DRIFTING EFFECT OF ELECTRONS ON DUST ION ACOUSTIC SOLITARY WAVES IN UNMAGNETIZED PLASMA

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ABSTRACT

Both compressive and rarefactive solitons are found to exist in the unmagnetized dusty plasma in presence of electrons' drift velocity (v'_e). Comparision with the earlier findings shows that the factor which is responsible for the existence both compressive and rarefactive solitons is the electrons' drift velocity (v'_e) in the plasma. It is observed that the compressive solitons exist for smaller values of dust charge $Z_d \left(= \frac{n_{d0}}{n_{i0}} = \frac{\text{equilibrium density of dust ions}}{\text{equilibrium density of ions}} \right)$ whereas the rarefactive solitons exist for higher values of Z_d .

Keywords: *Drifting effect, Ion acoustic, KdV, Soliton***.**

1. INTRODUCTION

The studies of dust ion acoustic (DIA) waves in plasma under variety of physical situations have created immense impact on the researchers. The presence of dust particles in a plasma influence the plasma properties leading to new significant results. By means of reductive perturbation method, Rao *et. al* (1990) have reported the existence of the dust acoustic waves for low frequency in unmagnetized dust plasma. D' Angelo(1995), Nakamura *et. al* (1999) and Duan *et. al* (2001) have studied the dust ion acoustic waves. Roychoudhury and S. Mukharjee (1997) have reported that the finite dust temperature restricts the region for the existence of nonlinear solitary waves. Shukla and Silin (1992) have reported about the existence of dust ion acoustic waves for higher frequency. Duan (2002) have investigated the dust acoustic solitary waves in hot dust plasma. Duan *et. al* (2003) have investigated dusty plasma with variable dust charge. El-Labany and El-Taibany (2003) have investigated the effects of variable dust charge, dust temperature and an arbitrary streaming ion beam on small amplitude dust acoustic waves. In their investigation they found that both compressive and rarefactive solitons as well as double layers exist. El-Labany and El-Taibany (2003) have investigated the effects of variable dust charge, dust temperature, and trapped electrons on small amplitude dust acoustic waves. They found that both compressive and rarefactive solitons as well as double layers exist depending on the nonisothermality parameter. Ghosh (2005) have investigated the role of negative ions in dusty plasma with variable dust charge. Zhang and Xue (2005) have studied the effects of the dust charge variation and non-thermal ions on the dust acoustic solitary structure in magnetized dusty plasmas. El-Labany *et. al* (2006) have studied the propagation of nonlinear dust acoustic waves in the dusty plasma consisting of a mixture of various charged dust particles, isothermal electrons and two-temperature isothermal ions. Das and Chatterjee (2009) have investigated the formation of large amplitude double layers in a dusty plasma constituting warm dust grains, non-thermal electrons and two temperature isothermal ions using Sagdeev's pseudopotential technique.

Recently Tiwari *et. al* (2011) have investigated characteristics of ion acoustic soliton in dusty plasma, including the dynamics of heavily charged massive dust grains using Sagdeev potential method.

In this paper, we have investigated the drifting effect of electrons on dust of ion-acoustic solitary waves in unmagnetized plasma. Various substantial and characteristic changes on solitons' amplitudes and growth are observed in this investigation due to the presence of the drift motion of electrons.

2. BASIC EQUATIONS AND DERIVATION OF KdV EQUATION:

The governing equations in one- dimension are given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0 \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -Z_d \frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}\left(n_e v_e\right)=0 \tag{3}$$

$$\frac{\partial v_e}{\partial t}+v_e\frac{\partial v_e}{\partial x}=\frac{Z_d}{Q}\left(\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \tag{4}$$

$$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial x}\left(n_d v_d\right)=0 \tag{5}$$

$$\frac{\partial v_d}{\partial t}+v_d\frac{\partial v_d}{\partial x}=Z_d\frac{\partial \phi}{\partial x} \tag{6}$$

Again for charge imbalances these equations are to be combined by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+Z_d n_d-n_i \tag{7}$$

where, *i*, e and d stand for positive ion, electron and dust ion respectively , $Q=\frac{m_e}{m_i}$ (= electron to ion mass ratio) and $Z_d=\frac{n_{d0}}{n_{i0}}=\frac{\text{equilibrium density of dust ions}}{\text{equilibrium density of ions}}$.

We have normalized densities n_i, n_e and n_d and by the unperturbed densities n_{d0}, n_{i0} and n_{e0} respectively, time t by the inverse of the characteristic ion plasma frequency i.e., $\omega_{pi}^{-1}=\left(\frac{m_i}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De}=\left(\frac{Z_d T_e}{4\pi n_{e0}e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s=\left(\frac{Z_d T_e}{m_i}\right)^{\frac{1}{2}}$, and the potential ϕ by $\frac{T_e}{e}$.

To derive the KdV equation from the set of equations (1) to (7), we use the stretched variables

$$\xi=\varepsilon^{\frac{1}{2}}\left(x-Ut\right),\quad \tau=\varepsilon^{\frac{3}{2}}t \tag{8}$$

with phase velocity *U* of the wave and the following expansions of the flow variables in terms of the smallness parameter ε:

$$\begin{aligned}
n_i &= n_{i0}+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\ldots\\
n_e &= 1+\varepsilon n_{e1}+\varepsilon^2 n_{e2}+\ldots\\
n_d &= n_{do}+\varepsilon n_{d1}+\varepsilon^2 n_{d2}+\ldots\\
v_i &= \varepsilon v_{i1}+\varepsilon^2 v_{i2}+\ldots\\
v_e &= v_e'+\varepsilon v_{e1}+\varepsilon^2 v_{e2}+\ldots\\
v_d &= \varepsilon v_{d1}+\varepsilon^2 v_{d2}+\ldots\\
\phi &= \varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\ldots
\end{aligned} \tag{9}$$

With the use of the transformations (8) and the expansions (9) in the normalized set of equations (1) – (7) subject to the boundary conditions, $n_{i1}=n_{e1}=n_{d1}=0$, $v_{i1}=v_{e1}=v_{d1}=0$, $\phi_1=0$ at $\left|\xi\right|\to\infty$, we get from ε order equations, the following quantities:

$$n_{i1}=\frac{Z_d n_{i0}\phi_1}{U^2},\quad n_{e1}=\frac{Z_d\phi_1}{Z_d-Q\left(U-v_e'\right)^2},\quad n_{d1}=-\frac{Z_d n_{d0}\phi_1}{U^2}$$

$$v_{i1} = \frac{Z_d \phi_1}{U}, \; v_{e1} = \frac{Z_d \phi_1}{Z_d - Q(U - v'_e)}, \; v_{d1} = -\frac{Z_d \phi_1}{U}, \tag{10}$$

$$n_{e1} + Z_d n_{d1} - n_{i1} = 0$$

Using the values of n_{i1}, n_{e1}, n_{d1} in the last equation of (10), we obtain the phase velocity equation as

$$\frac{Z_d}{Z_d - Q(U - v'_e)^2} - \frac{Z_d^2 n_{d0}}{U^2} - \frac{Z_d n_{d0}}{U^2} = 0 \tag{11}$$

This gives

$$U = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{12}$$

where $a = Z_d^2 - 1 - Q(1 + Z_d^2)$, $b = 2Qv'_e(1 + Z_d^2)$

$c = Z_d + Z_d^3 - Qv'^2_e(1 + Z_d^2)$

Using the relation (11) in the set of ε^2 - order equations obtained from equations (1) – (7), we have the KdV equation as,

$$\frac{\partial \phi_1}{\partial \tau} + p\phi_1 \frac{\partial \phi_1}{\partial \xi} + q\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{13}$$

where $p = \frac{A}{2B}$ and $q = \frac{1}{2B}$

with $A = \frac{3QZ_d^2(U - v'_e)^2 - Z_d^3}{\{Z_d - Q(U - v'_e)^2\}^3} - \frac{3n_{d0}Z_d^3}{U^4} + \frac{3n_{i0}Z_d^2}{U^4}$

and $B = \frac{QZ_d(U - v'_e) - Z_d^3}{\{Z_d - Q(U - v'_e)^2\}^2} + \frac{n_{d0}Z_d^2}{U^3} + \frac{n_{i0}Z_d}{U^3}$

3. SOLITARY WAVE SOLUTION

Using the transformation $\chi = \eta - V\tau$, the KdV equation (13) can be simplified to give the solitary wave solution as

$$\phi_1 = \frac{3V}{p} \sec h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\chi\right)$$

where V is the velocity with which the solitary waves travel to the right.

Thus, the wave amplitude of the soliton is given by $\phi_0 = \frac{3V}{p}$ and the corresponding width by $\Delta = 2\sqrt{\frac{q}{V}}$.

4. DISCUSSION

In this model of plasma, both compressive and rarefactive solitons are seen to be present which propagate depending on v'_e and Z_d. The electrons' drift velocity v'_e is found to be responsible for generating both compressive and rarefactive solitons. The amplitudes [Fig. 1(a)] of the rarefactive soliton decrease uniformly in magnitude with v'_e for $V = 0.10$ and $Z_d = 0.7, 0.8, 0.9$ and the corresponding widths [Fig. 1(b)] of rarefactive solitons exhibit their uniform decrease. Contrary to this, the amplitudes (ϕ_0) of the compressive solitons are seen to increase with v'_e [Fig. 2 (a)] for fixed $V = 0.05$ and for all $Z_d = 0.05,\ 0.10,\ 0.15$. Of course, it is to be noted that though the increase in amplitude is uniform for the smaller values of v'_e, the behaviour changes drastically for higher values of v'_e. But the corresponding widths (Δ) [Fig. 2(b)] decrease uniformly with v'_e for fixed $V = 0.05$ and for all

$Z_d = 0.05,\ 0.10,\ 0.15$. Interestingly, the amplitudes [Fig. 3(a)] of the compressive solitons become of parabolic nature showing uniform decrease upto a certain value of Z_d and then again shifting to uniform increase behaviour for $V = 0.10$ and $v_e = 4(1), 12(2)$. Moreover, the change of behaviour takes place almost in the mid values of Z_d for which the compressive solitons exist. The amplitude of rarefactive soliton is found to decrease in magnitude and tend to zero for higher values of Z_d. The change over from compressive to rarefactive soliton for each $v_e = 4(1), 12(2)$ is observed about the central part of the values of Z_d. On the other hand, the corresponding widths [Fig. 3(b)] of the solitons exhibit their uniform increase. Figure 4 shows the soliton solution ϕ_1 with $v'_e = 6$, $V = 0.10$ for different values of $Z_d = 0.7,\ 0.8,\ 0.9$. It is seen that ϕ_1 is maximum at $\chi = 0$.

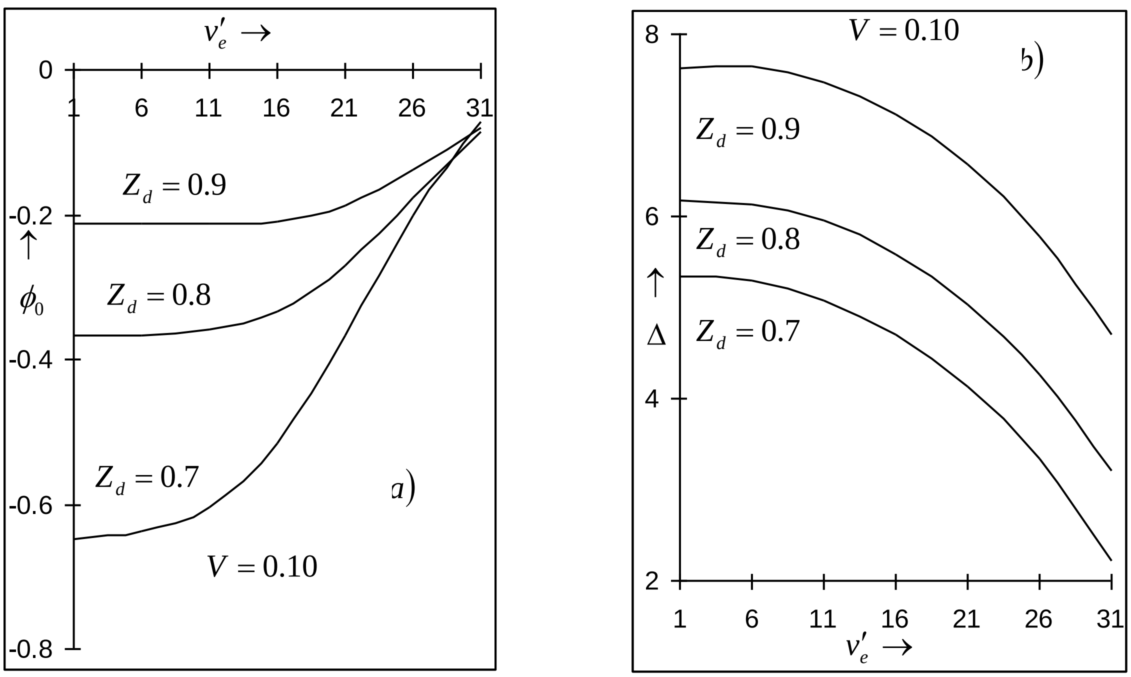

Figure 1. Amplitudes (a) and widths (b) of rarefactive for slow ion-acoustic solitons versus v'_e for fixed $V = 0.10$ and $Q = 0.54 \times 10^{-3}$ for different values of $Z_d = 0.7,\ 0.8,\ 0.9$.

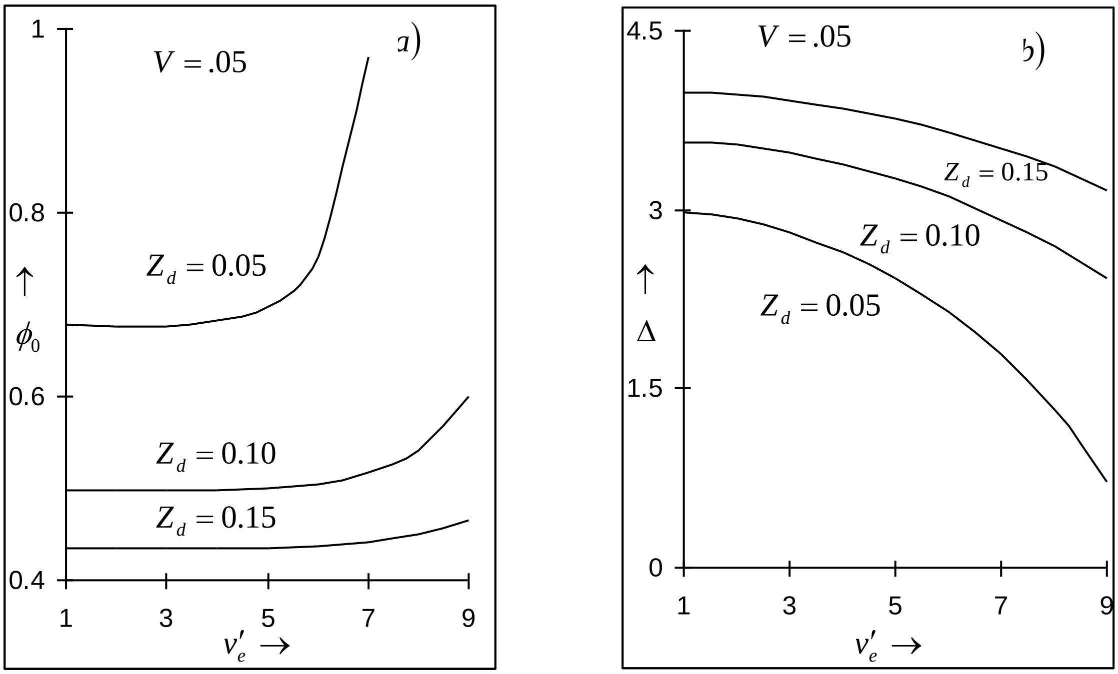

Figure 2. Amplitudes (a) and widths (b) of compressive for slow ion-acoustic solitons versus v_e' for fixed $V = 0.05$ and $Q = 0.54 \times 10^{-3}$ for different values of $Z_d = 0.05,\ 0.10,\ 0.15$.

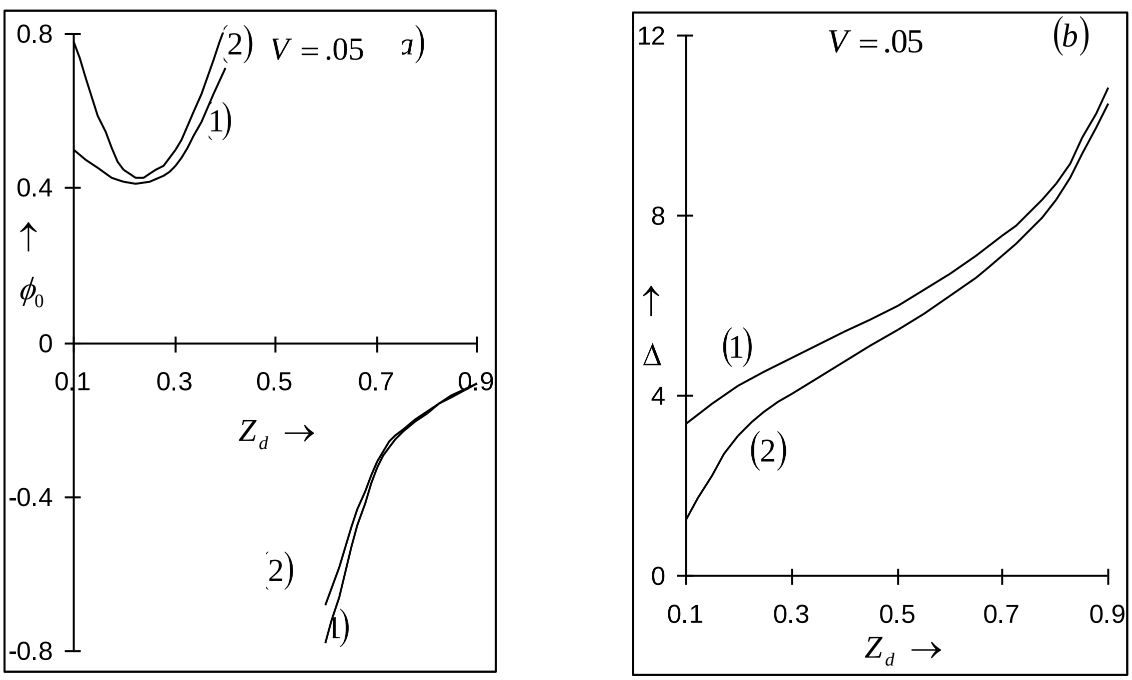

Figure 3. Amplitudes (a) and widths (b) of compressive for slow ion-acoustic solitons versus Z_d for fixed $V = 0.05$ and $Q = 0.54 \times 10^{-3}$ for different values of $v_e = 4(1), 12(2)$.

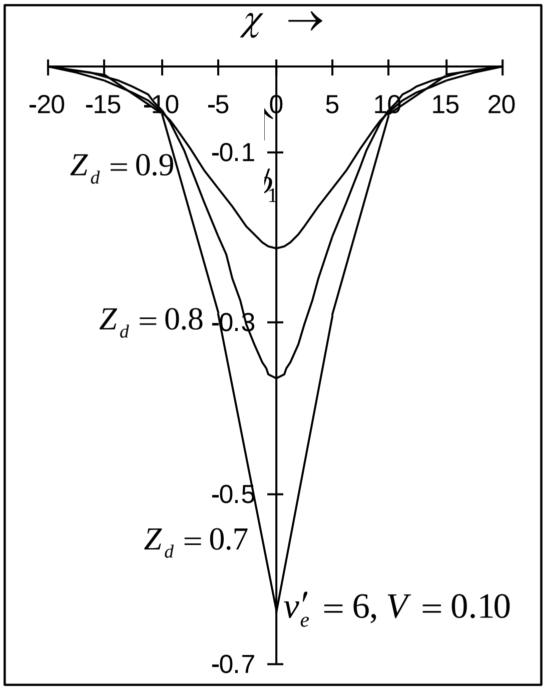

Figure 4. Plot of the amplitude of the KdV soliton solutions with $v'_e = 6$, $V = 0.10$ for different values of $Z_d = 0.7, 0.8, 0.9$.

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FIGURE CAPTION

Figure 1. Amplitudes (a) and widths (b) of rarefactive for slow ion-acoustic solitons versus v'_e for fixed V = 0.10 and Q= 0.54 X 10^{-3} for different values of Z_d=0.7, 0.8, 0.9.

Figure 2. Amplitudes (a) and widths (b) of compressive for slow ion-acoustic solitons versus v'_e for fixed V = 0.05 and Q= 0.54 X 10^{-3} for different values of Z_d=0.05, 0.10, 0.15.

Figure 3. Amplitudes (a) and widths (b) of compressive for slow ion-acoustic solitons versus Z_d for fixed V = 0.05 and Q= 0.54 X 10^{-3} for different values of V_e= 4(1), 12(2).

Figure 4. Plot of the amplitude of the KdV soliton solutions with $v'_e = 6$, V = 0.10 for different values of Z_d=0.7, 0.8, 0.9.