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ESTIMATION OF PERFORMANCE MEASURES FOR A STANDBY REDUNDANT COMPLEX SYSTEM

By

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INTRODUCTION

In this chapter, the author has done his efforts to estimate the performance measures of standby redundant complex system. The whole system contains two subsystems, namely A and B connected in series. The subsystem A has n identical units in series while subsystem B has two standby redundant similar units. On failure of any one unit of subsystem A, the whole system goes to failed state. On the other hand, after failing of online unit of subsystem B, the whole system remains operable with same efficiency with the help of standby unit, which can be online by using an imperfect switching device S. The whole system can also fail due to environmental reasons. If subsystem A fails, the system needs inspection before repair. Also, in case of failure of switching device S, we have to wait for repair facilities otherwise repair facilities are always available. All failures follow exponential time distribution whereas all repairs follow general time distribution.

Since the considered system is Non-Markovian, the author has used supplementary variables to convert this in Markovian. Mathematical model of the system has been solved by using Laplace transform. Fig-1.1 shows the system configuration and fig-1.2 shows the state-transition diagram. Reliability, M.T.T.F. and availability of considered system have been computed. Asymptotic behaviour and some particular cases have also been obtained to improve practical utility of the model. Graphical illustration followed by a numerical computation has been appended in the end to highlight important results of the study.

ASSUMPTIONS

The following assumptions have been associated with this model:

1. Initially, the whole system is new and operable.
2. Switching device S is imperfect.
3. Environmental reasons can fail the whole system.
4. The system needs inspection before the repair of subsystem A.
5. The system requires waiting in repair of switching device S.
6. All failures, inspection and waiting rate follow exponential time distribution.
7. All repairs follow general time distribution and after repair system works like new.
8. Failures are S-independent.

LIST OF NOTATIONS

λ_A / λ_B	:	Failure rate of any one unit of subsystem A/B.
$(1-\gamma)$	:	Failure rate of switching device S.
n	:	Number of units in subsystem A.
α / β	:	Inspection rate / waiting rate.
$\lambda_{E_1}, \lambda_{E_2}$	:	Environmental failure rates.
$\mu_i(j)\Delta$	:	The first order probability that i^{th} failure can be repaired in the time interval $(j, j+\Delta)$, conditioned that it was not repaired up to the time j.
$P_0(t)$	:	Pr {at time t, System is all operable}.
$P_i(j,t)\Delta$	:	Pr {at time t, system suffers with i^{th} failure}.Elapsed repair time lies in the interval $(j, j+\Delta)$.

$P_A^I(t) / P_{B_1A}^I(t)$	:	Pr {at time t, system is failed due to failure of subsystem $(A)/(B_1 and A)$ and an inspection is required}.
$P_A^R(x,t)\Delta / P_{B_1A}^R(x,t)\Delta$	:	Pr {at time t, system is ready for repair of subsystem A}. Elapsed repair time lies in $(x, x+\Delta)$.
$P_S^W(t)$	:	Pr {at time t, system is failed due to failure of switching device S and is waiting for repair}.
$P_S^R(z,t)\Delta$	:	Pr {at time t, system is ready for repair of switching device S}.Elapsed repair time lies in the interval $(z, z+\Delta)$.

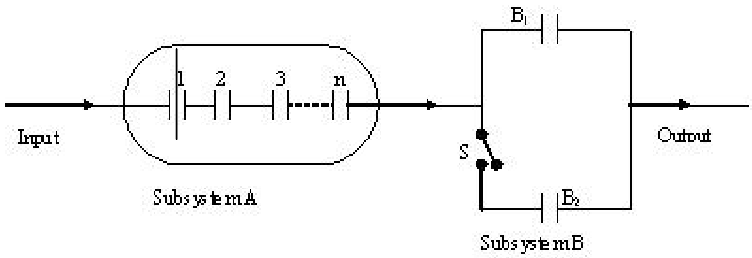

Fig-1.1: System Configuration

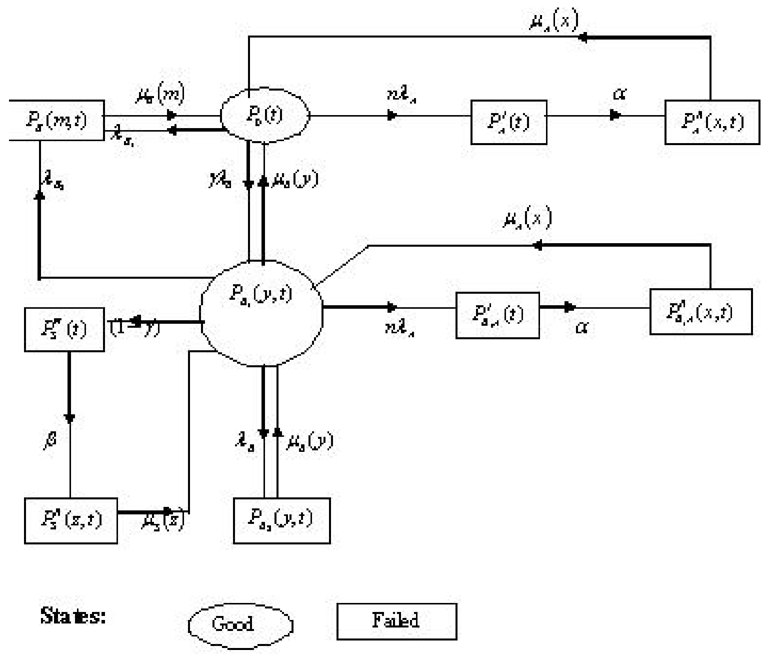

Fig 1.2: State-transition diagram

FORMULATION OF MATHEMATICAL MODEL

Using probability considerations and limiting procedure, we obtain the following set of difference-differential equations which is continuous in time, discrete in space and governing the behaviour of considered system:

$$\left(\frac{d}{dt} + n\lambda_A + \gamma\lambda_B + \lambda_{E_1}\right)P_0(t) = \int_0^\infty P_A^R(x,t)\mu_A(x)dx + \int_0^\infty P_{B_1}(y,t)\mu_B(y)dy \quad \ldots(1)$$

$$+\int_0^\infty P_E(m,t)\mu_E(m)dm$$

$$\left[\frac{d}{dt}+\alpha\right]P_A^I(t)=n\lambda_A\ P_0(t) \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_A^R(x,t)=0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+n\lambda_A+(1-\gamma)+\lambda_B+\lambda_{E_2}+\mu_B(y)\right]P_{B_1}(y,t)=0 \quad \ldots(4)$$

$$\left[\frac{d}{dt}+\alpha\right]P_{B_1A}^I(t)=n\lambda_A\ P_{B_1}(t) \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_{B_1A}^R(x,t)=0 \quad \ldots(6)$$

$$\left[\frac{d}{dt}+\beta\right]P_S^W(t)=(1-\gamma)P_{B_1}(t) \quad \ldots(7)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_S(z)\right]P_S^R(z,t)=0 \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_B(y)\right]P_{B_2}(y,t)=\lambda_B\ P_{B_1}(y,t) \quad \ldots(9)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\mu_E(m)\right]P_E(m,t)=0 \quad \ldots(10)$$

Boundary Conditions are:

$$P_A^R(0,t)=\alpha\ P_A^I(t) \quad \ldots(11)$$

$$P_{B_1}(0,t)=\gamma\lambda_B\ \ P_0(t)+\int_0^\infty P_{B_1A}^R(x,t)\mu_A(x)dx+\int_0^\infty P_S^R(z,t)\mu_S(z)dz+\int_0^\infty P_{B_2}(y,t)\mu_B(y)dy \quad \ldots(12)$$

$$P_{B_1A}^R(0,t)=\alpha\ P_{B_1A}^I(t) \quad \ldots(13)$$

$$P_{B_2}(0,t)=0 \quad \ldots(14)$$

$$P_S^R(0,t)=\beta\ P_S^W(t) \quad \ldots(15)$$

$$P_E(0,t)=\lambda_{E_1}P_0(t)+\lambda_{E_2}P_{B_1}(t) \quad \ldots(16)$$

Initial Conditions are:

$P_0(0)=1,$ otherwise zero at t = 0. …(17)

SOLUTION OF THE MODEL

In the order to solve the above mathematical model, we have to obtain various state probabilities depicted in fig-1.2. So, Taking L.T. of equations (1) through (16) subjected to initial conditions (17), we obtain

$$\left(s+n\lambda_A+\gamma\lambda_B+\lambda_{E_1}\right)\overline{P_0}(s)=1+\int_0^\infty \overline{P}_A^R(x,s)\mu_A(x)dx+\int_0^\infty \overline{P}_{B_1}(y,s)\mu_B(y)dy \quad \ldots(18)$$

$$+\int_0^\infty \overline{P}_E(m,s)\mu_E(m)dm$$

$$[s+\alpha]\overline{P_A}^{I}(s)=n\lambda_A\,\overline{P_0}(s) \quad \text{...(19)}$$

$$\left[\frac{\partial}{\partial x}+s+\mu_A(x)\right]\overline{P}_A^R(x,s)=0 \quad \text{...(20)}$$

$$\left[\frac{\partial}{\partial y}+s+n\lambda_A+(1-\gamma)+\lambda_B+\lambda_{E_2}+\mu_B(y)\right]\overline{P}_{B_1}(y,s)=0 \quad \text{...(21)}$$

$$[s+\alpha]\overline{P}_{B_1A}^I(s)=n\lambda_A\,\overline{P}_{B_1}(s) \quad \text{...(22)}$$

$$\left[\frac{\partial}{\partial x}+s+\mu_A(x)\right]\overline{P}_{B_1A}^R(x,s)=0 \quad \text{...(23)}$$

$$[s+\beta]\overline{P}_S^W(s)=(1-\gamma)\overline{P}_{B_1}(s) \quad \text{...(24)}$$

$$\left[\frac{\partial}{\partial z}+s+\mu_S(z)\right]\overline{P}_S^R(z,s)=0 \quad \text{...(25)}$$

$$\left[\frac{\partial}{\partial y}+s+\mu_B(y)\right]\overline{P}_{B_2}(y,s)=\lambda_B\,\overline{P}_{B_1}(y,s) \quad \text{...(26)}$$

$$\left[\frac{\partial}{\partial m}+s+\mu_E(m)\right]\overline{P}_E(m,s)=0 \quad \text{...(27)}$$

$$\overline{P}_A^R(0,s)=\alpha\;\overline{P}_A^I(s) \quad \text{...(28)}$$

$$\overline{P}_{B_1}(0,s)=\gamma\lambda_B\;\overline{P}_0(s)+\int_0^\infty \overline{P}_{B_1A}^R(x,s)\mu_A(x)dx+\int_0^\infty \overline{P}_S^R(z,s)\mu_S(z)dz+\int_0^\infty \overline{P}_{B_2}(y,s)\mu_B(y)dy \quad \text{...(29)}$$

$$\overline{P}_{B_1A}^R(0,s)=\alpha\;\overline{P}_{B_1A}^I(s) \quad \text{...(30)}$$

$$\overline{P}_{B_2}(0,s)=0 \quad \text{...(31)}$$

$$\overline{P}_S^R(0,s)=\beta\;\overline{P}_S^W(s) \quad \text{...(32)}$$

$$\overline{P}_E(0,s)=\lambda_{E_1}\overline{P}_0(s)+\lambda_{E_2}\overline{P}_{B_1}(s) \quad \text{...(33)}$$

Now, simplifying equation (19),(22) and (24), we get

$$\overline{P}_A^I(s)=\frac{n\lambda_A\overline{P_0}(s)}{s+\alpha} \quad \text{...(34)}$$

$$\overline{P}_{B,A}^I(s)=\frac{n\lambda_A\overline{P_{B_1}}(s)}{s+\alpha} \quad \text{...(35)}$$

$$\overline{P}_S^W(s)=\frac{(1-\gamma)\overline{P}_{B_1}(s)}{s+\beta} \quad \text{...(36)}$$

Integrating equation (20) with the help of boundary condition (28), we have

$$\overline{P}_A{}^R(x,s)=\alpha\;\overline{P}_A{}^I(s)\;\exp.\left\{-sx-\int\mu_A(x)dx\right\}$$

integrating this again w.r.t. x from 0 to ∞, we get

$$\overline{P}_A{}^R(s)=\alpha\;\overline{P}_A^I(s)\;\frac{1-\overline{S}_A(s)}{s}$$

or, $\overline{P}_A{}^R(s)=\alpha\;\overline{P}_A^I(s)\;\mathrm{D_A}(s)$ ...(37)

$$\Rightarrow \overline{P}_A^{\,R}(s) = \frac{\alpha\, n\lambda_A\, \overline{P}_0(s)}{s+\alpha}\; D_A(s) \qquad \text{[by (34)]}$$

Similarly, integrate equation (23) by using (30), we obtain

$$\overline{P}^{R}_{B_1A}(s) = \frac{\alpha\, n\lambda_A\, \overline{P}_{B_1}(s)}{s+\alpha}\; D_A(s) \qquad \ldots(38)$$

Again, integrate equation (25) subjected to (32), we have

$$\overline{P}^{R}_{S}(z,s) = \frac{\beta(1-\gamma)\, \overline{P}_{B_1}(s)}{s+\beta}\;\; \exp.\left\{-sz - \int \mu_S(z)dz\right\}$$

integrate this again w.r.t. z from 0 to ∞, we get ...(39)

$$\overline{P}^{R}_{S}(s) = \frac{\beta(1-\gamma)\, \overline{P}_{B_1}(s)}{s+\beta}\; D_S(s)$$

Equation (27) gives on integration subjected to (33):

$$\overline{P}_E(m,s) = \overline{P}_E(0,s)\;\; \exp.\left\{-sm - \int \mu_E(m)dm\right\} \qquad \ldots(40)$$

$$\Rightarrow \overline{P}_E(s) = \left[\lambda_{E_1}\overline{P}_0(s) + \lambda_{E_2}\overline{P}_{B_1}(s)\right]D_E(s)$$

Integrating (21), we obtain

$$\overline{P}_{B_1}(y,s) = \overline{P}_{B_1}(0,s)\; \exp.\left\{-\left(s + n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}\right)y - \int \mu_{B_1}(y)dy\right\} \qquad \ldots(41)$$

$$\Rightarrow \quad \overline{P}_{B_1}(s) = \overline{P}_{B_1}(0,s)\; D_B\left(s + n\lambda_A + 1 - \gamma + \lambda_B + \lambda_{E_2}\right)$$

Using the value of $\overline{P}_{B_1}(y,s)$ in equation (26) and then integrating, we have

$$\overline{P}_{B_2}(y,s)e^{sy+\int \mu_B(y)dy} = c + \gamma_B \int \overline{P}_{B_1}(y,s)e^{sy+\int \mu_B(y)dy}dy$$

$$= c + \gamma_B \int \overline{P}_{B_1}(0,s)e^{-\left(n\lambda_A + 1 - \gamma + \lambda_B + \lambda_{E_2}\right)y}dy$$

$$= c - \frac{\lambda_B \overline{P}_{B_1}(0,s)e^{-\left(n\lambda_A + 1 - \gamma + \lambda_B + \lambda_{E_2}\right)y}}{n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}}$$

Putting y=0, dy=0 and make use of (31), we get

$$c = \frac{\lambda_B \overline{P}_{B_1}(0,s)}{n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}}$$

Using this result in previous equation, we obtain

$$\overline{P}_{B_2}(y,s) = \frac{\lambda_B \overline{P}_{B_1}(0,s)}{n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}}\left[e^{-sy-\int \mu_B(y)dy} - e^{-\left(s + n\lambda_A + 1 - \gamma + \lambda_B + \lambda_{E_2}\right)y - \int \mu_B(y)dy}\right] \qquad \ldots(42)$$

$$\Rightarrow \overline{P}_{B_2}(s) = \frac{\lambda_B \overline{P}_{B_1}(0,s)}{n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}}\left[D_B(s) - D_B\left(s + n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}\right)\right]$$

where, $\overline{P}_{B_1}(0,s) = A(s)\overline{P_0}(s)$ ...(43)

where, $A(s) = \dfrac{\gamma\lambda_B}{1 - \dfrac{\lambda_B\left[\overline{S}_B(s) - \overline{S}_B(N)\right]}{N} - \left[\dfrac{\alpha n\lambda_A \overline{S_A}(s)}{s+\alpha} + \dfrac{\beta(1-\gamma)\overline{S_S}(s)}{s+\beta}\right]D_B(N)}$

and $N = \left(s + n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}\right)$

Finally, simplifying equation (18) by use of relevant expressions, one can obtain

$$\overline{P_0}(s) = \frac{1}{B(s)}$$

Thus, we obtain the following L.T of different state probabilities (depicted in fig-1.2) in terms of $B(s)$:

$$\overline{P}_0(s) = \frac{1}{B(s)} \quad \text{…(44)}$$

$$\overline{P}_A^I(s) = \frac{n\lambda_A}{(s+\alpha)B(s)} \quad \text{…(45)}$$

$$\overline{P}_A^R(s) = \frac{\alpha n\lambda_A\ D_A(s)}{(s+\alpha)B(s)} \quad \text{…(46)}$$

$$\overline{P}_{B_1}(s) = \frac{A(s)}{B(s)} D_B(N) \quad \text{…(47)}$$

$$\overline{P}_{B_1A}^I(s) = \frac{n\lambda_A\ A(s)D_B(N)}{(s+\alpha)B(s)} \quad \text{…(48)}$$

$$\overline{P}_{B_1A}^R(s) = \frac{\alpha n\lambda_A D_A(s)\ A(s)D_B(N)}{(s+\alpha)B(s)} \quad \text{…(49)}$$

$$\overline{P}_S^W(s) = \frac{(1-\gamma)\ A(s)D_B(N)}{(s+\beta)B(s)} \quad \text{…(50)}$$

$$\overline{P}_S^R(s) = \frac{\beta(1-\gamma)D_S(s)\ A(s)D_B(N)}{(s+\beta)B(s)} \quad \text{…(51)}$$

$$\overline{P}_{B_2}(s) = \frac{\lambda_B\ A(s)}{B(s)(N-s)}\left[D_B(s) - D_B(N)\right] \quad \text{…(52)}$$

$$\overline{P}_E(s) = \frac{1}{B(s)}\left[\lambda_{E_1} + \lambda_{E_2} A(s)D_B(N)\right]D_E(s) \quad \text{…(53)}$$

where, $N = \left(s + n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}\right)$ …(54)

$$A(s) = \frac{\gamma\lambda_B}{1 - \frac{\lambda_B}{N}\left[\overline{S}_B(s) - \overline{S}_B(N)\right] - \left[\frac{\alpha n\lambda_A \overline{S}_A(s)}{(s+\alpha)} + \frac{\beta(1-\gamma)\overline{S}_S(s)}{(s+\beta)}\right]D_B(N)} \quad \text{…(55)}$$

and

$$B(s) = s + n\lambda_A + \gamma\lambda_B + \lambda_{E_1} - \frac{\alpha n\lambda_A \overline{S}_A(s)}{(s+\alpha)} - \left[\lambda_{E_1} + \lambda_{E_2} A(s)D_B(N)\right]\overline{S}_E(s) - A(s)\overline{S}_B(N) \quad \text{…(56)}$$

VARIFICATION

It is worth noticing that

Sum of equations (44) through (53) $= \frac{1}{s}$ …(57)

ASYMPTOTIC BEHAVIOUR OF THE SYSTEM

By using final value theorem in L.T., viz; $\lim_{t\to\infty} P(t) = \lim_{s\to 0} s\ \overline{P}(s)$, provided the limit on L.H.S. exists, we can obtain the asymptotic behaviour of considered system from equations (44) through (53) as follows:

$$P_0 = \frac{1}{B'(0)} \quad \text{…(58)}$$

$$P_A^I = \frac{n\lambda_A}{\alpha\, B'(0)} \quad \text{…(59)}$$

$$P_A^R = \frac{n\lambda_A M_A}{B'(0)} \quad \text{…(60)}$$

$$P_{B_1} = \frac{A(0)}{B'(0)} D_B(N-s) \quad \text{…(61)}$$

$$P_{B_1A}^I = \frac{n\lambda_A\, A(0) D_B(N-s)}{\alpha\, B'(0)} \quad \text{…(62)}$$

$$P_{B_1A}^R = \frac{n\lambda_A M_A A(0) D_B(N-s)}{B'(0)} \quad \text{…(63)}$$

$$P_S^W = \frac{(1-\gamma)A(0)D_B(N-s)}{\beta.\, B'(0)} \quad \text{…(64)}$$

$$P_S^R = \frac{(1-\gamma)M_S A(0) D_B(N-s)}{B'(0)} \quad \text{…(65)}$$

$$P_{B_2} = \frac{\lambda_B A(0)}{(N-s)B'(0)}\left[M_B - D_B(N-s)\right] \quad \text{…(66)}$$

and $$P_E = \frac{1}{B'(0)}\left[\lambda_{E_1} + \lambda_{E_2} A(0) D_B(N-s)\right] M_E \quad \text{…(67)}$$

where,

$M_i = -\overline{S}_i{}'(0),$ = Mean time to repair i^{th} failure

$$B'(0) = \left[\frac{d}{ds}B(s)\right]_{s=0}$$

and $$A(0) = \frac{\gamma\lambda_B(N-s)}{\lambda_{E_2} + (N-s-\lambda_{E_2})\left[1-\overline{S_B}(N-s)\right]} \quad \text{…(68)}$$

SOME PARTICULAR CASES

Case I : When repairs follow exponential time distribution

In this case, setting $\overline{S}_i(j) = \mu_i / (j+\mu_i)$, $\forall$ i and j, in equations (44) through (53), we have obtained the following bared states probabilities of fig-1.2:

$$\overline{P}_0(s) = \frac{1}{E(s)} \quad \text{…(69)}$$

$$\overline{P}_A^I(s) = \frac{n\lambda_A}{(s+\alpha)E(s)} \quad \text{…(70)}$$

$$\overline{P}_A^R(s) = \frac{\alpha n\lambda_A}{(s+\alpha)(s+\mu_A)E(s)} \quad \text{…(71)}$$

$$\overline{P}_{B_1}(s)=\frac{C(s)}{E(s)(N+\mu_B)} \qquad \ldots(72)$$

$$\overline{P}^{I}_{B_1A}(s)=\frac{n\lambda_A C(s)}{(s+\alpha)(N+\mu_B)E(s)} \qquad \ldots(73)$$

$$\overline{P}^{R}_{B_1A}(s)=\frac{\alpha n\lambda_A C(s)}{(s+\alpha)(s+\mu_A)(N+\mu_B)E(s)} \qquad \ldots(74)$$

$$\overline{P}^{W}_{S}(s)=\frac{(1-\gamma)C(s)}{(s+\beta)(N+\mu_B)E(s)} \qquad \ldots(75)$$

$$\overline{P}^{R}_{S}(s)=\frac{\beta(1-\gamma)C(s)}{(s+\mu_S)(s+\beta)(N+\mu_B)E(s)} \qquad \ldots(76)$$

$$\overline{P}_{B_2}(s)=\frac{\lambda_B C(s)}{E(s)(N-s)}\left[\frac{1}{(s+\mu_B)}-\frac{1}{(N+\mu_B)}\right] \qquad \ldots(77)$$

$$\overline{P}_{E}(s)=\frac{1}{E(s)}\left[\lambda_{E_1}+\frac{\lambda_{E_2}C(s)}{(N+\mu_B)}\right]\frac{1}{(s+\mu_E)} \qquad \ldots(78)$$

where,

$$C(s)=\frac{\gamma\lambda_B}{1-\frac{\lambda_B\mu_B}{N}\left\{\frac{1}{(s+\mu_B)}-\frac{1}{(N+\mu_B)}\right\}-\left\{\frac{\alpha n\lambda_A\mu_A}{(s+\alpha)(s+\mu_A)}+\frac{\beta(1-\gamma)\mu_S}{(s+\beta)(s+\mu_S)}\right\}\frac{1}{(N+\mu_B)}} \qquad \ldots(79)$$

and $$E(s)=s+n\lambda_A+\gamma\lambda_B+\lambda_{E_1}-\frac{\alpha n\lambda_A\mu_A}{(s+\alpha)(s+\mu_A)}-\left[\lambda_{E_1}+\frac{\lambda_{E_2}C(s)}{(N+\mu_B)}\right]\frac{\mu_E}{s+\mu_E}-\frac{C(s)\mu_B}{(N+\mu_B)} \qquad \ldots(80)$$

Case II: When switching device S is perfect

In this case, the value of γ will be one, $P^{W}_{S}(t)=0$ and $P^{R}_{S}(t)=0$. Using these results, we can obtain the solution of considered system.

Case III: When subsystem A does not require inspection before repair

In this case, $\alpha=0, P^{I}_{A}(t)=0$ and $P^{R}_{A}(t)=0$. By using these results in (44) through (53), we may obtain the solution of considered system.

RELIABILITY AND M.T.T.F. OF THE SYSTEM

We have from equation (44),

$$\overline{R}(s)=\frac{1}{s+n\lambda_A+\gamma\lambda_B+\lambda_{E_1}}$$

Taking inverse L.T., we get

$$R(t)=\exp\left\{-\left(n\lambda_A+\gamma\lambda_B+\lambda_{E_1}\right)t\right\} \qquad \ldots(81)$$

Also, M.T.T.F. $=\int_0^\infty R(t)dt$

$$=\frac{1}{n\lambda_A+\gamma\lambda_B+\lambda_{E_1}} \qquad \ldots(82)$$

AVAILABILITY OF THE SYSTEM

We have from equation (44) and (72)

$$\overline{P}_{up}(s) = \frac{1}{s + n\lambda_A + \gamma\lambda_B + \lambda_{E_1}}\left[1 + \frac{\gamma\lambda_B}{s + n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}}\right] \quad \ldots(83)$$

taking inverse L.T., we obtain

$$P_{up}(t) = (1+Q)e^{-\left(n\lambda_A + \gamma\lambda_B + \lambda_{E_1}\right)t} - Qe^{-\left(n\lambda_A + (1-\gamma) + \lambda_B + \lambda_{E_2}\right)t} \quad \ldots(84)$$

where, $Q = \dfrac{\gamma\lambda_B}{(1-\gamma)(1+\lambda_B) + \lambda_{E_2} - \lambda_{E_1}}$

NUMERICAL EXAMPLE

For a numerical example, let us consider the values:

$$n = 4, \lambda_A = 0.005, \lambda_B = 0.008, \gamma = 0.3, \lambda_{E_1} = 0.006, \lambda_{E_2} = 0.007 \, and \; t = 0,1,2,---10.$$

Using these values in equations (81), (82) and (83), we compute the table-1, 2 and 3 respectively. The corresponding graphs have been shown in fig-2, 3 and 4 respectively.

RESULTS AND DISCUSSION

Table -1 computes the reliability of considered system for different time t. Its graph has been shown in fig-2. Its analysis reveals that reliability decreases in constant manner as we make increase in time t. Table-2 gives the values of M.T.T.F for different values of γ. It's graph has been sketched in fig-3. Critical examination of table-2 and fig-3 yields that M.T.T.F. decreases as we make increase in the value of γ. For a perfect switching device the value of M.T.T.F. is 29.41176.

Table-3 gives the values of availability of considered system for different time points. It is shown in fig-4. Examination of table-3 and fig-4 concludes that availability of considered system decreases smoothly with increase in time t. It should be noted that these are no sudden jumps in the values of R(t), M.T.T.F. and $P_{up}(t)$.

t	R(t)
0	1
1	0.971999
2	0.944783
3	0.918329
4	0.892615
5	0.867621
6	0.843327
7	0.819714
8	0.796761
9	0.774452
10	0.752767

Table-1

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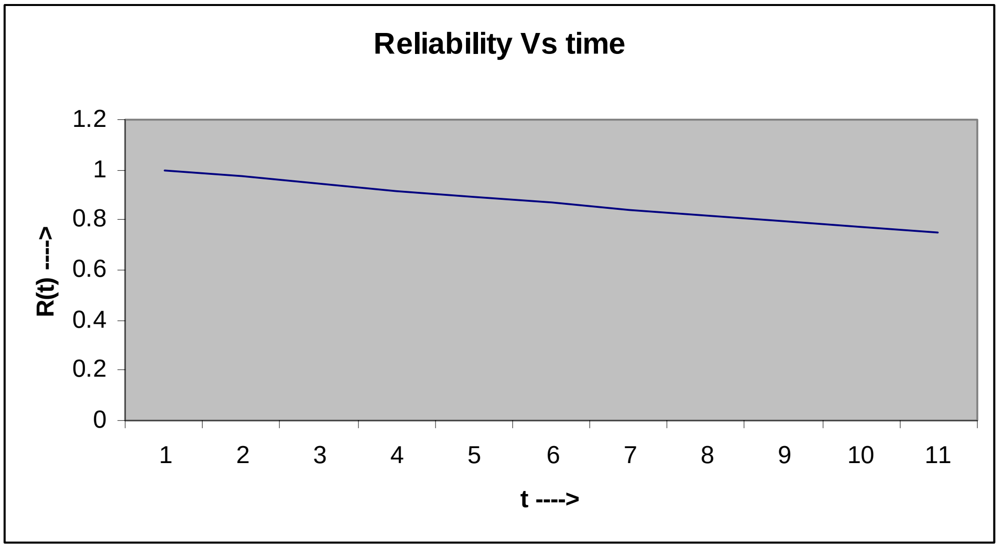

Fig-2

γ	**M.T.T.F.**
0	38.46154
0.1	37.31343
0.2	36.23188
0.3	35.21127
0.4	34.24658
0.5	33.33333
0.6	32.46753
0.7	31.64557
0.8	30.86420
0.9	30.12048
1.0	29.41176

Table-2

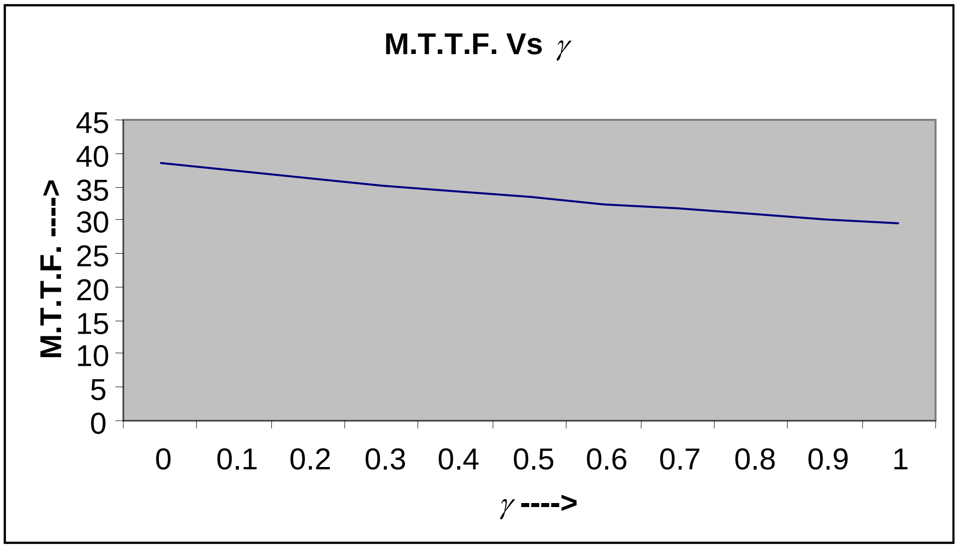

Fig-3

t	$P_{up}(t)$
0	1
1	0.973672
2	0.947211
3	0.921074
4	0.895468
5	0.870482
6	0.846151
7	0.822479
8	0.799459
9	0.777078
10	0.755322

Table -3

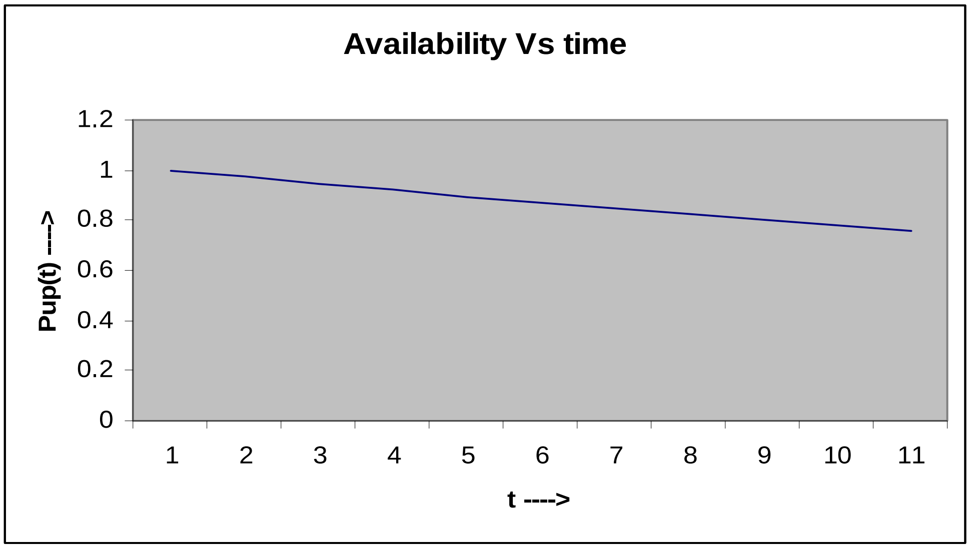

Fig-4

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