THREE STAGE DISCRTETE TIME EXTENDED KALMAN FILTER SCHEME FOR MICRO AIR VEHICLES

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ABSTRACT
Micro Air Vehicles (MAV) is one of the most interesting area of application of robotics. These are miniaturized Unmanned Air Vehicles (UAVs), usually defined to be less than fifteen centimeters (six inches) in length and below the weight of hundred grams which include twenty grams of payload as well to apply Micro Electro Mechanical System (MEMS) techniques for the navigation of Micro Air Vehicles (MAV) is an extremely challenging area. This paper presents an approach of designing an INS/GPS based navigation system using three stage discrete time Extended Kalman Filter which is a very strong mathematical tool, is used in this paper for the estimation of states of the Micro Air Vehicles (MAV). Kalman Filter is a set of mathematical equations, which provides a computational mean for estimation of the states of a process. This paper shows the mathematical work carried out for applying the three stage discrete time Extended Kalman Filter scheme.

Keywords: *Micro Electro Mechanical System (MEMS), Micro Air Vehicle (MAV), Measurement Covariance Matrix, Process Covariance Matrix.*

Nomenclature

P_N Inertial North Position of MAV
P_E Inertial East Position of MAV
W_N Wind from North
W_E Wind from East
V_{air} Total Airspeed
p Angular Rate about x-axis
q Angular Rate about y-axis
r Angular Rate about z-axis
ϕ Roll Angle
θ Pitch Angle
ψ Yaw Angle
• As superscript shows the rate of change
m_{ox} Northern Magnetic Field Component
m_{oy} Eastern Magnetic Field Component
m_{oz} Vertical Magnetic Field Component
Q Process Covariance Noise Matrix
R Process Covariance Noise Matrix

1. INTRODUCTION

The history of Micro Air Vehicles (MAVs) really began with the development of model airplanes in the 19th century and the development of radio controlled model airplanes in the 20th century [1]. The demand for small Unmanned Air Vehicles (UAVs), commonly termed micro air vehicles, is rapidly increasing. Driven by applications ranging from civil search-and-rescue missions to military surveillance and reconnaissance missions, there is a rising level of interest and investment in better vehicle designs [2], and miniaturized components [3] are enabling many rapid advances. Practical Micro Air Vehicle (MAV) missions, to be conducted in an outdoor urban environment, require the capability of slowly loitering over a target in order to capture and transmit clear images to the ground station.

Micro Air Vehicles (MAVs) is miniaturized version of Unmanned Air Vehicles (UAVs) which is very sensitive to the wind gust [4] and therefore proper navigation of Micro Air Vehicles (MAVs) is very they important because output of navigation acts as input for the control system of Micro Air Vehicles (MAVs). Micro Electro Mechanical Systems (MEMS) sensors are combined together in Inertial Measurement Unit (IMU) to estimate the states of Micro Air Vehicle (MAV). These sensors are very sensitive especially GPS because the performance of the low cost micro GPS receiver can be easily degraded in high maneuvering environments, fusing the navigation data with other sensors such as a magnetometer or barometer is necessary [5].

In this paper, the designing of INS/GPS based navigation system using three stage Discrete Time Extended Kalman Filter is carried out. Inertial Navigation System (INS) includes MEMS sensors [6] mainly MEMS gyro [7], MEMS accelerometer [8], MEMS magnetometer [9], GPS [10] and pressure sensor [11]. These MEMS sensors provide a complete description of attitude of Flapping Micro Air Vehicle. Pressure sensors gives air velocity and MEMS Gyro calculates angular rates (p,q,r) which are used as input to the MEMS accelerometer to calculate roll and pitch(ϕ and θ) respectively, of the FMAV. MEMS magnetometer gives the yaw (ψ) estimation of the FMAV. GPS gives the position as well as the heading and velocity of the FMAV. Kalman Filter is used for the estimation of roll (ϕ), pitch (θ), yaw (ψ) and position (P_N and P_E) mainly. The system considered is non-linear and it is first linearized for application of the Kalman Filter with the help of jacobian method. It is assumed that sampling rate (T_S) is 100 ms.

2. DISCRETE TIME EXTENDED KALMAN FILTER SCHEME FORMULATION

The system and measurement equations are given as follows:

$$
\begin{aligned}
x_k &= f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \\
y_k &= h_h(x_k, v_k) \\
w_k &\approx (0, Q_k) \\
v_k &\approx (0, R_k)
\end{aligned}
$$

Initialize the filter as follows:

$$
\begin{aligned}
\hat{x}_0^+ &= E(x_0) \\
P_0^+ &= E\left[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T\right]
\end{aligned}
$$

For k = 1, 2, 3 . . . .

Compute the following partial derivative matrices [12]:

$$
F_{k-1} = \left.\frac{\partial f_{k-1}}{\partial x}\right|_{\hat{x}_{k-1}^+} \qquad L_{k-1} = \left.\frac{\partial f_{k-1}}{\partial w}\right|_{\hat{x}_{k-1}^+}
$$

Perform the time update of the state estimate [13] and estimation error covariance [14] as follows:

$$
\begin{aligned}
P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T \\
\hat{x}_k^- &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0)
\end{aligned}
$$

Compute the following partial derivative matrices

$$
H_K = \left.\frac{\partial h_k}{\partial x}\right|_{\hat{x}_k^-} \qquad M_K = \left.\frac{\partial h_k}{\partial v}\right|_{\hat{x}_k^-}
$$

Perform the measurement update [15] of the state estimate and estimation error covariance as follows

$$
\begin{aligned}
K_k &= P_k^- H_k^T \left(H_k P_k^- H_k^T + M_k R_k M_k^T\right)^{-1} \\
\hat{x}_k^+ &= \hat{x}_k^- + K_k\left(y_k - h_k(\hat{x}_k^-, 0)\right) \\
P_k^+ &= (I - K_K H_k) P_k^-
\end{aligned}
$$

3. THREE STAGE EXTENDED KALMAN FILTER ESTIMATION SCHEME

In this scheme, three Extended Kalman Filters work independently, each imparting the information that it estimates to the stage below. This three stage filter assumes the least coupling.

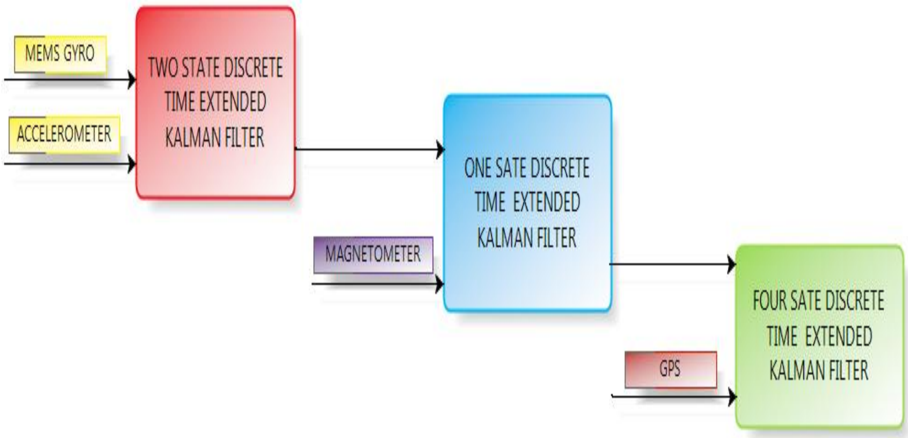

Figure 1: *Three stage Extended Kalman Filter Scheme*

First stage state variables and inputs can be shown as following:

$$x=\begin{bmatrix}\phi\\ \theta\end{bmatrix}, u=\begin{bmatrix}p\\ q\\ r\\ V_{air}\end{bmatrix}, y=\begin{bmatrix}acc_x\\ acc_y\\ acc_z\end{bmatrix}$$

Stage-two state variables and inputs can be shown as following:

$$x=[\psi], u=\begin{bmatrix}\phi\\ \theta\\ q\\ r\end{bmatrix}, y=[acc_z]$$

Stage-three state variables and inputs can be shown as following:

$$x=\begin{bmatrix}P_N\\ P_E\\ W_N\\ W_E\end{bmatrix}, u=\begin{bmatrix}\psi\\ V_{air}\end{bmatrix}, y=\begin{bmatrix}GPS_N\\ GPS_E\\ GPS_{Velocity}\\ GPS_{Heading}\end{bmatrix}$$

In three- stage discrete time Extended Kalman Filter state estimation scheme, the states can be related to the inputs in the following two steps as shown below:

Stage 1 consists of the following:

$$\dot{x}=\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\end{bmatrix}=f(x,u)=\begin{bmatrix}p+q\sin\phi\tan\theta+r\cos\phi\tan\theta\\ q\cos\phi+r\sin\phi\end{bmatrix}$$

Linearization through jacobian method

$$A=\frac{\partial f(\hat{x},u)}{\partial x}=\begin{bmatrix}q\cos\phi\tan\theta-r\sin\phi\tan\theta & \dfrac{q\sin\phi+rxos\phi}{\cos^2\theta}\\ -q\sin\phi+r\cos\phi & 0\end{bmatrix}$$

The output equations for the attitude estimator are shown below:

$$h(\hat{x},u)=\begin{bmatrix}\dfrac{V_{air}q\sin\theta}{g}+\sin\theta\\ \dfrac{V_{air}(r\cos\theta-p\sin\theta)}{g}-\cos\theta\sin\phi\end{bmatrix}$$

Linearization through jacobian method

$$h(\hat{x},u) = \begin{bmatrix} 0 & \frac{V_{air} q \cos\theta}{g} + \cos\theta \\ -\cos\theta\cos\phi & \frac{-V_{air}\left(r\sin\theta + p\cos\theta\right)}{g} + \sin\theta\sin\phi \end{bmatrix}$$

Process Covariance Matrix is given as follows:

$$Q = E\left(ww^T\right)$$

Where,

$$w = \begin{bmatrix} w_\phi \\ w_\theta \end{bmatrix}$$

and

$$w^T = \begin{bmatrix} w_\phi & w_\theta \end{bmatrix}$$

Noise is uncorrelated :

$$Q = E\begin{bmatrix} w_\phi^2 & 0 \\ 0 & w_\theta^2 \end{bmatrix}$$

Measurement Covariance Matrix is given as follows:

$$R = E(vv^T)$$

Where,

$$v = \begin{bmatrix} v_{acc_x} \\ v_{acc_y} \end{bmatrix}$$

And

$$v^T = \begin{bmatrix} v_{acc_x} & v_{acc_y} \end{bmatrix}$$

So

$$R = E\begin{bmatrix} v^2{}_{acc_x} & v_{acc_x} v_{acc_y} \\ v_{acc_y} v_{acc_x} & v_{acc_y}^2 \end{bmatrix}$$

Noise is uncorrelated

$$R = E\begin{bmatrix} v^2{}_{acc_x} & 0 \\ 0 & v_{acc_y}^2 \end{bmatrix}$$

Stage 2 consists of the following:

$$\dot{\psi} = f(x,u) = q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta}$$

Linearization through jacobian method

$$A = \frac{\partial f\left(\hat{x},u\right)}{\partial x} = 0$$

Process Covariance Matrix is given as follows:

$$Q = E\left(ww^T\right)$$

Where,

$$w = \begin{bmatrix} w_\psi \end{bmatrix}$$

And

$$w^T = \begin{bmatrix} w_\psi \end{bmatrix}$$

Noise is uncorrelated:

$$Q = E\left[w_\psi^2\right]$$

Output equations by heading estimator:

$$h(\hat{x},u) = \left[\frac{-V_{air} q \cos\theta}{g} - \cos\theta\cos\phi\right]$$

Linearization through jacobian method:

$$H_k = \frac{\partial h(\hat{x},u)}{\partial x} = [0]$$

Measurement Covariance Matrix is given as follows:

$$R = E(vv^T)$$

Where, $v = [acc_z]$

And

$$v^T = [acc_z]$$

Noise is uncorrelated:

$$R = E\left[acc_z^2\right]$$

Stage-three, Output equations by inertial estimator can be shown as following:

$$h(\hat{x},u) = \begin{bmatrix} GPS_N \\ GPS_E \\ GPS_{velocity} \\ GPS_{heading} \end{bmatrix} = \begin{bmatrix} P_N \\ P_E \\ \sqrt{V_{air}^2 + 2V_{air}(W_N\cos\psi + W_E\sin\psi) + (W_N^2 + W_E^2)} \\ \tan^{-1}\left(\frac{V_{air}\sin\psi + W_E}{V_{air}\cos\psi + W_N}\right) \end{bmatrix}$$

Stage 3 consists of the following:

$$\begin{bmatrix} \dot{P}_N \\ \dot{P}_E \\ \dot{W}_N \\ \dot{W}_E \end{bmatrix} = \begin{bmatrix} V_{air}\cos\psi - W_N \\ V_{air}\sin\psi - W_E \\ 0 \\ 0 \end{bmatrix}$$

Process Covariance Matrix is given as follows:

$$Q = E\left(ww^T\right)$$

Where,

$$w = \begin{bmatrix} w_{P_N} \\ w_{P_E} \end{bmatrix}$$

And

$$w^T = \begin{bmatrix} w_{P_N} & w_{P_E} \end{bmatrix}$$

Noise is uncorrelated:

$$Q = E\begin{bmatrix} w_{P_N}^2 & 0 \\ 0 & w_{P_E} \end{bmatrix}$$

Output equations by inertial estimator:

$$h(\hat{x},u) = \begin{bmatrix} GPS_N \\ GPS_E \end{bmatrix} = \begin{bmatrix} P_N \\ P_E \end{bmatrix}$$

Linearization through jacobian :

$$\frac{\partial h(\hat{x},u)}{\partial x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Measurement Covariance Matrix is given as follows:

$$R = E(vv^T)$$

Where,

$$v = \begin{bmatrix} GPS_N \\ GPS_E \\ GPS_{Velocity} \\ GPS_{Heading} \end{bmatrix}$$

and

$$v^T = \begin{bmatrix} GPS_N & GPS_E & GPS_{Velocity} & GPS_{Heading} \end{bmatrix}$$

So, The noise is uncorrelated

$$R = E\begin{bmatrix} GPS_N^2 & 0 & 0 & 0 \\ 0 & GPS_E^2 & 0 & 0 \\ 0 & 0 & GPS_{Velocity}^2 & 0 \\ 0 & 0 & 0 & GPS_{Heading}^2 \end{bmatrix}$$

4. CONCLUSIONS

The MAVs are playing a significant role military surveillance and reconnaissance, and civilian search and rescue. To control MAV, it is important to navigate it properly. Kalman Filter is one of the techniques, which can be used for the navigation of MAV. Three different schemes can be used for the discrete time Extended Kalman filter as shown in the paper above.

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