NON LINEAR ANOMALOUS SKIN – EFFECT IN METALS

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ABSTRACT
The theory of nonlinear phenomena taking place under conditions of the anomalous skin – effect is presented. The nonlinearity is caused by the influence of the wave magnetic field on the dynamics of effective electrons. In the case of small wave amplitude A the imaginary part of the surface Ƶ is proportional to A^2, while the real part Re Ƶ ≈ A^4. Under the strong nonlinearity conditions Ƶ ≈ $A^{-1/5}$. All the effects are different manifestations of the magnetodynamical nonlinearity which is characteristic for pure metals at low temperatures.

1. INTRODUCTION

The high value of electric conductivity in typical metals, especially at low temperatures, causes the electric field to be very small inside the sample. For this reason, a number of known mechanisms of nonlinear electro dynamical effects proves to be of no importance in metals (for example, the heating of electrons by electric field). This is one of essential features distinguishing the conductor electrons metals in respect to the gas discharge plasma, ionosphere, semiconductors and other media with appreciable electric resistance. In fact, the nonlinear electrodynamics of metals is due to the evident fact that the magnetic field in the wave in metals greatly exceeds the electric one, for the parameter 4πσ/ω is much grater than unity (σ is the conductivity, ω is the frequency). As the deflection from the linear response are caused by changes in the dynamics of electrons under the influence of the magnetic field of electric current, such a mechanism of nonlinearity a "magnetodynamical" one.

In this review the effect of the magnetodynamical nonlinearity is considered under conditions of the anomalous skin – effect, this phenomenon consists in that the non-uniform static magnetic field arises in a metal sample as a result of the strong electromagnetic wave.

The dependence of the induced field on the external magnetic field which is constant and uniform has a hysteretic form. Finally, it is necessary to stress that the magnetodynamical nonlinearity is by no means an exotic one, but it can be experimentally observed even easier than other nonlinear effects in metals.

2. THEORETICAL CONSIDERATIONS AND CALCULATIONS

Unlike all other conductors, metals show the well – manifested skin – effect due to which the electromagnetic field vanishes in the sample over a small distance δ. This quantity is called the skin – depth. It is important that the skin – depth is much less than the electromagnetic wavelength in vacuum, that is

$$\frac{\omega\delta}{c} = \left(\frac{\omega}{4\pi\sigma}\right)^{1/2} << 1 \qquad (1)$$

The parameter ωδ/c is very small up to infrared reign. For instance, in radiofrequency range, where ω is 106sec-1, and at low temperature where δ is of the order of 10-3 - 10-4 cm, ωδ/c. ≈ 10-7 - 10-8.

The inequality in equation (1) provides, in fact, an electro dynamical definition of a metal. Two consequences of principal importance follow from the condition in equation (1).

(i) the magnetic component $H(\vec{r}, t)$ of electromagnetic wave in a metal exceeds the electric one $E(\vec{r}, t)$ in direct ratio the parameter in equation (1)

(ii) the surface impedance of metal Z is small being c/wδ times less than the vacuum impedance.

The impedance Z(ω) is determined as the ratio of the total electric field on the metal surface to the total electric current in the sample. The both quantities must be taken in the principal harmonic approximation (at the frequency ω of the incident wave). Therefore

$$Ƶ(\omega) = \frac{4\Pi}{c} \bullet \frac{E\omega\,(o)}{H\omega\,(o)} \qquad (2)$$

The argument "0" donates the value taken on the surface.
Let us consider the reflection of a plane monochromatic wave with amplitude A which propagates perpendicular to the metal – vacuum boundary (along, the axis Ox), taking account nonlinear effects [4]. As the surface impedance is small, the wave is reflected mainly with the same frequency ω. All the other harmonics in the reflected wave are weaker with the parameter in equation (1)
At the same time, under the strong nonlinearity conditions all the field harmonics inside the metal may be of the same order of magnitudes as the principal one.
Indeed, at frequency ω, there are three waves: the incident, refracted and transmitted ones. It is well know that in the incident and refracted waves the amplitudes of electric field are equal to that of corresponding magnetic field, while in the transmitted wave the magnetic component is much larger than the electric one. The boundary conditions to Maxwell's equations demand continuity of the total tangential electric and magnetic fields on the sample's boundary. To provide these conditions, magnetic components of incident and refracted waves must be added, while electric ones are to be subtracted to compensate each other with the impedance accuracy. This means that on principal harmonic the amplitudes of incident and reflected waves are equal within the mentioned accuracy, while the magnetic field amplitude in the transmitted wave is 2A.
Let us pass to the generation of multiple harmonics $n\omega$, n =2, 3 - - -. At such frequencies there are only two waves, the reflected one in vacuum and penetrated one in the metal. According to Maxwell's equations, in the reflected wave, the electric and magnetic fields are equal, while inside the electric component $c/n\omega\delta$ times less than the magnetic one. Consequently, both reflected magnetic harmonics and electric components of reflected and transmitted waves are equally small. To ensure the magnetic field in transmitted wave to exceed the electric one, while satisfying the boundary conditions it is necessary to nullify the magnetic component on the boundary. In other words, the magnetic wave field component in the harmonics is found to be a nonmonotic function of the distance to the surface. Near the surface it increases over the length of order of the skin – depth δ, after which it decreases as a consequence of the skin – effect.
Thus, the incident electromagnetic radiation is nearly completely reflected at principal frequency even in the strong nonlinear regime, when all the harmonics are of the same order of magnitude.
In this meaning, a metal is always a good linear element, the reflection and absorption being characterized by unique quantity, the surface impedance (2). The accuracy of such description is given by the condition (1). Unlike a linear case, the surface impedance in nonlinear regime must depend on the amplitude of incident wave, Z = Z(A). The smallness of $c|z|/4\pi$ compared to unity does not mean at all the weak dependence on A or Z(A). Therefore, the condition (1) of applicability of the impedance description does not coinside with the condition of validity of the perturbation theory in powers of A.
The analysis carried out so far enables to reveal the origin and mechanism of nonlinearity under conditions of the anomalous skin – effect [4]. The high value of conductivity in metals makes it impossible to attain so strong electric fields as to distort essentially the equilibrium electron distribution. Therefore, the electric current, even if being very large, is always a linear function of the electric field. The origin of nonlinear effects in metals is in Lorentz's force which is determined by the magnetic field of the wave or self magnetic field (in static regime).
Under conditions of the anomalous skin-effect, the mechanism of nonlinearity is connected with the change in trajectories of small electron group due to Lorentz's force.
The anomalous skin – effect takes place when the skin depth δ, is much less than the effective mean free path

$$\bar{\lambda}^* = {}^{v}/|\upsilon - i\omega|,$$ that is

$$\delta << \bar{\lambda}^* \qquad (3)$$

Here v is the Fermi velocity, $\upsilon = 1/\tau$ is the collision frequency of electron; τ is the mean free time. The curvature of electron is trajectories plays an essential role only inside the skin – depth. The anomalous skin effect is determined only by the so – called "effective" electrons which move in the skinlayer during the time $|\upsilon - i\omega|$-1. The path of an effective electron in the nonuniform magnetic field of skin layer is of the order of

$$L \sim = (4c\, \rho_F\, \delta/eA)^{1/2} \qquad (4)$$

where ρ_F is the Fermi momentum, e is the absolute value of electron charge c is the light velocity. Evidently, the electron may travel L ~ if L ~ $\langle \overline{Xf} \rangle$ λ^*. In the opposite case, $\bar{\lambda}^* <$ L~, an electron cannot "feel" the influence of alternating nonuniform magnetic field of the wave. Note that in the static case ($\omega = 0$) the role of the skin – depth goes over to the sample's thickness d. Then the wave field A is replaced by the self magnetic field.

Thus, the character and intensity of nonlinear interaction of electrons with electromagnetic wave is determined by the relation between L~ and X*. This means that the nonlinearity of the anomalous skin – effect may be expressed by the dimensionless parameter

$$b = \left(\frac{h}{2A}\right)^{1/2} \qquad h = \frac{8c\rho_F\delta}{e\bar{\lambda}^{*2}} \tag{5}$$

Here the value of typical magnetic field is introduced, which corresponds to L~ $=\bar{\lambda}^*$. This determines the scale of amplitudes of incident wave, responsible for essentially nonlinear effects. If the magnetic component of transmitted wave, 2A, is much less than h, the parameter b is large compared to unity. In this case, the weak nonlinear regime takes place and the perturbation theory may be used in powers of b-4 ~ A2. On the contrary, the small values of b, when h << 2A, correspond to the strong nonlinear regime.

Let us give an estimation for the field h. In typical pure metals at helium temperature, in radio frequency band, δ is of the order of 10-3 – 10-4cm, λ^* ~ 10-1cm and we get for h. The values 0.5 - 5 Oe. In experiment, the amplitudes of electromagnetic wave may achieve several tens and even hundreds oersteds. This means that the cases of both weak and strong non-linearies may be realized experimentally. Note that the decrease in the mean free path $\bar{\lambda}^*$ results in a considerable increase in the field h. so, if $\bar{\lambda}^*$ ~ 10-2 - 10-3 cm, the value of h is 50 – 5000 Oe and only the weak nonlinear regime will take place with b-4 << 1.

In the weak nonlinear case, when the parameter b is larger than unity,

$$b4 >> 1 \tag{6}$$

the trajectories of effective electrons inside the skin layer are nearly straight lines. They are only slightly curved by Lorentz's force due to the wave magnetic field in metal. Therefore, in the main approximation for parameter b, the skin – effect is described by the linear theory. The non-linearity manifests itself by the small nonlinear term Δj is added to the linear current density jo. In Fourier re presentation, according to [4] and [1],

$$j(k) = J_o(k) + \Delta j(k)$$

$$J_o(k) = \frac{3\Pi\sigma_o}{4\ell}\left[\frac{\mathcal{E}(k)}{k} - \frac{2(1-\rho)}{\Pi^2}\int_0^\infty \frac{dk'£(k)}{k^2 - k'^2}\ln\frac{k}{k^1}\right] \tag{7}$$

$$\Delta j(k) = \frac{-9\sigma_o}{\ell}\left(\frac{\bar{\lambda}^{*2}}{8R}\right) \bullet \left[2 - (1-\rho)^2 \sum_{n=1}^{\infty} \frac{\rho^{n-1}}{n^2}\right] \bullet E(o, t) \tag{8}$$

Here Jo (k) is the usual linear current density, E(k) is the spatial Fourier transform of electric wave field, σo is the static conductivity, $\lambda = V/\upsilon$ is the mean free path of electron, R=cρ /e|H(o, t)| is the gyro – radius in the field H(o, t), the parameter ρ is the average probability of specular reflection of electrons on the metal boundary.

The nonlinear correction Δj(k) is small compared to Jo (k) in proportion to the parameter (kλ*2/8R)2 $\approx$ b-4. It does not depend on the wave number K and is determined by the electric field on the metal surface. This means that the correction Δj(x) has in the coordinate representation the form of the surface current localized immediately near the boundary. The calculation shows that Δj(x) changes at distances of the order $\bar{\lambda}$*2/R $\approx$ b-2. $\delta << \delta$ from the interface.

Note that the expression for Δj(x) has a form which is identical with the correction for the current density in a weak fixed magnetic field H(o, t) = constant. It is not striking, as the current Δj(x) flows along the interface and the wave magnetic field is quasistatic for effective electrons.

The surface impedance is easily found in the standard way, using the method of successive approximations. As a result we get

$$Z(A) = Z(o) - \frac{i3\sqrt{3}c^2}{8\Pi^2\omega\delta}|Z(o)|^2 \bullet \left(\frac{2A}{h}\right) \left[2 - (1-\rho)^2 \sum_{n=1}^{\infty} \frac{\rho^{n-1}}{n^2}\right]$$

$$Z(o) = 4\Pi\sqrt{3}\,\frac{\omega\delta}{c} \bullet \frac{\sin^2(\Pi Z_0/3)}{\sin^2(\Pi Z_0/2)} \bullet \exp\left(\frac{-\Pi i}{3}\right) \tag{9}$$

Here Π Zo = cos-1ρ and δ is the skin – depth in linear theory

$$\delta = \left(\frac{c^2 \ell}{3\Pi^2\omega\sigma_0}\right)^{1/3} \tag{10}$$

We see that the first linear correction to the linear theory impedance Z(o), is purely imaginary quantity. As the impedance Z(A) is analytical function of A, it follows that

$$\text{Re}\,[Z(A) - Z(o)\,] \approx b^{-8} \approx A^4 \tag{11}$$

Thus, in the weak nonlinear regime (6) the dependence of real and imaginary parts of surface impendence on the amplitude A proves to be essentially different.

Under the strong nonlinearity conditions, the parameter b is much less unity

$$b << 1 \tag{12}$$

In this case an important role plays the fact that the spatical distribution of magnetic field in metals has an oscillating form. In other words, the wave field H(x, t) is sign changing.

Owing to this, there arises a new group of the so called "captured electrons". The electrons move parallel to the sample surface along trajectories twisting around the plane X = Xo (t), where H(Xo t) = 0 (figure 1). They move in a cyclic way and have the typical half period.

$$T = \Pi \left(\frac{mc}{e\upsilon|H'(X_o\,(t)|}\right)^{1/2} \tag{13}$$

Here the prime denotes the derivative with respect to x, m is the effective electron mass, such character of motion of captured particles is connected with the change of the sign of Lorentz's force when an electron crosses the plane X = Xo (t). It is rather evident the Xo (t) is always of the order of δ. Therefore, the electrons captured in the wave potential well by Lorentz's force move in the skin layer range and interact effectively with the electromagnetic wave during the whole mean free time. It is easy to show that under the strong nonlinearity conditions (II) the skin effect is entirely determined by the captured electrons.

For this purpose, we use the nonlinear version of pippard's "ineffectiveness concept" which has been proved in [4]. The concept is used on the fact that the screening current in metal in the anomalous skin – effect is controlled by the small group of effective electrons. In the linear approximation, such electrons move along straight lines almost parallel to the interface during the whole mean free time τ. The relative number of the electrons is of the order δ/λ<< 1 and their contribution to conductivity is σeff ≈ σoδ/λ. Here and in what follows we restrict ourselves for simplicity with the low frequency case, ω << 0. In the strong nonlinear regime (II) the relative number of captured electrons of the order of σ/L~ = (σ/λ/) b.

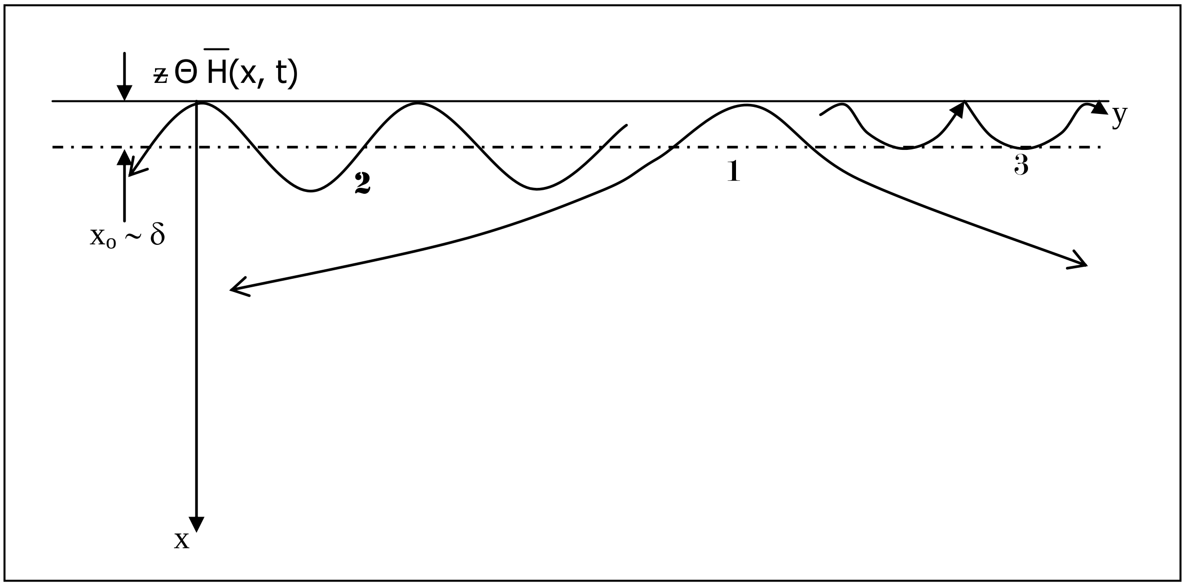

Figure 1 – The paths of effective electrons in the magnetic field of electromagnetic wave: 1 – the transit electron, 2 – the captured electron, 3 – the slipping electron.

Hence, the conductivity of captured electrons σ proves to be layer than the value σ_{eff}^{lin} in the linear theory. This means that under the anomalous skin – effect conditions the penetration depth δ and the surface impedance Z(A) are markedly decreases as compared with their values in the small amplitude range.

In the ineffectiveness concept, the true distribution of electromagnetic field in metal is replaced by the exponential one. It is characterized by an effective skin – depth δ which is generally speaking, a complex quantity. Then Maxwell's equations are transformed to an algebraically equation for determining δ. The relationship has the form.

$$\delta^2 = \frac{i\,c^2}{4\Pi\omega\sigma} \tag{14}$$

It is solved along with the equation for the captured electrons conductivity.

$$\sigma = \sigma_o \frac{\delta}{L\sim} = \sigma_o \left(\frac{e\,A\,\delta}{4c\rho_F}\right)^{1/2} \tag{15}$$

Finally, we get

$$\delta(A) = \left(\frac{c^2\rho_F}{4\Pi\omega^2\sigma_o^2 eA}\right) . \exp\left(\frac{\Pi i}{5}\right) \tag{16}$$

$$Z(A) = F\,\frac{4\Pi\omega|\delta(A)|}{c^2} . \exp\left(\frac{-3\Pi\iota}{10}\right) \sim A^{-1/5}\,\omega^{3/5} \tag{17}$$

Here F is a positive factor which depends neither on the amplitude A nor the frequency ω, and it must be found on the basis exact theory. Fig. 2 illustrates the schematic behaviour of impedance as a function of the magnetic field amplitude A of the incident wave.

In the range A ~ h there is a smooth maximum where the A – square dependence is replaced by the asymptotic (15). We see that even in the intermediate range A ≤ h the difference from the linear theory becomes noticeable.

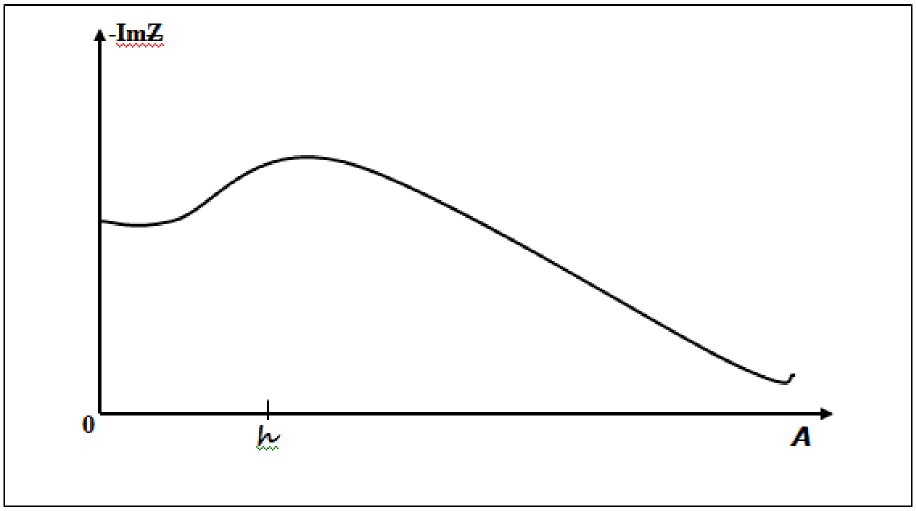

Figure 2 – The schematic behaviour of imaginary part of impedance as a function of the magnetic fields amplitude A of the incident wave.

3. RESULTS AND DISCUSSION

The analysis carried out in [4] shows that the captured electrons conductivity depends only on the module |H(x, t)|. This results in the absence of even harmonics 2ω, 4ω, … in the nonlinear anomalous skin – effect in metal. In particular, the dc component of the magnetic field does not arise in such situation. In other words, the incident wave of finite amplitude in metal generates only the odd harmonics.

The frequency dependence of impedance Z ~ ω3/5 and the value of its phase 3Π/10 differ from those know in the linear theory of anomalous skin – effect (8).

Let us note that the result (15) does not depend on the scattering of electrons on the interface because the captured electrons do not interact at all with the sample boundary. Only by specular reflection (ρ = 1) the value of F maybe slightly changed when the group of slipping electrons (group 3 on fig 1) appears having the conductivity of the same order as that of the captured particles.

It is necessary to stress that the mechanism of electromagnetic absorption has a collision nature under the strong nonlinearity conditions, in contrast to the collisionless damping in the linear regime. It is caused by the capture of electrons in the magnetic potential well and predominant contribution of captured particles to the formation of the screening current.

4. CONCLUSION

It is interesting to compare the nonlinear anomalous skin-effect with another known phenomena, that is the nonlinear landau damping [5] of own electromagnetic waves in a metal. The feature common for these two effects is the capture of electrons in potential profile of the wave, but the role of captured particles proves to be opposite. In the Landau damping the electrons comes out of action and do not contribute to collisionless absorption. Here the dynamics of captured electrons is no importance. On the contrary, the nonlinear anomalous skin effect is an example of strong nonlinear phenomenon when electromagnetic properties of conductor are completely determined by the dynamics of captured electron. In this case, the influence of the wave field on the conductivity of metal becomes so strong that the physical picture of skin – effect is significantly changed [2, 3].

5. REFERENCES

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