LOCALIZATION OF COMPACT INVARIANT SETS OF A 4D SYSTEM AND ITS APPLICATION IN CHAOS

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ABSTRACT
We combine the globally exponentially attractive set with the iterative theory to discuss the boundedness of a Lorenz-Stenflo chaotic system. Firstly, We get a exponentially attractive set for this system. Then, we use iterative theory to get a refined boundedness for this system. Finally, the boundedness for y, z is applied to chaos synchronization. Numerical simulations are presented to show the effectiveness of the proposed scheme.

Key words: *Chaotic system; iterative theorem; invariant sets.*

1. INTRODUCTION
Since the discovery of the Lorenz chaotic system in 1963 [1]. Chaos has been studied extensively. After that, many chaotic systems have been discovered such as Rössler system [2], Chen system [3], and Lü system [4]. And these chaotic systems have been widely studied [5-9]. Efforts of many researchers have been aimed towards investigations of bifurcations, chaos, boundary and related control problems for these new chaotic systems. In particular, the boundness plays an important role in chaotic systems. If we can show that a system under consideration has a globally attractive set, then we know that the system cannot have equilibrium points, periodic solutions, quasi–periodic solutions, or other chaotic attractors outside the globally attractive set. This great -ly simplifies the analysis of the dynamical properties of the system. A noticeable prog -ress has been achieved in spite of substantial complexity of mathematical models of such systems. The paper [10] and [11] has been devoted to analysis the pitchfork and Hopf bifurcations based on using bifurcations theory and the central manifold theorem, obtaining an approximate stability boundary and some other topics. Recently, theoretical efforts have been performed to find localization domains containing all comp -act invariant sets of a nonlinear continuous-time system possessing complex behavior [12], [13], [14] and [15]. Here, we recall studies of Lorenz system [16] and the perma nent-magnet motor system [13]. Bounds for a domain containing all compact invaria –nt sets obtained in these papers in many cases can be used not only for theoretical stu -dies of chaotic attractors, e.g. but also for estimating for the Hausdorff Dimension [17], or for the numerical search of attractors. The ultimate bound also plays an important role in designing scheme for chaos control and chaos synchronization [18].

Recently, a new Lorenz-Stenflo chaotic system has the following equation [19]:

$$\begin{cases} \dot{x} = ay - ax + dw \\ \dot{y} = cx - xz - y \\ \dot{z} = xy - bz \\ \dot{w} = -x - aw \end{cases} \qquad (1)$$

Where a>0, b>0, c>0, d>0 are parameters; a, c, d are the Rayleigh, and the rotation numbers respectively, and b>0 is a geometric parameter. When parameters a=1, b=0.7, c=26, d=1.5, the Lyapunov exponent for system is $(3.665, 0.004. -0.589, -4.296)$. The largest Lyapunov exponent is 3.665>0. So the system (1) is chaotic as a=1, b=0.7, c=26, d=1.5.[19]. Fig.1. shows the phase portraits of system (1) in three dimensional spaces. see Fig.1
Fig.2. shows Hopf bifurcation diagram for system (1) with system parameters a=1,b=0.7,c=26 and parameter d ranges from 0.1 to 3. 5. see Fig.2

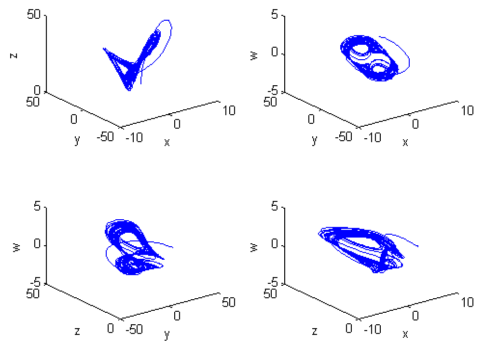

Fig.1. Phase portraits of system (1) with system parameters a=1,b=0.7,c=26, d=1.5.

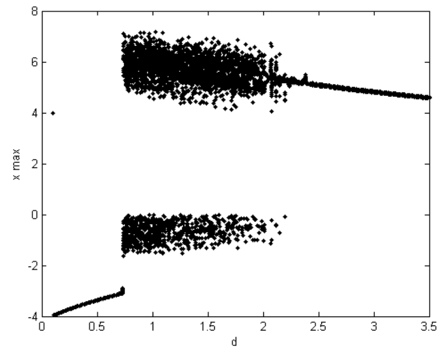

Fig.2. Hopf bifurcation diagram for system (1) with system parameters a=1,b=0.7,c=26 and parameter d ranges from 0.1 to 3. 5.

Moreover, in the sense defined by Vaneˇcˇek and Cˇelikovský [20]. It is immediately clear that the system is topologically nonequivalent to the original Lorenz and the other Lorenz-like systems. Therefore, it is interesting is to further find out what kind of new dynamics this system has. In the following, we will discuss the ultimate bound and the localization set of chaotic system (1). Some basic dynamical properties were studied in [19]. But many properties of this new systems remains to be uncovered. In this paper, we investigate the ultimate bound and the compact invariant sets for the new chaotic system via Lyapunov functions theory, extremum theory and iteration theorem [13-14].
The rest of this paper is organized as follows: Section 1 is some notations. In section 2, we give an ellipsoidal localization and we denote $\gamma(\alpha)$ the radius for the ellipsoid. In section 3, it is about applications of the iteration theorem. In section 4 and 5, we use another two localizing functions. In section 4, we Localize by using the circular cylinder. In section 5, we Localize with help of cylindrical surface. In section 6, we give one example of the localization of a chaotic attractor for the Lorenz-Stenflo system. At the same time, our work mainly focus in these sections. It is evident that we can get a smaller bound of the chaotic attractor for this chaotic system. In section 7, we study synchronization using the bound for y, z. Section 8 is simulation study. The conclusion is drawn in Section 9.

2. SOME NOTAIONS AND PRELIMINARIES

Let us introduce a real polynomial right-side system

$$\dot{x} = f(x) \qquad (2)$$

Here $x \in R^n$ is the state vector. Let us take a real polynomial h of n real variables wh -ich is not a first integral of (2). The function h that used in the solution of the localiz- ation problem of the compact invariant sets is called localizing. By $h|_B$ we denote the restrictions of function *h* on a set $B \subset R^n$. By $L_f h$ we denote the Lie derivative of the function *h*. And here $L_f(h(x))$ is the Lie derivative [21]:

$$L_f h(x) = \sum_{i=1}^{n} f_i(x) \frac{\partial h(x)}{\partial x_i}, \ (f_1(x), f_2(x), \cdots, f_n(x))^T = f(x)$$

Let us define

$$h_{\inf} = \inf\left(h(x)\middle| x \in S_h\right),\ h_{\sup} = \sup\left(h(x)\middle| x \in S_h\right).$$

A localization of all compact invariant sets of the system (2) is described by the follo- wing results [13-14].

Proposition 1. Let $W \cap S(h) \neq \varnothing.$ Then each compact invariant set Γ of the system (2) contained in W is located in the set

$$K = \left\{x \middle| h_{\inf}(W) \le h(x) \le h_{\sup}(W)\right\} \cap W \tag{3}$$

If $W \cap S(h) = \varnothing$, then system has no compact invariant sets that contained in W.

Lemma 1. Each compact invariant set of (2) that contained in W has common points with the set $W \cap S(h)$.

The function *h* used in the formulation of these results is called localizing.

Theorem 1. Let $h_m(x)$($m = 1, 2, \ldots$) be a sequence of functions from $C^\infty\left(R^n\right)$.

Set

$$K_1 = K_{h_1},\ K_m = K_{m-1} \cap K_{m-1,m},\ m > 1.\ \text{with}$$

$$K_{m-1,m} = \left\{x \middle| h_{m,\inf} \le h_m(x) \le h_{m,\sup}\right\},$$

$$h_{m,\sup} = \sup_{S_{h_m} \cap K_{m-1}} h_m(x),$$

$$h_{m,\inf} = \inf_{S_{h_m} \cap K_{m-1}} h_m(x),$$

Then we have $K_1, K_2, \cdots, K_m \cdots$contain all compact invariant sets of the system (2)

and $K_1 \supseteq K_2 \supseteq \ldots \supseteq K_m \supseteq \ldots$ (4)

3. ELLIPSOIDAL LOCALIZATION WITH PRECISE BOUNDS

System (1) has an ellipsoidal globally exponentially attractive set as follows:

$\Omega = \left\{(x, y, z, w) \middle| \lambda x^2 + y^2 + (z - \lambda a - c)^2 + \lambda d w^2 \le \gamma^2(\alpha)\right\}$, where λ is a real number and $\lambda > 0$.

where

$$\gamma^2(\alpha) = \begin{cases} \dfrac{(\lambda a + c)^2 b^2}{4(b-1)} & a \ge 1, b \ge 2 \\ (\lambda a + c)^2 & a > \dfrac{b}{2}, b < 2 \\ \dfrac{(\lambda a + c)^2 b^2}{4a(b-a)} & a < 1, b \ge 2a \end{cases}$$

Proof: Define the following positive definite and radically unbounded Lyapunov function:

$$h_1(x, y, z, w) = V(x, y, z, w) = \frac{1}{2}\left[\lambda x^2 + y^2 + (z - \lambda a - c)^2 + \lambda d w^2\right]$$

Then its derivative along the orbits of system (1) is

$$\begin{aligned} \dot{V} &= \lambda x\dot{x} + y\dot{y} + (z - \lambda a - c)\dot{z} + \lambda d w\dot{w} \\ &= -\lambda a x^2 - y^2 - \lambda a d w^2 - b z^2 + \lambda a b z + b c z \qquad (5) \\ &= -\lambda a x^2 - y^2 - \lambda a d w^2 - b\left(z - \frac{\lambda a + c}{2}\right)^2 + \frac{b(\lambda a + c)^2}{4} \end{aligned}$$

Let $\dot{V} = 0$, then we can get the following ellipsoidal surface Γ:

$$\lambda a x^2 + y^2 + \lambda a d w^2 + b\left(z - \frac{\lambda a + c}{2}\right)^2 = \frac{b(\lambda a + c)^2}{4}$$

Outside Γ , $\dot{V} < 0$, while inside Γ , $\dot{V} > 0$. Therefore, the ultimate bound for system (1) can be reached on Γ . In the following, we will discuss the system chaotic attractor bo -und about system different parameter.

（i）When $a \ge 1, b \ge 2$, let

$$f(z) = (1-b)z^2 + (b-2)(\lambda a + c)z$$

In the following, we estimate the maximum value of $f(z)$ on Γ . From (5) we can get $z \in I = [0, \lambda a + c]$.

Then let $f'(z) = 2(1-b)z + (b-2)(\lambda a + c) = 0$ leads to $z = \tilde{z} = \dfrac{(\lambda a + c)(b-2)}{2(b-1)} \in I$.

Furthermore, we notice that $f''(\tilde{z}) < 0$, so we can obtain that

$$\sup_{z \in R} f(z) = \frac{(\lambda a + c)^2 (b-2)^2}{4(b-1)} .$$

From (5) we can derive

$$\begin{aligned}
\dot{V} &= -\lambda a x^2 - y^2 - (z - \lambda a - c)^2 - \lambda a d w^2 + (1-b)z^2 + (b-2)(\lambda a + c)z + (\lambda a + c)^2 \\
&\le -\lambda x^2 - y^2 - (z - \lambda a - c)^2 - \lambda d w^2 + f(z) + (\lambda a + c)^2 \\
&\le -\lambda x^2 - y^2 - (z - \lambda a - c)^2 - \lambda d w^2 + f(\tilde{z}) + (\lambda a + c)^2 \\
&\le -\lambda x^2 - y^2 - (z - \lambda a - c)^2 - \lambda d w^2 + \gamma^2(\alpha) \\
&= -2V + \gamma^2(\alpha)
\end{aligned}$$

When $\lambda x^2 + y^2 + (z - \lambda a - c)^2 + \lambda d w^2 \ge \gamma^2(\alpha)$, According to the comparison theorem, we have

$$V(X(t)) \le V(X_0) e^{-2(t-t_0)} + \int_{t_0}^{t} e^{-2(t-\tau)} \gamma^2(\alpha) d\tau = V(X_0) e^{-2(t-t_0)} + \frac{\gamma^2(\alpha)}{2}\left(1 - e^{-2(t-t_0)}\right)$$

When $V(X(t)) > \dfrac{\gamma^2(\alpha)}{2} (t > t_0)$, there exists exponential estimation given as follows:

$$\left\langle V(X(t)) - \frac{\gamma^2(\alpha)}{2} \right\rangle \le \left\langle V(X_0) - \frac{\gamma^2(\alpha)}{2} \right\rangle e^{-2(t-t_0)} \qquad (6)$$

(ii) When $a > \dfrac{b}{2}, b < 2$, from (5) we can get

$$\begin{aligned}
\dot{V} &= -\lambda a x^2 - y^2 - b z^2 - \lambda a d w^2 + \lambda a b z + b c z \\
&\le -\lambda \frac{b}{2} x^2 - \frac{b}{2} y^2 - \lambda \frac{b}{2} d w^2 - \frac{b}{2} z^2 + \frac{b}{2}\left[2(\lambda a + c)\right] z \\
&= \frac{b}{2}\left[-\lambda x^2 - y^2 - (z - \lambda a + c)^2 - \lambda d w^2\right] + \frac{b}{2}(\lambda a + c)^2 \\
&= -\frac{b}{2}\left[2V + \gamma^2(\alpha)\right] \le 0 \text{ (when } \lambda x^2 + y^2 + (z - \lambda a - c)^2 + \lambda d w^2 \ge \gamma^2(\alpha) \text{)}
\end{aligned}$$

Similarly, when $a < 1, b \ge 2a$, $V(X(t)) > \dfrac{\gamma^2(\alpha)}{2} (t > t_0)$, we can get an exponential estimation given as follows:

$$\left\langle V(X(t)) - \frac{\gamma^2(\alpha)}{2} \right\rangle \le \left\langle V(X_0) - \frac{\gamma^2(\alpha)}{2} \right\rangle e^{-b(t-t_0)} \qquad (7)$$

(iii)When $a < 1, b \ge 2a$. Similarly to (i). (ii), we can also get:

When $V\left(X\left(t\right)\right) > \frac{\gamma^2\left(\alpha\right)}{2}\left(t > t_0\right)$, we have

$$\left\langle V\left(X\left(t\right)\right) - \frac{\gamma^2\left(\alpha\right)}{2}\right\rangle \le \left\langle V\left(X_0\right) - \frac{\gamma^2\left(\alpha\right)}{2}\right\rangle e^{-a\left(t-t_0\right)} \qquad (8)$$

From formula (6), (7), (8) we obtain

$$\lim_{t\to+\infty} V\left(X\left(t\right)\right) \le \frac{\gamma^2\left(\alpha\right)}{2}$$

Hence $\Omega = \left\{\left(x,y,z,w\right)\middle| \lambda x^2 + y^2 + \left(z-\lambda a - c\right)^2 + \lambda d w^2 \le \gamma^2\left(\alpha\right)\right\}$ is the globally expone- ntially attractive set of system (1). At the same time, according (6), (7), (8), it implies that Ω is also the ultimate bound for system (1). Especially, let us take $\lambda = 1$, we can get the following theorem.

Theorem 2. All compact invariant sets of the system (1) are contained in the ellipsoid defined by

$$K\left(h_1\right) = \left\{\left(x,y,z,w\right)\middle| x^2 + y^2 + \left(z-a-c\right)^2 + dw^2 \le \gamma^2\left(\alpha\right)\right\}$$

With

$$h_{1\sup} = \gamma^2\left(\alpha\right) = \begin{cases} \dfrac{\left(a+c\right)^2 b^2}{4\left(b-1\right)} & a \ge 1, b \ge 2 \\ \left(a+c\right)^2 & a > \dfrac{b}{2}, b < 2 \\ \dfrac{\left(a+c\right)^2 b^2}{4a\left(b-a\right)} & 0 < a < 1, b \ge 2a \end{cases}$$

4. APPLICATIONS OF THE ITERATION THEOREM

We note that $K\left(h_1\right)$ is contained in the polytope Π defined by

$$\left|x\right| \le \gamma\left(\alpha\right); \left|y\right| \le \gamma\left(\alpha\right); \left|z\right| \le a + c + \gamma\left(\alpha\right); \left|w\right| \le \frac{\gamma\left(\alpha\right)}{\sqrt{d}}$$

Now we can apply additional localizing functions, let us take $h_2\left(x,y,z,w\right) = w$.

Then $S\left(h_2\right)$ is given by $w = -\frac{x}{a}$, $h_2\big|_{S\left(h_2\right)} = -\frac{x}{a}$.

Therefore the set $S\left(h_2\right) \cap K\left(h_1\right) = \left\{\left(x,y,z,w\right)\middle| w = -\frac{x}{a}, x^2 + y^2 + \left(z-a-c\right)^2 + dw^2 \le \gamma^2\left(\alpha\right)\right\}$. which is contained in the set $S\left(h_2\right) \cap \Pi = \left\{\left(x,y,z,w\right)\middle| w = -\frac{x}{a}, x^2 \le \gamma^2\left(\alpha\right)\right\}$.

Therefore

$$h_{2\sup} = \frac{\gamma\left(\alpha\right)}{a}, h_{2\inf} = -\frac{\gamma\left(\alpha\right)}{a}.$$

And

$$K_{12} = \left\{\left(x,y,z,w\right)\middle| -\frac{\gamma\left(\alpha\right)}{a} \le w \le \frac{\gamma\left(\alpha\right)}{a}\right\}.$$

It is clearly that, the latter can be refined by the formula

$$K_{12} = \left\{\left(x,y,z,w\right)\middle| w \le \min\left(\frac{\gamma\left(\alpha\right)}{a}, \frac{\gamma\left(\alpha\right)}{\sqrt{d}}\right)\right\}$$

Here and below we use the same notations for sets K_{ij} and sets with approximate corresponding bounds.

Hence, $K_2 = \left\{ (x,y,z,w) \middle| x^2 + y^2 + (z-a-c)^2 + dw^2 \le \gamma^2(\alpha), w \le \min\left(\frac{\gamma(\alpha)}{a}, \frac{\gamma(\alpha)}{\sqrt{d}} \right) \right\}$. (9)

Also, concerning the lower bound on w, it follows (9) that we can get

$$|w| \le \frac{\gamma(\alpha)}{a} \quad 0 < d < a^2$$

for

$$|w| \le \frac{\gamma(\alpha)}{\sqrt{d}} \quad d > a^2$$

5. LOCALIZING BY USING THE CIRCULAR CYLINDER

let us take $h_3(x,y,z,w) = y^2 + z^2 - 2cz$, then $S(h_3)$ is given by $y^2 = -bz^2 + bcz$

and

$$h_3\big|_{S(h_3)} = (1-b)\left(z + \frac{c(b-2)}{2(1-b)} \right)^2 + R(b,c),$$

with

$$R(b,c) = -\frac{c^2(b-2)^2}{4(1-b)}.$$

So we get

$$h_{3\sup} = R(b,c), \text{ if } 0 < b < 1 \quad (10)$$

$$h_{3\inf} = R(b,c), \text{ if } b > 1 \quad (11)$$

Hence all compact invariant sets of system (1) are located in parabolic cylinder $K(h_3)$ defined by $K_3 = \left\{ (x,y,z,w) \middle| h_3 \le h_{3\sup} \right\}$; $K_3 = \left\{ (x,y,z,w) \middle| h_3 \ge h_{3\inf} \right\}$, in the dependence on which pair of inequalities (10), (11) is fulfilled.

6. LOCALIZING WITH HELP OF CYLINDRICAL SURFACE

Here we take a localizing function $h_4(x,y,z,w) = x^2 + dw^2 - 2az$, then the set $S(h_4)$ is given by the equation $z = \frac{x^2}{b} + \frac{dw^2}{b}$, therefore by using this formula we can get that

$$h_4\big|_{S(h_4)} = x^2 + dw^2 - 2a\left(\frac{x^2}{b} + \frac{dw^2}{b} \right)$$

$$= \frac{b-2a}{b} x^2 + \frac{(b-2a)d}{b} w^2$$

So

$$h_{4\sup} = 0, \text{ if } 0 < b < 2a. \quad (12)$$

$$h_{4\inf} = 0 \text{ .,if } b > 2a. \quad (13)$$

Hence all compact invariant sets of system (1) are located in the parabolic cylinder $K(h_4)$ defined by $K_4 = \left\{ (x,y,z,w) \middle| h_4 \le h_{4\sup} \right\}$; $K_4 = \left\{ (x,y,z,w) \middle| h_4 \ge h_{4\inf} \right\}$, in the dep -endence on which pair of inequalities (12), (13) is fulfilled

Theorem 3. All compact invariant sets of the system (1) are contained in the set defined by

$$\left\{(x,y,z,w)\middle| x^2+y^2+(z-a-c)^2+dw^2\le\gamma^2(\alpha), w\le\min\left(\frac{\gamma(\alpha)}{a},\frac{\gamma(\alpha)}{\sqrt{d}}\right)\right\}\cap K_3\cap K_4$$

Provided restrictions that imposed on parameters hold. That is to say in the dependence on which pair of inequalities (10), (11), (12), (13) is fulfilled.
With

$$h_{1\sup}=\gamma^2(\alpha)=\begin{cases}\dfrac{(a+c)^2b^2}{4(b-1)} & a\ge 1, b\ge 2\\ (a+c)^2 & a>\dfrac{b}{2}, b<2\\ \dfrac{(a+c)^2b^2}{4a(b-a)} & 0<a<1, b\ge 2a\end{cases}$$

7. ONE EXAMPLE OF THE LOCALIZATION OF A CHAOTIC ATTRACTOR

In this section, we describe one example of the localization of a chaotic attractor which was found [19], Parameters of the system (1) are chosen a=1, b=0.7,c=26, d = 1.5. For convenience, we depict some ellipsoid according to Theorem 2. According to Theorem 2 , we can attain $\gamma(\alpha)=a+c=27$ .We show a figure of the chaotic attractor estimated in Theorem 2 and the phase portrait for system (1) that projected into the (x, y, z), (x, y, w), (x, z, w), (y, z,w) space. see Fig.3

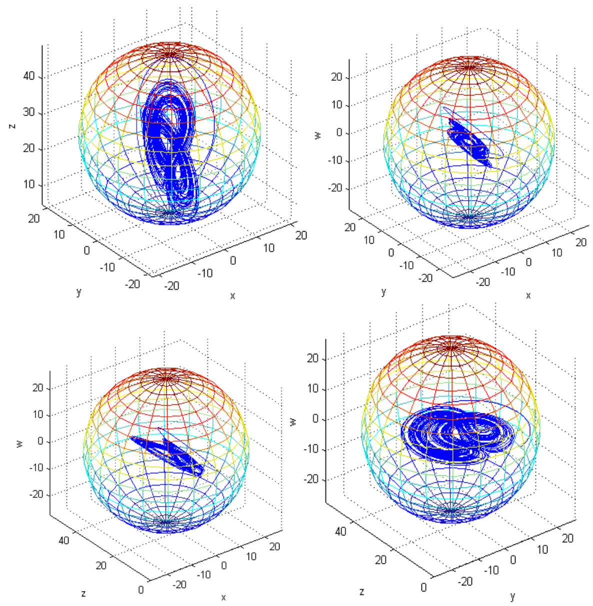

Fig.3 . The projections of the chaotic attractor into the (x, y, z), (x, y, w), (x, z, w), (y, z,w) space according to in Theorem 2.

8. SYNCHRONIZATION OF THE NEW CHAOTIC SYSTEM

Let the following the system (1.1) be the driver system,

$$\begin{cases}\dot{x}=ay-ax+cw\\ \dot{y}=dx-xz-y\\ \dot{z}=xy-bz\\ \dot{w}=-x-aw\end{cases}\qquad(1.1)$$

and the response system is:

$$\begin{cases} \dot{y}_1 = a\left(y_2 - y_1\right) + cy_4, \\ \dot{y}_2 = dy_1 - y_1y_3 - y_2 - k\left(y_2 - x_2\right), \\ \dot{y}_3 = y_1y_2 - by_3, \\ \dot{y}_4 = -y_1 - ay_4, \end{cases} \tag{14}$$

System (1.1) can synchronize the system (14) by adjusting parameter k. From theorem 2, then we can get the boundness of y, z that is $|y| \le \gamma(\alpha)$, $|z| \le \gamma(\alpha) + |d| + |a|$. For convenience, let us denote $M_y = \gamma(\alpha)$, $\gamma(\alpha) + |d| + |a| = M_z$, then we have the following theorem.

Theorem 4 . System (1.1) and system (14) are globally completely synchronize. When

$$k > \frac{b\left(\rho a + d + M_z\right)^2}{4\rho ab - M_y^2} - 1 \text{ (here } \rho > \frac{M_y^2}{4ab} > 0 \text{)}$$

Proof: Let the state errors be $e_1 = y_1 - x, e_2 = y_2 - y, e_3 = y_3 - z, e_4 = y_4 - w$, then the error dynamics of system (1.1) and (14) is

$$\begin{cases} \dot{e}_1 = a\left(e_2 - e_1\right) + ce_4, \\ \dot{e}_2 = de_1 - y_1e_3 - ze_1 - \left(k+1\right)e_2, \\ \dot{e}_3 = y_1e_2 + ye_1 - be_3, \\ \dot{e}_4 = -e_1 - ae_4, \end{cases} \tag{15}$$

Let $V = \frac{1}{2}\left(\rho e_1^2 + e_2^2 + e_3^2 + \rho c e_4^2\right)$ where ρ is a real parameter and $\rho > \frac{M_y^2}{4ab}$. Then its time derivative along the system (15) is

$$\begin{aligned} \dot{V} &= \rho\dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \rho c\dot{e}_4e_4 \\ &= \rho a e_1e_2 - \rho a e_1^2 + \rho c e_1e_4 + de_1e_2 - y_1e_2e_3 - ze_1e_2 - \left(k+1\right)e_2^2 \\ &\quad + y_1e_2e_3 + ye_1e_3 - be_3^2 - \rho c e_1e_4 - \rho ac e_4^2 \\ &= -\rho a e_1^2 - \left(k+1\right)e_2^2 - be_3^2 - \rho ac e_4^2 + \left(\rho a + d - z\right)e_1e_2 + ye_1e_3 \\ &\le -\rho a e_1^2 - \left(k+1\right)e_2^2 - be_3^2 - \rho ac e_4^2 + \left(\rho a + d + M_z\right)|e_1||e_2| + M_y|e_1||e_3| \\ &= -E'PE \end{aligned}$$

Where $E = \left[|e_1|, |e_2|, |e_3|, |e_4|\right]^T$,

$$P = \begin{pmatrix} \rho a & -\frac{\rho a + d + M_z}{2} & -\frac{M_y}{2} & 0 \\ -\frac{\rho a + d + M_z}{2} & k+1 & 0 & 0 \\ -\frac{M_y}{2} & 0 & b & 0 \\ 0 & 0 & 0 & \rho ac \end{pmatrix}$$

By some elementary calculation, we know that the matrix P is positively definite when

$$k > \frac{b\left(\rho a + d + M_z\right)^2}{4\rho ab - M_y^2} - 1, \rho > \frac{M_y^2}{4ab}.$$

This implies that the origin of the error system (15) is asymptotic stable, which is equivalent to say that the system (1.1) can synchronize the system (14) completely.

9. SIMULATION STUDIES

The numerical simulations are carried out using the MATLAB 7.4. The initial conditions of the driver (1.1) and response systems (14) are $(0.2,2,0.5,0.9)$ and $(1,1.8,0.9,0.3)$. When a=1, b=0.7, c=1.5, d=26.[19], it is easy to obtain $\gamma(\alpha)=27$, $M_y=27$, $M_z=54$, So then the bound of the coefficients of feedback control k could be obtained according to the Theorem 4. Choose k=286, The response system synchronizes with the drive system as shown in Fig.4.

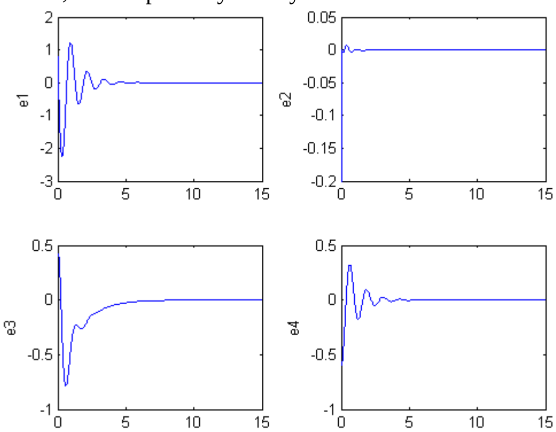

Fig.4. Synchronization error of the two systems under linear feedback control

10. CONCLUSIONS

To estimate a domain containing all compact invariant sets of a dynamical system i- s an important but quite challenging task in general. In this paper we study the localization problem of compact invariant sets of the new chaotic system with the help of the iteration theorem and the first order extremum theorem. Conclusions about iteration theorem is not obtained from results of this paper, it was obtained in papers of Luis N. Coria and Konstantin E. Starkov. We also establish that all compact invariant sets of this system are located in the intersection of a ellipsoid with a frusta and we also compute its parameters. In addition, localization by using the two-parameter set of the circular cylinder is described. Then, a domain containing all compact invariant sets has been established. Finally, the bound for y, z is applied to chaos synchronization. Numerical simulations show the effectiveness and the advantage of our methods.

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