + \dot{V}_{32} + \dot{V}_{33} + \dot{V}_{34}\right\}\bar{\xi}(t) \quad (22)$$

where,

$$\dot{V}_{31} = \begin{bmatrix} L_1 + L_1^T + M_1 + M_1^T & L_2^T - M_1 + M_2^T + N_1 & -L_1 + L_3^T + M_3^T - N_1 & L_4^T + M_4^T & 0 \\ * & -M_2 - M_2^T + N_2 + N_2^T & -L_2 - M_3^T - N_2 + N_3^T & -M_4^T + N_4^T & 0 \\ * & * & -L_3 - L_3^T - N_3 - N_3^T & -L_4^T - N_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

$$\dot{V}_{32} = \bar{\tau}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}^T, \quad \dot{V}_{33} = \bar{\tau}_1 \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix}^T, \quad \dot{V}_{34} = \bar{\tau}_2 \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix} R_3^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix}^T$$

Then, from (11), (12), (15) and (22) we have

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \Sigma$$

$$\dot{V}(t) \le \bar{\xi}^T(t) \left\{ \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \bar{\phi}_{15} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\ * & * & \phi_{33} & \phi_{34} & \bar{\phi}_{35} \\ * & * & * & \phi_{44} & \phi_{45} \\ * & * & * & * & \bar{\phi}_{55} \end{bmatrix} + \bar{\tau}\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix} R_1^{-1} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ 0 \end{bmatrix}^T + \bar{\tau}\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix} R_2^{-1} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ 0 \end{bmatrix}^T + \bar{\tau}_2 \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix} R_3^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ 0 \end{bmatrix}^T \right\} \bar{\xi}(t) \tag{23}$$

where ϕ_{11}, ϕ_{12}, ϕ_{13}, ϕ_{14}, ϕ_{22}, ϕ_{23}, ϕ_{24}, ϕ_{25}, ϕ_{33}, ϕ_{34}, ϕ_{44} and ϕ_{45} are given in (7), and $\bar{\phi}_{15} = G_1 E$, $\bar{\phi}_{35} = G_3 E$, $\bar{\phi}_{55} = 0$.

Applying Schur complement [14], (23) is equivalent to

$$\dot{V}(t) \le \bar{\xi}^T(t) \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \bar{\phi}_{15} & \varphi_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \varphi_2 \\ * & * & \phi_{33} & \phi_{34} & \bar{\phi}_{35} & \varphi_3 \\ * & * & * & \phi_{44} & \phi_{45} & \varphi_4 \\ * & * & * & * & \bar{\phi}_{55} & \varphi_5 \\ * & * & * & * & * & \varphi_6 \end{bmatrix} \bar{\xi}(t) \tag{24}$$

where $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5$, φ_6 are given in (7) and other parameters are given in (23).

First, we consider the asymptotic stability of system (4) satisfying (5) with $w(t) = 0$. For this case, from (24) we have

$$\dot{V}(t) \le \bar{\xi}^T(t) \Phi \bar{\xi}(t) \tag{25}$$

where

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \varphi_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \varphi_2 \\ * & * & \phi_{33} & \phi_{34} & \varphi_3 \\ * & * & * & \phi_{44} & \varphi_4 \\ * & * & * & * & \varphi_6 \end{bmatrix}$$

It is noticed that condition (7) implies $\Phi < 0$ and hence $\dot{V}(t) < 0$. Therefore, we can conclude that system (4) satisfying (5) with $w(t) = 0$ is asymptotically stable.

Now, we consider quadratic supply function (3). By substituting the output equation of the system (4) into supply function (3), we get quadratic supply function in the variable x and w,

$$s(w(t), z(t)) = s(w(t), Cx(t) + C_d x(t - \tau(t)) + Fw(t))$$

$$\begin{aligned} s(w(t), x(t)) = {} & (Cx(t) + C_d x(t - \tau(t)) + Fw(t))^T Q (Cx(t) + C_d x(t - \tau(t)) + Fw(t)) \\ & + w^T(t) S^T (Cx(t) + C_d x(t - \tau(t)) + Fw(t)) \\ & + (Cx(t) + C_d x(t - \tau(t)) + Fw(t))^T S w(t) + w^T(t) R w(t) \end{aligned} \tag{26}$$

$$s(w(t),x(t))=\bar{\xi}^T(t)\begin{bmatrix} C^TQC & 0 & C^TQC_d & 0 & C^TQF+C^TS \\ * & 0 & 0 & 0 & 0 \\ * & * & C_d^TQC_d & 0 & C_d^TQF+C_d^TS \\ * & * & * & 0 & 0 \\ * & * & * & * & R+S^TF+F^TS+F^TQF \end{bmatrix}\bar{\xi}(t) \tag{27}$$

From (24) and (27) we have,

$$\dot{V}(t)-s(w(t),x(t))\le\bar{\xi}^T(t)\begin{bmatrix} \bar{\phi}_{11} & \phi_{12} & \bar{\phi}_{13} & \phi_{14} & \bar{\bar{\phi}}_{15} & \varphi_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \varphi_2 \\ * & * & \bar{\phi}_{33} & \phi_{34} & \bar{\bar{\phi}}_{35} & \varphi_3 \\ * & * & * & \phi_{44} & \phi_{45} & \varphi_4 \\ * & * & * & * & \bar{\bar{\phi}}_{55} & \varphi_5 \\ * & * & * & * & * & \varphi_6 \end{bmatrix}\bar{\xi}(t) \tag{28}$$

where,

$\bar{\phi}_{11}=\phi_{11}-C^TQC$, $\bar{\phi}_{13}=\phi_{13}-C^TQC_d$, $\bar{\bar{\phi}}_{15}=\phi_{15}-C^TQF$, $\bar{\phi}_{33}=\phi_{33}-C_d^TQC_d$, $\bar{\bar{\phi}}_{35}=\phi_{35}-C_d^TQF$, $\bar{\bar{\phi}}_{55}=\phi_{55}-F^TQF$.

By Schur complement, (28) can be written as follows

$$\dot{V}(t)-s(w(t),x(t))\le\bar{\xi}^T(t)\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \varphi_1 & C^T(-Q)^{1/2} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \varphi_2 & 0 \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & \varphi_3 & C_d^T(-Q)^{1/2} \\ * & * & * & \phi_{44} & \phi_{45} & \varphi_4 & 0 \\ * & * & * & * & \phi_{55} & \varphi_5 & F^T(-Q)^{1/2} \\ * & * & * & * & * & \varphi_6 & 0 \\ * & * & * & * & * & 0 & -I \end{bmatrix}\bar{\xi}(t) \tag{29}$$

From the above inequality, the condition in (7) guarantees that $\dot{V}(t)-s(w(t),x(t))<0$. By using Definition 1, the continuous-time system with two additive time-varying delays (4) satisfying (5) with respect to quadratic supply function (3) is asymptotically stable and dissipative. The proof is completed.

Theorem 1 provides condition under which the time delays system is asymptotically stable and dissipative. The condition is represented in terms of solution to LMI (7). It can be numerically solved using any standard LMI solver.

Remark 2. As stated previously, systems with two additive time-varying delays can be found in practical situation in Networked Control Systems (NCSs) [9, 10], where $\tau_1(t)$ represents the time-delay from sensor to the controller while $\tau_2(t)$ represents the time-delay from controller to the actuator. Due to the condition of network transmission, those two delays may not be identical [9]. In the following, we use the result derived in this paper to analyze the stability and dissipativity of NCS.

We further consider the model of Networked Control Systems in [10, 15], as shown in Figure 1. The two additive delays $\tau_1(t)$ and $\tau_2(t)$ have very different properties in that $\tau_1(t)$ and $\tau_2(t)$ are assumed to be constant and non-differentiable, respectively. The assumption in (5) reads

$$\tau_1(t)\equiv\bar{\tau}_1<\infty,\ 0\le\tau_2(t)\le\bar{\tau}_2<\infty \tag{30}$$

Then, we have the following corollary.

Corollary 1. *Given real matrices Q, S, R where Q and R are symmetric, and consider system (4) subject to assumption 1. Then the system (4) satisfying (30) is asymptotically stable and (Q, S, R)-dissipative if there exist matrices* $P=P^T>0,\quad Q_2=Q_2^T>0,\ R_1=R_1^T>0,\quad R_2=R_2^T>0,\ R_3=R_3^T>0$ *and* G_i, L_i, M_i, N_i, *i = 1,...,4 are free matrices satisfying,*

$$\begin{bmatrix} \bar{\bar{\phi}}_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \varphi_1 & C^T(-Q)^{1/2} \\ * & \bar{\bar{\phi}}_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \varphi_2 & 0 \\ * & * & \bar{\bar{\phi}}_{33} & \phi_{34} & \phi_{35} & \varphi_3 & C_d^T(-Q)^{1/2} \\ * & * & * & \phi_{44} & \phi_{45} & \varphi_4 & 0 \\ * & * & * & * & \phi_{55} & \varphi_5 & F^T(-Q)^{1/2} \\ * & * & * & * & * & \varphi_6 & 0 \\ * & * & * & * & * & 0 & -I \end{bmatrix} < 0 \qquad (31)$$

where,

$\bar{\bar{\phi}}_{11} = Q_2 + G_1 A + A^T G_1^T + L_1 + L_1^T + M_1 + M_1^T$,

$\bar{\bar{\phi}}_{22} = -Q_2 - M_2 - M_2^T + N_2 + N_2^T$,

$\bar{\bar{\phi}}_{33} = G_3 A_d + A_d^T G_3^T - L_3 - L_3^T - N_3 - N_3^T$, and other parameters are given in (7).

Proof.
Define a Lyapunov-Krasovski functional, $V(t) = V_1(t) + V_2(t) + V_3(t)$, where $V_1(t)$ and $V_3(t)$ are given in (8) and (10), respectively and

$$V_2(t) = \int_{t-\bar{\tau}_1}^{t} x^T(s) Q_2 x(s) ds$$

Then, the proof is derived along similar lines as in the proof of Theorem 1. □

Corollary 1 provides condition under which the time delays system is asymptotically stable and dissipative for special case in Networked Control Systems (NCSs). The result obtained here will be used to investigate dissipative controller for NCS.

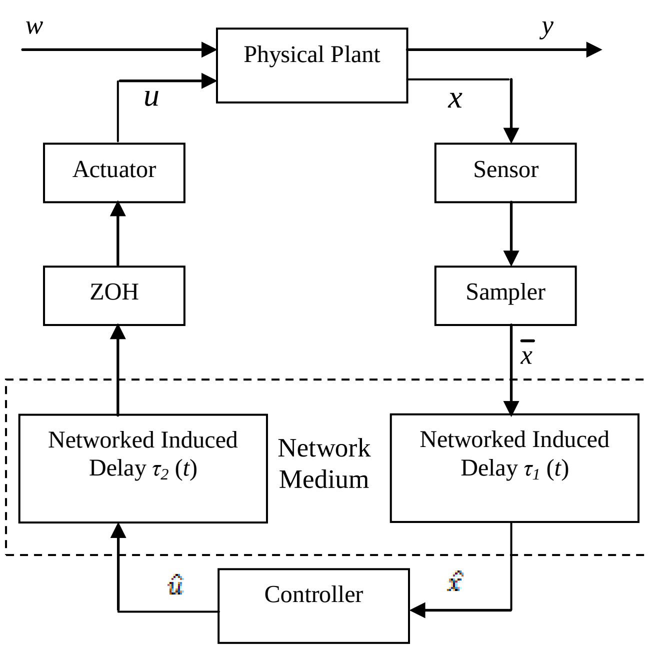

Figure 1. Networked Control Systems

4. ILLUSTRATIVE EXAMPLE

In this section, we give example in case of H_∞ performance analysis, that is $Q = -I$, S = 0 and $R = \gamma^2 I$. We consider time delay dynamical system in [10]. The system matrices A, A_d, C, C_d, E, F in (4) are given as follows,

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ E = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ C_d = \begin{bmatrix} 2 & 1 \end{bmatrix},\ F = 0.5.$$

Suppose we know that $\dot{\tau}_1 \le 0.1$ and $\dot{\tau}_2 \le 0.8$.

We assume the delay upper bound $\bar{\tau}_1$ and $\bar{\tau}_2$. Our objective is to find the minimum guaranteed H_∞ performance, γ_{min} , that makes condition in Theorem 1 feasible.

First, we assume $\bar{\tau}_1 = 1$ and $\bar{\tau}_2 = 0.1$. By solving LMI (7), we get the minimum guaranteed H_∞ performance, γ_{min} = 1.31. Comparison of different cases with varying time delay upper bounds $\bar{\tau}_1$ and $\bar{\tau}_2$ is provided in Table 1. It shows that the resulting γ_{min} decreases as the upper bounds of the delay decrease.

Table 1 Minimum guaranteed H_∞ performance, γ_{min}, for different cases of $\bar{\tau}_1$ and $\bar{\tau}_2$

$\bar{\tau}_1$ (s)	1			1.2			1.5		
$\bar{\tau}_2$ (s)	0.1	0.2	0.3	0.1	0.2	0.3	0.1	0.2	0.3
γ_{min}	1.31	2.16	4.07	1.72	3.05	6.26	2.7	5.23	14.51

5. CONCLUSIONS

In this paper, we have investigated the asymptotic stability and dissipativity of continuous-time system with two additive time-varying delays in the state. By exploiting Lyapunov-Krasovski functional and introducing free weighting matrix variables, the stability condition and dissipative were derived by using linear matrix inequality (LMI) techniques. The result obtained in this paper will be used to design dissipative controller for Networked Control Systems (NCS). An illustrative example is provided in case of H_∞ performance requirement to show effectiveness of the analysis method.

6. ACKNOWLEDGEMENTS

Authors are grateful to the Laboratory of Control and Computer Systems, School of Electrical Engineering and Informatics, Institute of Technology Bandung Indonesia and to the Faculty of Engineering, Gorontalo State University Indonesia for providing facilities, academic atmosphere and financial support to complete this research article.

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