COMMON FIXED POINT THEOREM FOR FOUR MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE USING ABSORBING MAPS

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ABSTRACT
The aim of this paper is to obtain a common fixed point theorem for four mappings by using absorbing maps in intuitionistic fuzzy metric space.

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1. INTRODUCTION
Zadeh [12] introduced the notion of fuzzy sets. Later many authors have extensively developed the theory of fuzzy metric sets and application.The idea of fuzzy metric space introduced by Kramosil and Michalek [7] was modified by George and Veeramani [6]. Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a gener-alization of fuzzy sets and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [3]. In 2004, Park [8] introduced a notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [7].The aim of this paper is to introduce the new notion of absorbing maps, it is not necessary that absorbing maps commute at their coincidence points however if the mapping pair satisfy the contractive type condition then point wise absorbing maps not only commute at their coincidence points but it becomes a necessary condition for obtaining a common fixed point of mapping pair.Let A and B are two self maps on a intuitionistic fuzzy metric space (X, M,N, *, ◊) then f is called B - absorbing if there exists a positive integer R > 0 such that

$M(Bx, BAx, t) \geq M(Bx, Ax, t/R)$

$N(Bx, BAx, t) \leq N(Bx, Ax, t/R)$ for all $x \in X$

Similarly B is called A - absorbing if there exists a positive integer R > 0 such that $M(Ax, ABx, t) \geq M(Ax, Bx, t/R)$ & $N(Ax, ABx, t) \leq N(Ax, Bx, t/R)$ for all x. The map A is called point wise B - absorbing if for given x $\in$ X, there exists a positive integer R > 0 such that $M(Bx, BAx, t) \geq M(Bx, Ax, t/R)$

$N(Bx, BAx, t) \leq N(Bx, Ax, t/R)$ for all $x \in X$,similarly we can defined point wise
A - absorbing maps.

2. Preliminaries and Notations
In this section we recall some definitions and known results in fuzzy metric space.

Definition 2.1. [12] Let **X** be any non empty set. A fuzzy set **A** in **X** is a function with domain **X** and values in [0,1].

Definition 2.2. [2] Let a set E be fixed. An intuitionistic fuzzy set (IFS) A of E is an object having the form,A = {<x, μA(x), VA(x) >/x ∈ E}
where the function $\mu_A : E \to [0, 1]$, $V_A : E \to [0, 1]$ define respectively, the degree of membership and degree of non-membership of the element x $\in$ E to the set A, which is a subset of E, and for every x $\in$ E, $0 \leq \mu A(x)+VA(x) \leq 1$

Definition 2.3. [9] A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval [0,1] such that for all a, b, c, d $\in$ [0, 1] the following conditions are satisfied:
(1) $a * 1 = a$;
(2) $a * b = b * a$;
(3) $a * b = c * d$ whenever a = c and b = d ;
(4) $a * (b * c) = (a * b) * c$.

Definition 2.4. [9] A binary operation $\Diamond$:[0, 1] × [0, 1] → [0, 1] is a continuous t-conorm if it satisfies the following conditions:

(a) $\Diamond$ is commutative and associative;

(b) $\Diamond$ is continuous;

(c) $a \Diamond 0 = a$ for all $a \in [0, 1]$;

(d) $a \Diamond b = c \Diamond d$ whenever $a \geq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.5. [1] A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$;

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$;
(IFM-2) $M(x, y, 0) = 0$;
(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$;
(IFM-4) $M(x, y, t) = M(y, x, t)$;
(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(IFM-6) $M(x, y, .): [0, \infty) \rightarrow [0, 1]$ is left continuous;
(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$
(IFM-8) $N(x, y, 0) = 1$;
(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$;
(IFM-10) $N(x, y, t) = N(y, x, t)$;
(IFM-11) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$;
(IFM-12) $N(x, y, .): [0, \infty) \rightarrow [0, 1]$ is right continuous;
(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions M (x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.6. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space if X of the form $(X, M, 1 - M, *, \Diamond)$ such that t- norm $*$ and t-conorm $\Diamond$ are associated, that is, $x \Diamond y = 1-((1 - x) * (1 - y))$ for any $x, y \in X$. But the converse is not true.

Example 2.7. [7] Let (X, d) be a metric space. Denote $a * b = ab$ and $a \Diamond b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows;

$M_d(x, y, t)= t/t + d(x, y)$, $N_d(x, y , t)= d(x, y)/ t + d(x, y)$.

Then (M_d,N_d) is an intuitionistic fuzzy metric on X . We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.8. Note the above example holds even with the t-norm $a * b = \min \{a, b\}$ and the t-conorm $a \Diamond b = \max\{a, b\}$ and hence (M_d,N_d) is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t- conorm.

Example 2.9. Let X = N. Define $a * b = \max \{0, a+ b - 1\}$ and $a \Diamond b = a + b - ab$ for all $a,b \in [0, 1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows;

$$M(x, y, t) = \begin{cases} x / y, x \leq y \\ y / x, y \leq x \end{cases} \text{ and } N(x,y,t) = \begin{cases} y - x / y, x \leq y \\ x - y / x, y \leq x \end{cases}$$

all $x, y, z \in X$ and $t > 0$. Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzymetric space.

Definition 2.11. [1] Let (X, M, N, $*$, $\Diamond$) be an intuitionistic fuzzy metric space. (a) A sequence $\{x_n\}$ in X is called cauchy sequence if for each t >0 and P>0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$

(b)A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each t >0.

(c) An intuitionistic fuzzy metric space is said to be complete if every Cauchy sequence is convergent.

Definition 2.12. Let A and S be mappings from an intuitionistic fuzzy metric space (X, M, N, $*$, $\Diamond$) into itself. Then the mappings are said to be reciprocally continuous if $\lim_{n\to\infty} ASx_n = Az$, and $\lim_{n\to\infty} SAx_n = Sz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} ASx_n = \lim_{n\to\infty} SAx_n = z$, for some $z \in X$.

Remark 2.20. If A and S are both continuous then they are obviously reciprocally continuous. But the converse need not be true.We shall use the following lemmas to prove our next result without any further citation:

Lemma 2.12. [8] In an intuitionistic fuzzy metric space X , M(x, y, .) is non-decreasing and N (x, y, .) is non-increasing for all $x, y \in X$.

Lemma 2.13. [10] Let (X, M, N, $*$, $\Diamond$) be an intuitionistic fuzzy metric space. If there exists a constant k . (0, 1) such that

$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$, $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

For every t >0 and n =1, 2,.. then $\{y_n\}$ is a cauchy sequence in X .

Lemma 2.14. [10] Let (X, M, N, $*$, $\Diamond$) be an intuitionistic fuzzy metric space.If there exists a constant $k \in (0, 1)$ such that

$M(x, y, kt) \geq M(x, y, t)$, $N(x, y, kt) \leq N(x, y, t)$,for $x, y \in X$. Then x = y.

3. Main Results

Theorem 1. Let P be point wise S - absorbing and Q be point wise T – absorbing self maps on a complete Intuitionistic fuzzy metric space (X, M, N, $*$, $\Diamond$) with continuous t- norm defined by $a * b = \min\{a, b\}$ where $a, b \in [0, 1]$, satisfying the conditions:

(1.1) $P(X) \subseteq T(X)$, $Q(X) \subseteq S(X)$

(2.2) There exists $k \in (0, 1)$ such that for every $x, y \in X$ and t > 0,

$$M(Px, Qy, kt) \geq \min.\{M(Sx, Ty, t), M(Px, Sx, t), M(Qy, Ty, t), M(Px, Ty, t)\ M(Px, Qy, t)\ M(Sx, Qy, t)\}$$

$$N(Px, Qy, kt) \leq \max.\{N(Sx, Ty, t), N(Px, Sx, t), N(Qy, Ty, t), N(Px, Ty, t)\ N(Px, Qy, t)\ N(Sx, Qy, t)\}$$

(3.3) for all $x, y \in X$, $\lim_{t\to\infty} M(x, y, t) = 1$, $\lim_{t\to\infty} N(x, y, t) = 0$

If the pair of maps (P, S) is reciprocal continuous compatible maps then P, Q, S and T have a unique common fixed point in X.

Proof: let x_0 be any arbitrary point in X, construct a sequence $y_n \in X$ such that $y_{2n-1} = Tx_{2n-1} = Px_{2n-2}$ and $y_{2n} = Sx_{2n} = Qx_{2n+1}$, n = 1, 2, 3……..This can be done by the virtue of (1.1). By using contractive condition we obtain,

$M(y_{2n+1}, y_{2n+2}, kt) = M(Px_{2n}, Qx_{2n+1}, kt) \geq \min\{M(Sx_{2n}, Tx_{2n+1}, t),$
$M(Px_{2n}, Sx_{2n}, t), M(Qx_{2n+1}, Tx_{2n+1}, t), M(Px_{2n}, Tx_{2n+1}, t), M(Px_{2n}, Qx_{2n+1}, t), M(Sx_{2n}, Qx_{2n+1}, t)\}$
$\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n+1}, t),$
$M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n}, t)\}$
$\geq \min\{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), 1, M(y_{2n+1}, y_{2n}, t), 1\}$
$\geq M(y_{2n}, y_{2n+1}, t)$

$N(y_{2n+1}, y_{2n+2}, kt) = N(Px_{2n}, Qx_{2n+1}, kt) \leq \max.\{N(Sx_{2n}, Tx_{2n+1}, t),$
$N(Px_{2n}, Sx_{2n}, t), N(Qx_{2n+1}, Tx_{2n+1}, t), N(Px_{2n}, Tx_{2n+1}, t), N(Px_{2n}, Qx_{2n+1}, t), N(Sx_{2n}, Qx_{2n+1}, t)\}$

$\leq$ max.{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t),N(y_{2n+1}, y_{2n+1}, t),
N (y_{2n+1}, y_{2n}, t), N (y_{2n}, y_{2n}, t)}
$\leq$ max.{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t), N(y_{2n}, y_{2n+1}, t),0, N(y_{2n+1}, y_{2n}, t), 0}
$\leq$ N (y_{2n}, y_{2n+1}, t) which implies
M (y_{2n+1}, y_{2n+2}, kt) $\geq$ M (y_{2n}, y_{2n+1}, t)
N (y_{2n+1}, y_{2n+2}, kt) $\leq$ N (y_{2n}, y_{2n+1}, t)
In general, M (y_n, y_{n+1}, kt) $\geq$ M (y_{n-1}, y_n, t)
N (y_n, y_{n+1}, kt) $\leq$ N(y_{n-1}, y_n, t)................................ (1)
To prove {y_n } is a Cauchy sequence, we have to show M (y_n, y_{n+1}, t) $\rightarrow$ 1 and
N (y_n, y_{n+1}, t) $\rightarrow$ 0 (for t > 0 as n $\rightarrow \infty$ uniformly on p $\in$ N), for this from (2.3) we have, M (y_n , y_{n+1}, t) $\geq$ M (y_{n-1}, y_n , t/k) $\geq$ M (y_{n-2} , y_{n-1} , t/k^2) $\geq$ .... $\geq$ M (y_0 , y_1, t/k^n)$\rightarrow$1 as n $\rightarrow \infty$
N(y_n , y_{n+1} , t) $\leq$ N (y_{n-1} , y_n , t/k) $\leq$ N (y_{n-2} , y_{n-1} , t/k^2) $\leq$ .... $\leq$ N (y_0 , y_1 , t/k^n)$\rightarrow$0 as n $\rightarrow \infty$
for p $\in$ N , by (1) we have
M (y_n , y_{n+p} , t) $\geq$ M (y_n , y_{n+1} , (1 - k)t) * M(y_{n+1} , y_{n+p} , kt)
$\geq$ M (y_0 , y_1 , (1 - k)t/k^n) * M (y_{n+1} , y_{n+2} , t) * M(y_{n+2} , y_{n+p} , (k - 1)t)
$\geq$ M (y_0 , y_1 , (1 - k)t/k^n) * M (y_0 , y_1 , t/k^n) * M (y_{n+2} , y_{n+3} , t)
* M (y_{n+3} , y_{n+p} , (k - 2)t)
$\geq$ M (y_0 , y_1 , (1 - k)t/k^n) * M (y_0 , y_1 , t/k^n) * M (y_0 , y_1 , (1 - k)t/k^{n+2})
. . . * M (y_0, y_1, (k - p) t/ k^{n+p+1})
And N (y_n , y_{n+p} , t) $\leq$ N (y_n , y_{n+1} , (1 - k)t) * N(y_{n+1} , y_{n+p} , kt)
$\leq$ N (y_0 , y_1 , (1 - k)t/k^n) * N (y_{n+1} , y_{n+2} , t) * N(y_{n+2} , y_{n+p} , (k - 1)t)
$\leq$ N (y_0 , y_1 , (1 - k)t/k^n) * N (y_0 , y_1 , t/k^n) * N (y_{n+2} , y_{n+3} , t)
* N(y_{n+3}, y_{n+p} , (k - 2)t)
$\leq$ N (y_0 , y_1 , (1 - k)t/k^n) * N (y_0 , y_1 , t/k^n)* N (y_0 , y_1 , (1 - k)t/k^{n+2})
. . . * N (y_0, y_1, (k - p)t/ k^{n+p+1})
Thus M (y_n , y_{n+p} , t) $\rightarrow$ 1 & N(y_n , y_{n+p} , t) $\rightarrow$ 0 (for all t > 0 as n$\rightarrow \infty$ uniformly on p $\in$ N). Therefore {y_n } is a Cauchy sequence in X. But (X, M, N, * , $\Diamond$) is complete so there exists a point (say) z in X such that{y_n } $\rightarrow$ z. Also, using (1.1) we have {Px_{2n-2} },{T x_{2n-1}},{Sx_{2n}},{Qx_{2n+1}}$\rightarrow$z. Since the pair (P, S) is reciprocally continuous mappings, then we have, $\lim_{n\to\infty}$ PSx_{2n} = P z and $\lim_{n\to\infty}$ SPx_{2n}= Sz and compatibility of P and S yields,

$\lim_{n\to\infty}$ M (PSx_{2n} , SP x_{2n} , t) = 1 and $\lim_{n\to\infty}$ N(PSx_{2n} , SP x_{2n} , t) = 0
i.e. M (Pz, Sz, t) = 1, N (Pz, Sz, t) = 0.
Hence Pz = Sz.
Since P (X) $\subseteq$ T (X) then there exists a point u in X such that Pz = Tu. Now by contractive condition, we get,
M (Pz, Qu, kt) $\geq$ min.{M (Sz, Tu, t), M (Pz, Sz, t), M (Qu, Tu, t),M (Pz, Tu, t)
M (Pz, Qu, t), M (Sz, Qu, t)}
$\geq$ min.{M (Pz, Pz, t), M (Pz, Pz, t), M (Qu, Pz, t), M(Pz, Pz, t) M (Pz, Qu, t),
M (Pz, Qu, t)}
> M (Pz, Qu, t)
N (P z, Qu, kt) $\leq$ max.{N (Sz, Tu, t), N (Pz, Sz, t), N (Qu, Tu, t),
N (P z, Tu, t) N (Pz,Qu, t), N (Sz, Qu, t }
$\leq$ max.{N (Pz, Pz, t), N (Pz, Pz, t), N (Qu, Pz, t), N(Pz, Pz, t), N (Pz, Qu, t),
N (Pz, Qu, t)}
< N (P z, Qu, t)
i.e. Pz = Qu. Thus Pz = Sz = Qu =Tu. Since P is S - absorbing then for R > 0
we have, M(Sz, SPz, t) $\geq$ M (Sz, Pz, t/R) = 1
N (Sz, SPz, t) $\leq$ N (Sz, Pz, t/R) = 0

i.e. Pz = SPz = Sz. Now by contractive condition, we have,

M (Pz, PPz, t) = M (PPz, Qu, t) $\geq$ min.{M (SPz, Tu, t), M (PPz, SPz, t),
M (Qu, T u, t), M (PPz, Tu, t) M (PPz, Qu, t), M (SPz, Qu, t)}
= min.{M (Pz, Pz, t), M (PP z, Pz , t), M (Qu, Qu, t),M (PP z, P z, t)
M (PPz, Pz, t), M (Pz, Pz, t)}
= M (PP z, P z, t)

N (Pz, PPz, t) = N (PPz, Qu, t) $\leq$ max.{N (SPz, Tu, t), N (PPz, SPz, t),
N (Qu, T u, t), N (PPz, Tu, t) N (PPz, Qu, t), N (SPz, Qu, t)}

= max.{N (Pz, Pz, t), N (PPz, Pz , t), N (Qu, Qu, t),N (PPz, P z, t), N(PPz, Pz, t),
N (Pz, Pz, t)}
= N (PP z, P z, t)

i.e. PPz = Pz = SPz. Therefore Pz is a common fixed point of P and S. Similarly,
T is Q - absorbing therefore we have, *M (T u, T Qu, t)* $\geq$ *M (T u, Qu, t/R) = 1*
N (T u, T Qu, t) $\leq$ *N (T u, Qu, t/R) = 0*
i.e. Tu = TQu = Qu. Now by contractive condition, we have

M (QQu, Qu, t) = M (P z, QQu, t) $\geq$ *min.{M (Sz, TQu, t), M (Pz, Sz, t),*
M (QQu, TQu, t), M (Pz, TQu, t), M (Pz, QQu, t), M (Sz, QQu, t)}
= min.{M (Sz, Qu, t), M (Pz , P z, t), M (QQu, Qu, t), *M(P z, Qu, t),*
M (Pz, QQu, t), M(Sz, QQu, t) }
= min.{M(QQu, Qu, t), *M(Pz, QQu, t),M(Sz, QQu, t)}*.
= M(QQu, Qu, t)

N (QQu,Qu, t) = N (P z,QQu, t) $\leq$ *max.{N(Sz, TQu, t), N (Pz, Su, t),*
N (QQu, T Qu, t), N (Pz, T Qu, t), N (Pz, QQu, t),N (Sz, QQu, t)}
= max.{ N (QQu, Qu, t), *N(P z ,QQu, t), N(Sz,QQu, t)}*
= N (QQu, Qu, t).

i.e. QQu = Qu = TQu . Hence Qu = Pz is a common fixed point of P, Q, S and T. Uniqueness of Pz can easily follows from contractive condition. The proof is similar when Q and T are assumed compatible and reciprocally continuous. This completes the proof. Now we prove the result by assuming the range of one of the mappings P,Q, S or T to be a complete subspace of X.

Theorem 2. Let P be point wise S-absorbing and Q be point wise T–absorbing *self maps on a* Intuitionistic *fuzzy metric space (X, M, N,* $*$ *,* $\Diamond$ *) with continuous t-norm defined by a* $*$ *b = min{a, b} where a, b* $\in$ *[0, 1], satisfying the conditions:*

(1.1) *P (X)* $\subseteq$ *T (X), Q(X)* $\subseteq$ *S(X)*

(2.2) *There exists k* $\in$ *(0, 1) such that for every x, y* $\in$ *X and t > 0,*

M (P x, Qy, kt) $\geq$ *min . {M (Sx, T y, t), M (Px, Sx, t), M (Qy, T y, t), M (P x, T y, t),*
M (Px, Qy, t), M (Sx, Qy, t)}

N (P x, Qy, kt) $\leq$ *max .{N(Sx, T y , t), N (Px, Sx, t), N (Qy, T y,t), N (P x, T y, t),*
N (Px, Qy,t), N(Sx, Qy, t)}

(3.3) *for all x, y* $\in$ *X,* $\lim_{t\to\infty}$ *M (x, y, t) = 1,* $\lim_{t\to\infty}$ *N (x, y, t) = 0*

If the range of one of the mappings maps P, Q, S or *T be a complete subspace of X then P, Q, S* and *T have a unique common fixed point in X.*

Proof: let x_0 be any arbitrary point in X, construct a sequence $y_n \in$ X such that $y_{2n-1} = Tx_{2n-1} = P x_{2n-2}$ *and* $y_{2n} = Sx_{2n} = Qx_{2n+1}$ *, n = 1, 2, 3, . . . (2.4)*
This can be done by the virtue of (2.1) and by using the same techniques of above theorem we can show that { y_n } is a Cauchy sequence. Let S(X) the range of X be a complete metric subspace than there exists a point Su such that $\lim_{n\to\infty} Sx_{2n} = Su$. *By (2.4) we get* $Qx_{2n+1} \to Su$, $P x_{2n-2} \to Su$, $T x_{2n-1} \to Su$ *and* $\{ y_n \} \to Su$ as $n \to \infty$. By using contractive condition we obtain,

$M (Pu, Qx_{2n+1}, kt) \geq \min\{M (Su, T x_{2n+1}, t), M (P u, Su, t), M(Qx_{2n+1}, T x_{2n+1}, t),$

$M(Pu, Tx_{2n+1}, t), M(Pu, Qx_{2n+1}, t), M(Su, Qx_{2n+1}, t)\}$

$N(Pu, Qx_{2n+1}, kt) \leq \max\{N(Su, Tx_{2n+1}, t), N(Pu, Su, t), N(Qx_{2n+1}, Tx_{2n+1}, t),$
$N(Pu, Tx_{2n+1}, t), N(Pu, Qx_{2n+1}, t), N(Su, Qx_{2n+1}, t)\}$

Letting $n \to \infty$ we get

$M(Pu, Su, kt) \geq \min.\{M(Su, Su, t), M(Pu, Su, t), M(Su, Su, t), M(Pu, Su, t),$
$M(Pu, Su, t), M(Su, Su, t)\}$

$N(Pu, Su, kt) \leq \max.\{N(Su, Su, t), N(Pu, Su, t), N(Su, Su, t), N(Pu, Su, t),$
$N(Pu, Su, t), N(Su, Su, t)\}$

i.e. P u = Su. Since P (X) $\subset$ T (X) then there exists w $\in$ X such that Su = Tw.
Again by using contractive condition we get,

$M(Pu, Qw, kt) \geq \min.\{M(Su, Tw, t), M(Pu, Su, t), M(Qw, Tw, t), M(Pu, Tw, t),$
$M(Pu, Qw, t), M(Su, Qw, t)\}$

$N(Pu, Qw, kt) \leq \max.\{N(Su, Tw, t), N(Pu, Su, t), N(Qw, Tw, t), N(Pu, Tw, t),$
$N(Pu, Qw, t), N(Su, Qw, t)\}$

i.e Pu = Su = Qw = T w.
Since P is pointwise S-absorbing then we have

$M(Su, SPu, t) \geq M(Su, Qu, t/R)$

$N(Su, SPu, t) \leq N(Su, Qu, t/R)$

i.e. Su = SPu = SSu, and similarly Q is pointwise T -absorbing then we have

$M(Tw, TQw, t) \geq M(Tw, Qw, t/R)$

$N(Tw, TQw, t) \leq N(Tw, Qw, t/R)$

i.e. Tw = TQw = QQW . Thus Su (= T w) is a common fixed point of P, Q, S and T .
Uniquness of common fixed point follows from contractive condition. The proof is similar when T (X), the range of T is assumed to be a complete subspace of X. Moreover, Since P (X) $\subset$ T (X) and Q(x) $\subset$ S(X). The proof follows on similar line when either the range of P or the range of Q is assumed complete. This completes the

3.2. Let X = [2, 20] and $(X, M, N, *, \Diamond)$ be a intuitionistic fuzzy *metric*. *Define* mappings P, Q ,R, S **:** X→X by

$P(x) = \begin{cases} 2, x = 2 \\ 3, x > 2 \end{cases}$,$S(x) = \begin{cases} 2, x = 2 \\ 6, x > 2 \end{cases}$

$Q(x) = \begin{cases} 2, x = 2 \\ 8, 2 < x \leq 5 \end{cases}$, $T(x) = \begin{cases} 2, 2 \leq x \leq 5 \\ x - 3, x > 5 \end{cases}$

Also, We Define,

$M(Ax, By, t) = t/(t + |x - y|)$, $N(Ax, By, t) = |x - y| / t/(t + |x - y|)$

for all x, y $\in$ X, t > 0. Then P, Q, S and T satisfy all the conditions of the above Theorem with k =(0,1) and have a unique common fixed point x =2. Here, P and S are reciprocally continuous compatible maps. But neither P nor S is continuous, even at the common fixed point x =2. The mapping Q and T are non-compatible but Q is pointwise T - absorbing. To see Q and T are non compatible let us consider the sequence $\{x_n\}$ in X defined by

$x_n = 5+1/n$, $n \geq 1$. Then $T\{x_n\} \to 2$, Bxn =2, TBxn =2, BTxn =6.

Hence B and T are noncompatible.

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