INFERENCE AND HYPOTHESIS TESTING FROM METHODS OF PARAMETERS ESTIMATION

Oseni B.A[1]& Oyenuga I.F[2]
Department of Mathematics & Statistics, The Polytechnic, Ibadan
Email: bolaoseni,2007@yahoo.com

ABSTRACT
In this paper, we present the general problem of statistics, that of making inferences and parameters estimation. Inferences are drawn based on two classical methods of estimations namely, moment method and maximum likelihood estimation. Inferences are drawn from theoretical and numerical examples when population variances are known.

1. INTRODUCTION

Estimation of parameters and drawing of inferences is one of the aspect of classification of statistics (Inferentials statistics). On the other hand, hypothesis testing which is defined as a formal procedure not too different from that taken by a skilled gambler. Statistics is concerned with problems having elements of uncertainty. An experiment involving random variables is performed, and decisions are based on the values assumed by the random variables. The experiment may be a survey, a poll, a sample of product output from a manufacturing process, an examination and so forth. The uncertainty is reflected in the fact that the same experiment repeated again under the same experimental conditions usually gives different sample results. However, it is difficult and often impossible to obtain a truly random sample. In this paper, we are going to discuss the general problem of statistics, that of making inferences about the distribution function f(X) from the sample observation ($x_1, x_2, \ldots, x_n$). This problem can be classified into two closely related categories of problems: (i) testing certain hypothesis regarding the nature of a population, i.e either the nature of the function representing the population, or the values of the parameters of the function and (ii) specifying f(X) when the type of function or family is known but certain parameters are unknown- this is called estimation.

In this paper, we are going to draw inferences based on two classical methods of estimation viz: moment method and maximum likelihood estimation (m.l.e). Let $X_1, X_2, \ldots, X_n$ be a sample of observations from a population with the distribution function $F(x/\theta_1,\ldots\ldots,\theta_k)$ where $\theta_1,\ldots\ldots,\theta_k$ are unknown parameters to be estimated based on the sample.

Keywords: Likelihood, estimators, inference, parameters, LRT, pivotal quantity.

1.1: Moment Estimation:

Definition: Let $f(x/\theta_1,\theta_2,\ldots\ldots,\theta_k)$ denote the probability density function(pdf) or probability mass function (pmf) of a random variable X with cumulative distribution function (cdf) $F(x/\theta_1,\theta_2\ldots\ldots,\theta_k)$. The moment about the origin are usually functions of $\theta_1,\theta_2\ldots\ldots,\theta_k$. It can be noted that $E\left[X_i^k\right]=E(X_1^k)$, i =2,3,. . . ., n because the X_i's are identically distributed. The moment estimators can be obtained by solving the following system of equations for $\theta_1,\theta_2\ldots\ldots,\theta_k$.

$$\frac{1}{n}\sum_{i=1}^{n}X_i = E(X_1)$$

$$\frac{1}{n}\sum_{i=1}^{n}X_i^2 = E(X_1^2) \qquad \ldots\ldots\ldots\ldots\ldots..(1)$$

. .
. .
. .

$$\frac{1}{n}\sum_{i=1}^{n}X_i^k = E(X_1^k)$$

, for X a discrete random variable

and

$E\left[X_1^j\right] = \int_{-\infty}^{\infty} x^j f(x/\theta_1,\theta_2,...,\theta_k)dx,$ j=1,2,…,k for X a continuous random variable.

1.2 Maximum Likelihood Estimation

Definition: For a given sample x = (x_1, x_2,…, x_n), the function defined by

$L\left(\theta_1,\theta_2,...,\theta_k / x_1,x_2,...,x_n\right) = \prod_{i=1}^{n} f\left(x_i / \theta_1,\theta_2,...,\theta_k\right)$ is called the likelihood function. The maximum likelihood estimators are the values of $\theta_1,\theta_2,...,\theta_k$ that maximimize the likelihood function.

To obtain the parameters $\theta_1,\theta_2,...,\theta_k$ we differentiate partially the log of m. l. e. with respect to $\theta_1, i = 1,2,...,k$ and equates to zero and solved

1.3 Inferences

Let X= (x_1, x_2, …,x_n) be a random sample from a population, and let x=(x_1, x_2,…, x_n), where x_i is an observed value of X_i,i=1,2,…,n. for simplicity, let us assume that the distribution function $F(x/\theta)$ of the population depends only on a single parameter θ. In the sequel, p($X \le x/\theta$) means the probability that X is less than or equal to x when θ is the parameter of the distribution of X.

1.4 The Likelihood Ratio Test (LRT)

Let X= (x_1, x_2, …,x_n) be a random sample from a population with the pdf ($F(x/\theta)$). And further let $x = (x_1, x_2,..., x_n)$ be an observed sample. Then the likelihood function is given by

$$L(\theta / x) = \prod_{i=1}^{n} f\left(x_i / \theta\right)$$

The LRT statistic for testing hypotheses $H_o : \theta \in \theta_0$ versus $H_\alpha : \theta \in \theta_0^c$ = where H_0 is called the null hypothesis, H_α is called the alternative or research hypothesis, θ_0^c denotes the complement set of θ_0 and $\theta_0 \cup \theta_0^c = \theta$ is given by

$$\lambda(x) = \frac{\sup \theta_o L(\theta/x)}{\sup \theta L(\theta/x)} \quad \text{……………………(2)}$$

and $\lambda(x)$ lies in the interval $0 < \lambda(x) < 1,$ and the LRT rejects H_o for smaller values of $\lambda(x)$.

Inferential procedures are usually developed based on a statistic T(X) called pivotal quantity. The distribution of T(X) can be used to make inferences on θ . The value of T(X) is called the observed value of T(X). That is, T(X) is the numerical based on the observed sample x.

1.5.1 Theoretical example

Given that y_1, y_2,…., y_n is a sample of observations from a population $N(\mu, \sigma_0^{\ 2})$ where $\sigma_0^{\ 2}$ is known. Test the hypothesis $H_0 : \mu = \mu_o$ versus $H_\alpha : \mu = \mu_\alpha > \mu_0$.

Solution,

Since $f_j(y_j, \mu, \sigma_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \ell^{-\frac{1}{2}\left(\frac{yj-\mu}{\sigma_0}\right)^2}$

and

$$\ell(y_j, \mu, \sigma_0) = \frac{1}{\sigma_0{}^n (2\pi)^{n/2}} \ell^{-1/2 \sum \left(\frac{yj-\mu}{\sigma_0}\right)^2}$$

$$l_{\alpha/0} = \frac{\frac{1}{\sigma_0{}^n (2\pi)^{n/2}} \ell^{-1/2 \sum \left(\frac{yj-\mu_\alpha}{\sigma_0}\right)^2}}{\frac{1}{\sigma_0{}^n (2\pi)^{n/2}} \ell^{-1/2 \sum \left(\frac{yj-\mu_0}{\sigma_0}\right)^2}},$$

simplified to

$$\ell_{\alpha/0} = \ell^{-\frac{1}{2\sigma_0{}^2}\left(\sum (y_j - \mu_\alpha)^2 - \sum (y_j - \mu_0)^2\right)} \quad \text{………………..(3)}$$

By Neyman Pearson Lemma, best critical region is given by $\ell_{(\alpha/0)} > C_{\alpha.}$
Then

$$\log \ell_{\alpha/0} = -\frac{1}{2\sigma_0{}^2}\left[\sum (y_j - \mu_\alpha)^2 - \sum (y_j - \mu_0)^2\right] > \log C_\alpha$$

simplifies to

$$\log \ell_{\alpha/0} = -\frac{1}{2\sigma_0{}^2}\left\{\sum \left[2y_j - (\mu_\alpha + \mu_0)\right](\mu_\alpha - \mu_0)\right\} > \log C_\alpha$$

after simplification we then have

$$\bar{Y} > \frac{1}{2n}\left[\frac{2\sigma_0{}^2 \log C_\alpha}{\mu_\alpha - \mu_0} + n(\mu_0 + \mu_\alpha)\right] \quad \text{………….. (4)}$$

That is

$\bar{Y} > d\alpha$ where $d\alpha$ is the expression on the right hand side to which we want to find an alternative $P(\bar{Y} > d\alpha / H_\alpha) = \alpha$

Since $y_j \sim N(\mu, \sigma_0{}^2)$

$$\therefore Z = \frac{\bar{Y} - \mu}{\sigma_0 / \sqrt{n}} \sim N(0,1)$$

Here, we require Kα such that $P(Z \geq k_\alpha / H_\alpha) = \alpha$

i.e $P\left(\frac{\bar{Y} - \mu}{\sigma_0 / \sqrt{n}} > k_\alpha / H_\alpha\right) = \alpha$

Hence $P(\bar{Y} > \mu_0 + \frac{k_\alpha \sigma_0}{\sqrt{n}}) = \alpha$

Therefore, test-statistic is $\bar{Y}$.

$d_\alpha = \mu_0 + \frac{k_\alpha \sigma_0}{\sqrt{n}}$ and

$$W_\alpha = \left(\bar{Y} : \bar{Y} > \mu_0 + \frac{k_\alpha \sigma_0}{\sqrt{n}} \right) \quad \text{..................(5)}$$

1.5.2 Numerical example

The following observations: 105.7, 91.9, 101.8, 100.4, 89.0, 100.2, 105.5, 105.1, 107.6 and 96.3 are from normal population with variance 16 i.e $y_j \sim N(\mu, 16)$.

Test $H_0 : \mu = 80$ versus $H_\alpha : \mu = 80$ at α = 0.05

Using equation (5) above

$\bar{Y} = 100.35$

But
$$W\alpha = \left(\bar{Y} : \bar{Y} \geq \mu_0 + \frac{k_\alpha \sigma_0}{\sqrt{n}} \right)$$

where

$\mu_0 + \frac{k_\alpha \sigma_0}{\sqrt{n}}$ gives 82.09

Hence, $W\alpha = \left(\bar{Y} : \bar{Y} > 82.09 \right)$

Implies that

$\bar{Y} \in W\alpha$

$\therefore$ Reject H_o and conclude that $\mu > 80$

REFERENCES

[1]. K. Krishnamoorthy (2006): Handbook of Statistical Distributions with Applications. Chapman & Hall/CRC, Taylor & Francis Group New York.

[2]. Odeyinka, J. A. & Oseni, B.A (2008): Basic Tools in Statistical Theory. Highland Publishers, Ibadan.

[3]. Elwood G. Kirkpatrick (1974): Introductory Statistics and Probability for Engineering, Science and Technology. U. S. A.