NUMERICAL INVESTIGATION OF FIN EFFICIENCY AND TEMPERATURE DISTRIBUTION OF CONDUCTIVE, CONVECTIVE AND RADIATIVE STRAIGHT FINS

D.D.Ganji*, M.Rahgoshay, M.Rahimi & M.Jafari

Department of Mechanical Engineering, Babol University of Technology, Babol, Iran, P.O.Box 484
E–mail: ddg_davood@yahoo.com , mirgang@nit.ac.ir

ABSTRACT

In this study, fin efficiency, temperature distribution and effectiveness of conductive, convective and radiative straight fins with temperature dependent thermal conductivity are solved using Galerkin Method (GM). The concept of galerkin method is briefly introduced, and then it is employed to derive solution of nonlinear governing equation of fin with a highly nonlinear term because of existing radiation in this study. The obtained results from GM are compared with numerical Boundary Value problem Method (BVP) to verify the accuracy of the proposed methods. The effects of some physical appropriate parameters in this problem such as thermo-geometric fin parameter and thermal parameters are analyzed.

Keywords: Fin Efficiency; Fin effectiveness; Thermal Conductivity; Galerkin Method (GM),

1. INTRODUCTION

Heat transfer is a very popular science in mechanical engineering, since it can be necessary or harmful in various objects. When heat transfer rate is smaller than needed, there are different ways to solve the problem; one of them to improve heat transfer is using the extended surface called fin.

In this study the communication between rate of heating and length of fin as a controller of the convection rate is analyzed. The heat transfer mechanism of fin is to conduct heat from heat source to the fin surface by its thermal conduction, and then dissipate heat to the air by the effect of thermal convection and radiation

Nomenclature		***Greek symbols***	
A	area(m^2)	α	thermal diffusivity(m^2/s)
BVP	Boundary value problem method	β	constant, volumetric thermal expansion coefficient (1/K)
C	specific heat (J/kg K)	ε	effectiveness
C_a	specific heat at temperature Ta (J/kg K)	ρ	density (kg/m^3)
E_{ff}	Fin effectiveness	σ	Stefan–Boltzman constant
E_g	surface emissivity (W)	η	fin efficiency
GM	Galerkin method		
H	coefficient of natural convection(W/m^2.K)	***Subscripts***	
K	thermal conductivity (W/mK)	A	air
k_a	thermal conductivity in T = Ta (W/mK)	G	surface emissivity
L	latent heat length(m)	B	base temperature
N	Geometric parameter		
T	Temperature(K)		
T_a	environment temperature (K)		
T_s	effective sink temperature (K)		
T_b	temperature at the base(K)		
T_i	initial temperature (K)		

In this manuscript, the resulting nonlinear differential equation is solved by Galerkin to evaluate the temperature distribution within the fin and compared with Boundary Value Problem Method [1, 4]. Using the temperature

distribution, the efficiency of the fins is expressed through a term called thermo-geometric fin parameter (N) and thermal conductivity parameter (ε_1), thermal radiative parameter (ε_2), describing the variation of the thermal conductivity [5-8].
Galerkin method is perfectly universal and applied to elliptic, hyperbolic and parabolic equations, nonlinear problems as well as complicated boundary conditions. The results obtained by this method are then compared with those of BVP solution [9- 11].
Recently this kind of problems has been analyzed by some researchers using different methods [12- 20].
In this letter, analytical and numerical solutions of fin efficiency and Temperature Distribution of Conductive, Convective and Radiative Straight fins with Temperature Dependent Thermal Conductivity have been studied by Galerkin Method and one numerical method. For this purpose, after description of the problem and brief introduction for GM, they are applied to find the approximate solution. Obtaining the analytical solution of the model and comparing with numerical methods the capability, effectiveness, convenience and high accuracy of this method can be seen.

DESCRIPTION OF THE PROBLEM
The studied problem is the one-dimensional heat transfer in a straight fin with the length of L and the cross section area of A and the perimeter of P (see Fig. 1).

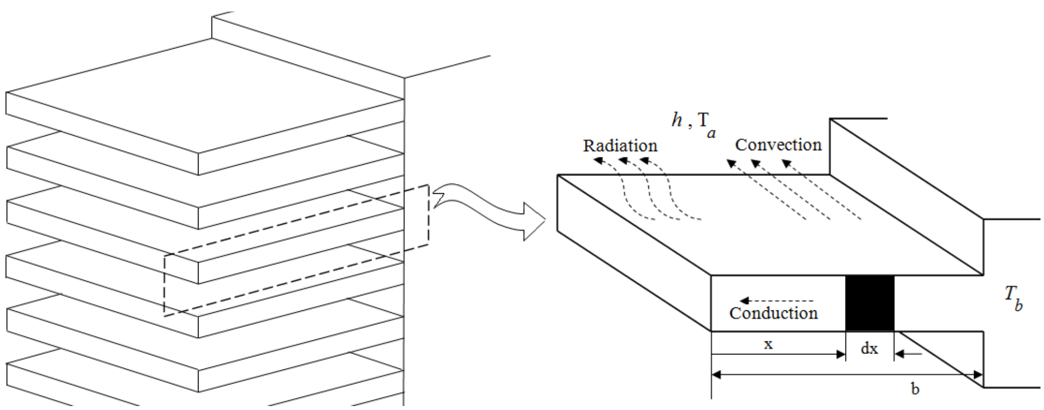


Fig.1.schematic geometry of fin

The fin surface transfers heat through both convection and radiation. Suppose the temperature of the surrounding air is T_a and the effective sink temperature for the radiative heat transfer is T_s . We assume that base temperature of the fin is T_b and there is no heat transfer of the tip of the fin. It is also assumed that the convection heat transfer coefficient, h, and the emissivity coefficient of surface, E_g are both constant while conduction coefficient, k, can be variable. The energy equation and the boundary conditions for the fin are as follows:

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{hP}{A}(T-T_a)-\frac{E_g\sigma}{A}(T^4-T_s^4)=0 \quad (1)$$

$$\frac{dT}{dx}=0 \ at \ \mathrm{x}=0 \quad (2)$$

$$T=T_b \ at \ x=l \quad (3)$$

Assuming k as a linear function of temperature, we have:

$$k(T)=k_a\left[1+\beta(T-T_a)\right] \quad (4)$$

After making the equation dimensionless and changing parameters, we have:

$$\theta = \frac{T}{T_b} \quad \theta_a = \frac{T_a}{T_b} \quad \theta_s = \frac{T_s}{T_b} \quad X = \frac{x}{L} \quad N^2 = \frac{hpL^2}{k_a A} \quad \varepsilon_1 = \beta T_b \quad \varepsilon_2 = \frac{E_g \sigma T_b^3 pL^3}{k_a A} \tag{5}$$

And substituting Eq. (5) in Eq. (1) we have:

$$\frac{d}{dX}\left\{[1+\varepsilon_1(\theta-\theta_a)]\frac{d\theta}{dX}\right\} - N^2(\theta-\theta_a) - \varepsilon_2(\theta^4-\theta_s^4) = 0 \tag{6}$$

$$\frac{d\theta}{dX} = 0 \ at \ \mathrm{X=0} \ , \ \theta = 1 \ at \ X = 1 \tag{7}$$

by assuming

$$\theta_a = \theta_s = 0 \tag{8}$$

the final fin differential equation will be:

$$\frac{d^2\theta}{dX^2} - N^2\theta + \varepsilon_1(\frac{d\theta}{dX})^2 + \varepsilon_1\theta(\frac{d^2\theta}{dX^2}) - \varepsilon_2\theta^4 = 0 \tag{9}$$

FIN EFFICIENCY

The heat transfer rate from the fin is found by using Newton's law of cooling.

$$Q = \int_0^b P(T - T_a)dx \tag{10}$$

The ratio of actual heat transfer from the fin surface to the other side while whole fin surface is at the same temperature, commonly called fin efficiency.

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T-T_a)dx}{Pb(T_b - T_a)} = \int_{\zeta=0}^{1} \theta(\zeta)d\zeta \tag{11}$$

FIN EFFECTIVENESS

The heat transfer rate from the fin is found by using Foureir's law as follow:

$$Q = k_a A \left.\frac{\partial\theta}{\partial x}\right|_{x=L} \tag{12}$$

The ratio of actual heat transfer from the fin surface to the other side while the fin is absent, commonly called Fin Effectiveness

$$E_{ff} = \frac{k_a A \left.\frac{\partial\theta}{\partial x}\right|_{x=L}}{hA(T_b - T_a)} \tag{13}$$

By assuming following parameter according to Eq.(5)

$$\theta = \frac{T}{T_b} \ X = \frac{x}{L} \ T_a = 0 \tag{14}$$

Finally, The Effectiveness of fin will be as follow:

$$E_{ff} = \frac{k_a \frac{\partial \theta}{\partial X}}{L} \tag{15}$$

2. BASIC IDEA OF APPLIED METHOD

2.1. Galerkin Method (GM)

To illustrate the basic background of this method, let us consider the following equation [21]:

$$\nabla^2 T(r) + A\,T(r) + \frac{1}{\mathrm{k}} g(r) = 0 \tag{16}$$

That covers a wide range of steady state heat conduction problems.

Let $\phi_j(r)$, $j = 1, 2, 3, \ldots$ be a set of basic functions. According to galerkin method we can construct the *n*-term trial solution, $\tilde{T}_n(r)$ in the form of [21]:

$$\widetilde{\mathrm{T}}(\mathrm{r}) = \psi_0(r) + \sum_{j=1}^{n} c_j \phi_j(r) \tag{17}$$

Where the function $\psi_0(r)$ is included to satisfy the non-homogeneous part of the boundary condition and the basis function $\phi_j(r)$ satisfy the homogeneous part. When all the boundary conditions are homogeneous, the function $\psi_0(r)$ is not needed. The subscript *n* in the trial solution denotes that it is an *n*-term trial solution.

When the trial solution, i.e., the Eq (16), is substituted into the differential equation, Eq. (15), a residual $R(c_1, c_2, \ldots, c_n; r)$ is left, because $\tilde{T}_n(r)$ is not an exact solution:

$$\nabla^2 \tilde{T}_n(r) + A\,\tilde{T}_n(r) + \frac{1}{\mathrm{k}} g(r) \equiv R(c_1, c_2, \ldots, c_n; r) \neq 0 \tag{18}$$

Now, the galerkin method for determination of the *n* unknown coefficient $c_1, c_2, \ldots, c_n$ is given by [21]:

$$\int_R \varphi_j(r) \left\{ \nabla^2 \tilde{T}_n(r) + A\,\tilde{T}_n(r) + \frac{1}{\mathrm{k}} g(r) \right\} dv = 0, \quad j = 1, 2, 3, \ldots, n. \tag{19}$$

That is written more compactly in the form of

$$\int_R \varphi_j(r) R(c_1, c_2, \ldots, c_n; r) dv = 0, \quad \mathrm{j}=1,2,3,\ldots,n. \tag{20}$$

Equation (19) provides *n* algebraic equations for determination of *n* unknown coefficient $c_1, c_2, \ldots, c_n$.

If the problem can be solved by the separation of variables and the basis functions, i.e., $\phi_j(r)$,are taken to be the Eigen-functions for the problem , then the solution obtained by the galerkin method becomes the exact solution for the problem as the number of terms approaches infinity.

SOLUTION WITH GALERKIN METHOD (GM)

For applying galerkin method trial function $\tilde{T}_n(r)$ is assumed as follows:

$$T(x) = c_0 + c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + c_5 X^5 + c_6 X^6 + \ldots \tag{21}$$

By considering weight functions in following form:

$$w_i = \frac{d\tilde{T}_i(t)}{dc_i} \tag{22}$$

The general galerkin integral is made for different i as follows:

$$\int_0^1 w_i R(c_1, c_2, ..., X)dX \quad (23)$$

The constants of integral will be obtained from the physics of problem. Then by setting obtained equation in a system of equations and considering boundary conditions and solving system of equations, the values of c constants will be obtained. These values of c constants by considering ε_1=0.2,ε_2=0.2,N=1 are shown in follows:

$$c_1 = 0, c_2 = 0.3118652298,\ c_3 = -0.001244517743,$$
$$c_4 = 0.02362896007,\ c_5 = -0.005265529866,\ c_6 = 0.004002839502 \quad (24)$$

Temperature profile of fin will be as follows:

$$T(x) = 0.6670130182 + 0.3118652298X^2 - 0.001244517743X^3$$
$$+0.02362896007X^4 - 0.005265529866X^5 + 0.004002839502X^6 + ... \quad (25)$$

The obtained efficiency value for fin is:

$$\eta = 0.7750770033 \quad (26)$$

RESULTS AND DISCUSSION

In Figure 2 the result of Galerkin Method (GM) is compared with boundary value problem (BVP). High accuracy of this method can be seen in this figure and table.2. Also above theme is shown in table 2.It is very interesting that GM with second degree's trial function converge to result with a good accuracy. By increasing length of fin the change of its influence on heat transfer rate will be decreased. In length of fin by increasing X, surface temperature of fin approach to ambient temperature that cause to decrease the rate of heat transfer in this region of fin. In Fig.4 the effect of fin's length increasing on Effective is investigate that a linear relevance between length and Effective is seen.

Fig.5 show by increasing the length of fin, its efficiency will decrease and the more its length increase the less its influence will be.

Table.2.Comparison of the solutions via GM and Numerical solution for $\theta(X)$

Degree of trial function		2		5		6	
X	**NM**	**GM**	**Err (e-6)**	**GM**	**Err (e-6)**	**GM**	**Err (e-6)**
0	0.6670135976	0.636624	5.92	0.6670169	3.3	0.6670130	0.6
0.1	0.6701327453	0.639967	0.46	0.6701322	0.5	0.6701327	0.0
0.2	0.6795140050	0.650044	5.62	0.6795104	3.6	0.6795140	0.0
0.3	0.6952292542	0.666955	2.90	0.6952273	2.0	0.6952288	0.4
0.4	0.7173997763	0.690857	3.36	0.7174015	1.8	0.7173992	0.6
0.5	0.7461987342	0.721976	6.31	0.7462023	3.5	0.7461986	0.2
0.6	0.7818550610	0.760617	2.77	0.7818566	1.5	0.7818553	0.2
0.7	0.8246593362	0.807176	3.94	0.8246569	2.4	0.8246594	0.0
0.8	0.8749723806	0.862157	5.74	0.8749689	3.5	0.8749719	0.5
0.9	0.9332377964	0.926181	1.62	0.9332386	0.8	0.9332376	0.2
1	1.0000000000	1.000000	0.0	1.0000000	0. 0	1.0000000	0. 0
$\overline{Err}$			3.51		2.1		0.3

Fig.2. comparison of the solutions via GM and Numerical solution for $\theta(X)$

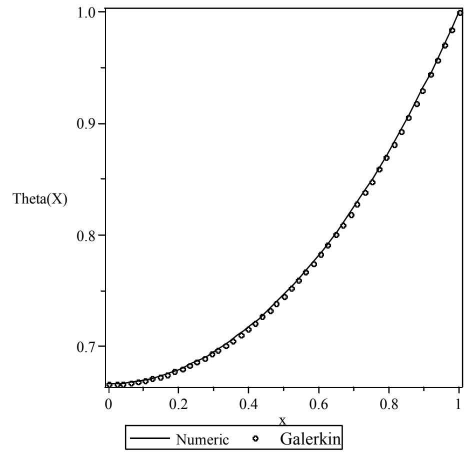


Fig.2 (a) $\varepsilon_1 = 0.2, \varepsilon_2 = 0.2, N = 1$

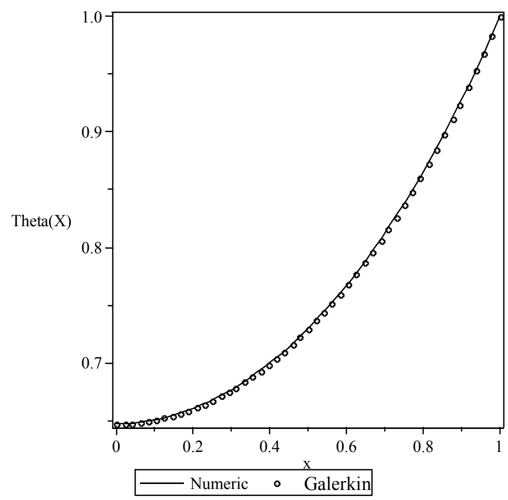


Fig.2 (b) $\varepsilon_1 = 0.3, \varepsilon_2 = 0.675, N = 1$

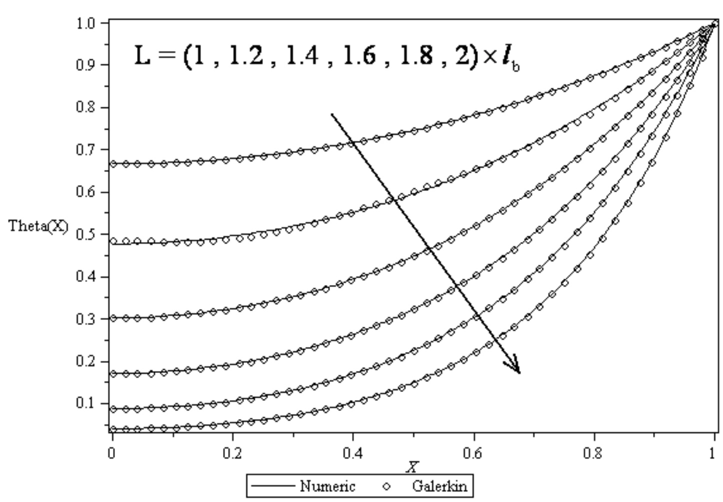


Fig.3 .The effect of length's increase on temperature distribution

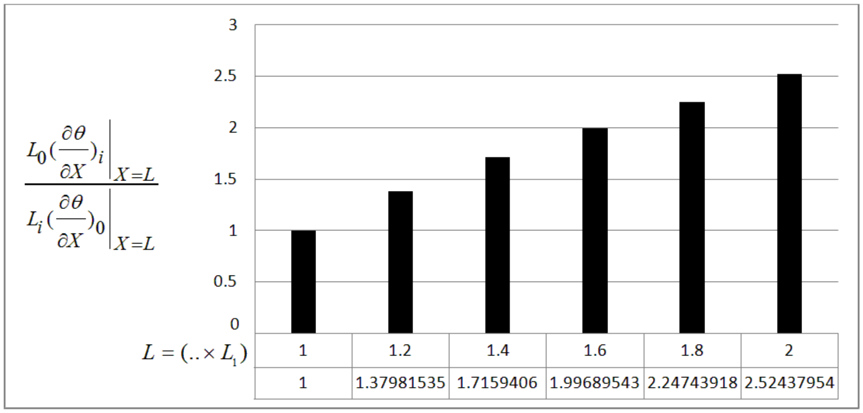
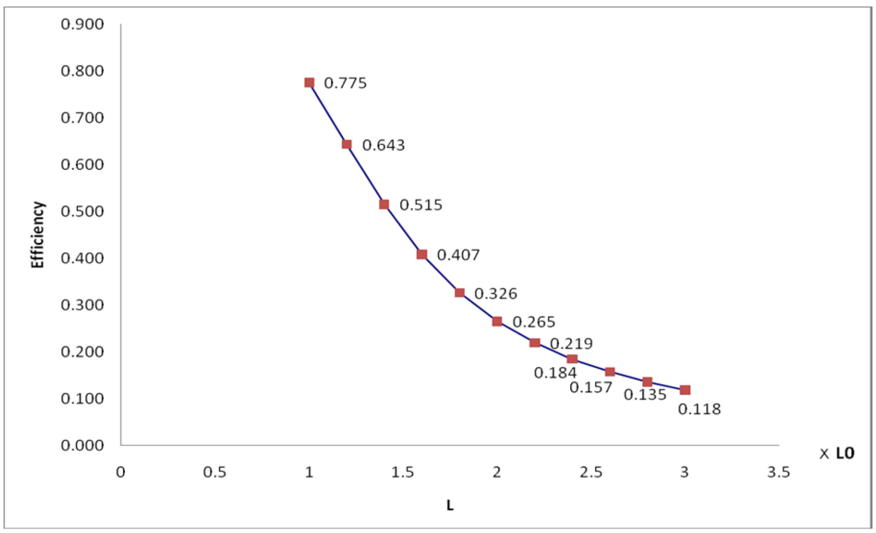


Fig.4. The effect of length increasing on efficiency



CONCLUSION

In this study, the definition and operation of Galerkin Method (DTM) is presented.. This method has applied to solve nonlinear differential equation arising in Conductive, Convective and Radiative Straight fins temperature-dependent thermal conductivity problem. The obtained fin Efficiency and fin Effective can be applied to advise a good geometry.

This exerting of GM is compared to BVP. The figures and tables clearly show high accuracy of GM to solve heat transfer problems in engineering.

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