OPTIMIZATION OF THE REINFORCEMENT TEXTURE OF A CYLINDRICAL COMPOSITE SHELL UNDER STATIC LOADS

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ABSTRACT

This work is devoted to a method of optimization of the composite shell woven reinforcements. This method is based on Goldenblat-Kopnov's anisotropic model which is suitable for this material. This study involves three steps: Firstly, it is carried out experimental characterization of the strength properties of the material; then it is proposed a method of optimization of the winding angle of composite shell woven reinforcements based on the strength anisotropy of the material. The third step consist in establishing the variation ranges of the static strength tensor components of this material in three planes of symmetry from the Sylvester's criterion of positive definition of symmetric matrices. Obtained results allow improve the structural performance of composite shells with woven reinforcements.

Keywords: *optimization, winding angle, texture, variation ranges*.

1. INTRODUCTION

Optimization of structures is a very challenge in the industry in general and in civil engineering in particular where composite parts are increasingly growing due to their physico-mechanical properties more interesting than those of traditional materials such as concrete or steel. To do this, one of ways is to work on the material. This work is particularly interested in a two-dimensional cylindrical composite shell with woven reinforcements. The advantages of this type of material compared to the one-way composite laminates are [1]: firstly, a significant flexibility allowing the design of preforms close to the shape of the final object reducing manufacturing costs; secondly, a better tolerance for defects. Indeed, it is now necessary to design the priori material to adapt it locally for its function within the overall structure. It is with this approach that the potential for optimization of woven composite materials becomes particularly interesting especially through optimal management of their anisotropy. However, the multitude of design parameters and material parameters influencing the performance of a composite structure and the absence of direct relations between themselves make complex any structural optimization. This is why it is now essential to implement a calculation chain linking mechanical properties and process to the final structural performance. Woven composites applications into the industrial sector (mechanical engineering, civil engineering) are varied. In particular, in civil engineering for example, the liquid materials sector (bentonite, bituminous mixtures, and liquefied gas) includes different key stages: production: distribution, storage and operation. Nowadays the storage of these sensitive materials is still a major technological and scientific lock. One of the solutions to this challenge is for example the design and manufacture of high-pressure composite tanks consisting by a "polymer liner" and a structuring composite shell filed by winding filament on the entire surface. In this type of tank, the 'liner' does not support the load; it provides only its seal. At present no 'high pressure' solution satisfied fully with technical and economic industrial specifications. Since 1998 work on the development of storage tanks (70 MPa operating pressure) have already obtained promising results [2, 3]. Most current sizing methods are carried out considering the initial mechanical properties of the material and are based on empirical safety coefficients [4,5]. First experimental monitoring by acoustic emission studies dating back 20 years had allowed giving a certain empirical idea of accumulation of fiber breakages in these structures. Blassiau [6] went further to develop a means of control and characterization of composite tanks, but also to determine the life time of these. Regarding optimization of the tanks many works are carried out on the architecture of the reinforcements [7.8, 9]. These optimizations are based on feedback and targeted performance but not on taking account of the actual damage mechanisms.

The purpose of this work is to propose a method of optimization of the winding angle of the woven reinforcements based on anisotropy of strength of the material by which can be established the variation ranges of the static strength tensor components of the material.

2. MATERIALS AND METHOD

The purpose of the study structure is orthotropic cylindrical shell with woven reinforcement: weft direction is radial (corresponding to index 1 in formulas) and branch warp direction is longitudinal (corresponding to index 2). The loads are static but of different natures:

- uniform distributed internal hydrostatic loading q
- external applied torsion moment M_k
- combination of hydrostatic load and torsion moment

In this work, we use the thin-walled shell theory; therefore we will adopt the following relationships:

$$\sigma_{11}=\frac{qR}{h},\ \sigma_{22}=\frac{qR}{2h},\ \ \sigma_{12}=\frac{M_k}{2\pi R^2 h} \tag{1}$$

σ_{11} and σ_{22} are respectively the radial and normal longitudinal stresses, σ_{12} - the shear stress in the shell cross section, R - mean radius, h - shell thickness. For evaluation of the complex stress state, we will use Goldenblat-Kopnov criterion for anisotropic materials [10], which in the case of combination of hydrostatic load and torsion moment, take following form:

$$\Pi_{11}\sigma_{11}+\Pi_{22}\sigma_{22}+(\Pi_{1111}\sigma_{11}^2+\Pi_{2222}\sigma_{22}^2+2\Pi_{1122}\sigma_{11}\sigma_{22}+4\Pi_{1212}\sigma_{12}^2)^{0,5}=1, \tag{2}$$

Π_{ik} and Π_{ikmn} are the static strength tensor components of anisotropic material calculated by its strength limit in traction, compression and shear [10].

Expressions (1) and (2) allow establish the parameters of the breaking load, for example the breaking pressure q_p or the breaking moment of torsion M_k:

$$q_p=2h\{R[2\Pi_{11}+\Pi_{22}+(4\Pi_{1111}+\Pi_{2222}+4\Pi_{1122})^{0,5}]\}^{-1} \tag{3}$$

$$M_k=\pi R^2 h(\Pi_{1212})^{-0,5} \tag{4}$$

For loading combination of internal pressure and torsional moment the break load limit q_p is calculated by the establishment of relationship between normal stress and shear, for example by writing $\sigma_{12}=k\,\sigma_{11}$:

$$q_p=2h\{R[2\Pi_{11}+\Pi_{22}+(4\Pi_{1111}+\Pi_{2222}+4\Pi_{1122}+16k^2\Pi_{1212})^{0,5}]\}^{-1} \tag{5}$$

If you vary the reinforcement winding angle φ, due to the strength anisotropy of the material, on the one hand there is variation of the value of q_p and especially variation of the strength tensor components which is necessary to determine the variation limits of φ.

In addition, can proceed with a prediction of these variations using formulae Π_{ik} and Π_{ikmn} of tensor analysis. These formulas have the following expressions:

$$\Pi_{11}(\varphi)=\Pi_{11}cos^2\varphi+\Pi_{22}sin^2\varphi+2\Pi_{12}sin\varphi cos\varphi\ ;$$

$$\Pi_{22}(\varphi)=\Pi_{11}sin^2\varphi+\Pi_{22}cos^2\varphi-2\Pi_{12}sin\varphi cos\varphi\ ;$$

$$\Pi_{1111}(\varphi)=\Pi_{1111}cos^4\varphi+\Pi_{2222}sin^4\varphi+(\Pi_{1212}+0.5\Pi_{1122})sin^2 2\varphi\ ; \tag{6}$$

$$\Pi_{2222}(\varphi)=\Pi_{1111}sin^4\varphi+\Pi_{2222}cos^4\varphi+(\Pi_{1212}+0.5\Pi_{1122})sin^2 2\varphi\ ;$$

$$\Pi_{1122}(\varphi)=(\Pi_{1111}+\Pi_{2222}-4\Pi_{1212})sin^2\rho\ cos^2\varphi++\Pi_{1122}(sin^4\varphi+cos^4\varphi)\ ;$$

$$\Pi_{1212}(\varphi)=(\Pi_{1111}+\Pi_{2222}-2\Pi_{1122})sin^2\rho\ cos^2\varphi+\Pi_{1122}cos^2 2\varphi\ ;$$

The next step is to calculate relations (3), (4) and (5) by substituting expressions (6).

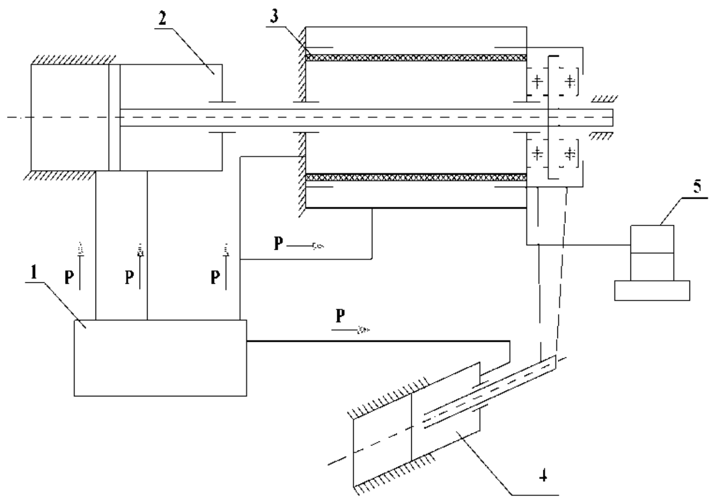
As result we obtain for each angle φ the corresponding value of q_p or M_k.

3. RESULTS AND DISCUSSION

The purpose of the study material is cylindrical polymeric composite shell reinforced by woven fiberglass CT-10. Basis mechanical characteristics for this composite are obtained by traction-compression and shear tests and are given in table 1. The values of the strength tensor components are reported in table 2.

3.1 Description of the test

The mechanical characteristics are determined by uniaxial traction- compression and shear tests using an experimental device used for deformation testing. This device is equipped with an electronic sensor to calculate values of stresses (figure 1). The tests are performed on reinforced with fiberglass composite test tubes (figure 2): reinforcements warp directions are prepared following the longitudinal axis while reinforcements meaning weft are arranged following circular direction of the test piece, the all drowned in polymer polyester matrix.



Legend**:**

1, 2, 4 - loading system of the test piece

3 -test tube

5 –electronic data logger

Figure1. Experimental testing device on fiberglass/polyester composite CT-10

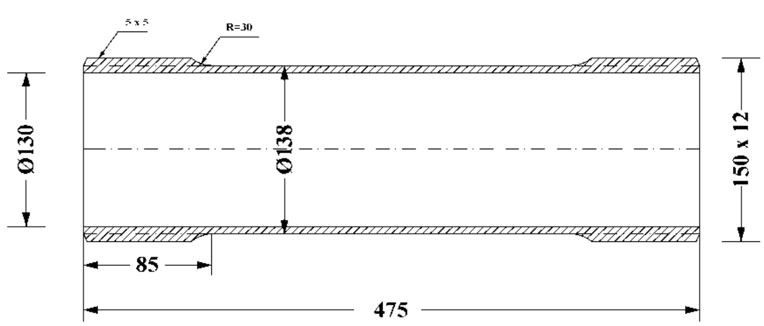


Figure2. Tubular fiberglass/polyester composite CT-10

Table1. Mechanical characteristics of the woven fiberglass composite CT-10
Table1a. Resistance in traction, compression and shear

Mechanical characteristics	σ_{t1}^{+}	σ_{C1}^{-}	σ_{t2}^{+}	σ_{C2}^{-}	τ_{t45}^{+}	τ_{t45}^{-}	τ_{t0}
Values, (MPa)	605	285	325	250	285	247	155

Table 1 b. Elastic and shear modules and Poisson's coefficient

Elastic characteristics of the composite CT-10	Under static loading
E_x , MPa	$2.31 \cdot 10^4$
E_y , MPa	$1.86 \cdot 10^4$
E_Z , MPa	$0.35 \cdot 10^4$
G_{xy} , MPa	$4.1 \cdot 10^3$
G_{yZ} , MPa	$2.4 \cdot 10^3$
G_{Zx} , MPa	$1.3 \cdot 10^3$
μ_{xy}	0.21
μ_{yZ}	0.39
μ_{Zx}	0.15

Table2. Values of the static strength tensor components $(MPa)^{-1}$

Π_{11}	Π_{22}	Π_{1111}	Π_{2222}	Π_{1212}	Π_{1122}
$-0.957 \cdot 10^{-3}$	$-0.177 \cdot 10^{-3}$	$8.765 \cdot 10^{-5}$	$15.06 \cdot 10^{-5}$	$13.74 \cdot 10^{-5}$	$3.465 \cdot 10^{-5}$

The static strength tensor components are calculated according to [10].

In fig.ure3 it is showed the curve of relation $M_k = f(\varphi)$ and in figure4 the curve $q_p = f(\varphi)$ under action of internal hydrostatic pressure (curve 1) and for combination of internal hydrostatic load and torsional applied moment. Curve 2 corresponds to the value k = 0. 25, curve 3 corresponds to k = 0. 5 and the curve 4 to k = 1.

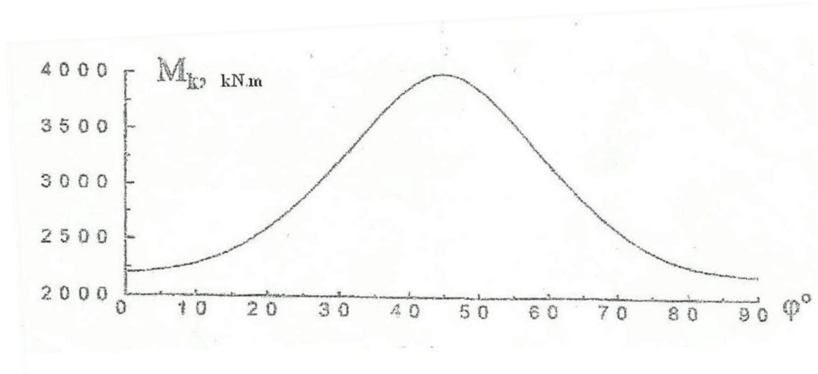


Figure3. Curve $M_k = f(\varphi)$.

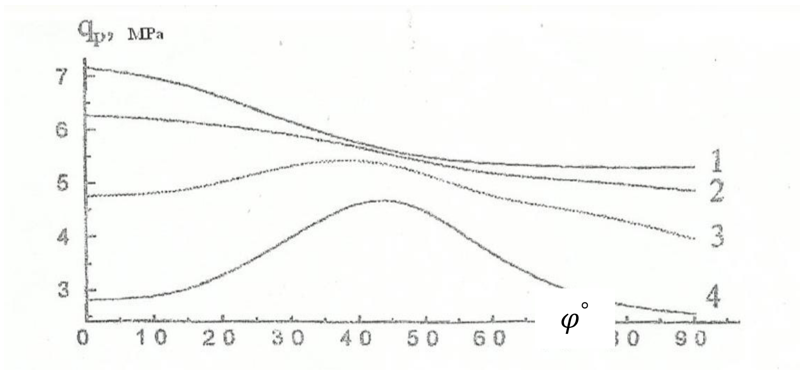


Figure4. Function curves $q_p = f(\varphi)$

3.2 Determination of the variations ranges of the static strength tensor components of the material

As mentioned earlier the need to determine the variation ranges of the strength tensor components is obvious. This aspect was addressed in [11] for the establishment of the ranges of variability of the Mises-Hill resistance tensor components for anisotropic elastic material.

Studied material is orthotropic material in three planes of symmetry of elastic properties, we will write the terms of positive definition of symmetric matrices as follow:

a)	1	μ_{12}	μ_{13}	μ_{14}	0	0
		1	μ_{23}	μ_{24}	0	0
			1	μ_{34}	0	0
				1	0	0
					1	μ_{56}
						1

b)	1	μ_{12}	μ_{13}	0	0	0
		1	μ_{23}	0	0	0
			1	0	0	0
				1	0	0
					1	0
						0

where $\mu_{kk} = \frac{\Pi_{kk}}{\sqrt{\Pi_{ii}\Pi_{kk}}}$, Π_{kk} – Mises-Hill's tensor components.

The diagonal components of the matrix Π_{ik} for this material are positive and are reference values for the determination of the permissible variation ranges of other non-diagonal components. From tests in traction or compression in the principal directions of anisotropy and shear in the main plans of anisotropy we find:

$$\Pi_{11} = \sigma_{t1}^{-2} \underset{(1.2.3)}{\overrightarrow{\longleftarrow}}, \qquad \Pi_{44} = 0.25\sigma_{t4}^{-2} \underset{(4.5.6)}{\overrightarrow{\longleftarrow}} \tag{7}$$

$\sigma_{t1}^{-2} \underset{(1.2.3)}{\overrightarrow{\longleftarrow}}$ and $\sigma_{t4}^{-2} \underset{(4.5.6)}{\overrightarrow{\longleftarrow}}$ are the parameter limits of the corresponding stress states (for example σ_{t4} is pure shear stress in anisotropy plane with the anisotropy axes 1 and 2, σ_{t5} - the pure shear in the anisotropy plane of with anisotropy axes 2 and 3, etc.), $\underset{(1.2.3)}{\overrightarrow{\longleftarrow}}$ and $\underset{(4.5.6)}{\overrightarrow{\longleftarrow}}$ are the symbols of circular index permutation.

From the Sylvester's criterion of positive definition of symmetric matrices, for the first up-diagonal components of matrices (a) and (b), we find:

$$\mu_{i(i+1)} = sin\emptyset_{i(i+1)} \tag{8}$$

$$\left(-\frac{\pi}{2} < \emptyset_{i(i+1)} < \frac{\pi}{2}\right) \text{ i=1, 2, 3, 5.}$$

For the second up-diagonal components of these matrices, we have:

$$\boldsymbol{\mu}_{i(i+2)} = \boldsymbol{sin}\emptyset_{i(i+1)}\boldsymbol{sin}\emptyset_{(i+1)(i+2)} + \boldsymbol{cos}\emptyset_{i(i+1)}\boldsymbol{cos}\emptyset_{(i+1)(i+2)}\boldsymbol{sin}\emptyset_{i(i+1)} \tag{9}$$

$$\left(-\frac{\pi}{2} < \emptyset_{i(i+1)},\ \emptyset_{i+1)(i+2)}, \emptyset_{i(i+2)} < \frac{\pi}{2}\right), \text{ i=1, 2.}$$

For the component μ_{14} of matrix (a) its explicit expression is more complicated

$$\mu_{14} = \left[-D_{241} + (D_{241}^2 + D_{141}D_{341})^{\frac{1}{2}} sin\emptyset_{14}\right]\frac{1}{D_{141}} \tag{10}$$

where $-\frac{\pi}{2} < \emptyset_{14} < \frac{\pi}{2}$, $D_{41} = \begin{bmatrix} 1 & \mu_{23} \\ \mu_{23} & 1 \end{bmatrix}$,

$$D_{241} = \begin{bmatrix} \mu_{12} & \mu_{13} & 0 \\ 1 & \mu_{23} & \mu_{24} \\ \mu_{12} & 1 & \mu_{34} \end{bmatrix}, \quad D_{241} = \begin{bmatrix} 1 & \mu_{12} & \mu_{13} & 0 \\ \mu_{21} & 1 & \mu_{23} & \mu_{24} \\ \mu_{31} & \mu_{32} & 1 & \mu_{34} \\ 0 & \mu_{42} & \mu_{43} & 1 \end{bmatrix}.$$

Case of the transverse isotropic material:

For this particular case should be substitute in matrix (b) expression:

$\mu_{13} = \mu_{23}$,

hence, referring to the expressions (3) and (4) we end up to the limitations of the angles $\emptyset_{12}$ and $\emptyset_{23}$:

$$-1 < tg\emptyset_{23}\frac{1-sin\emptyset_{12}}{cos\emptyset_{12}} < 1 \tag{11}$$

For example when $\emptyset_{12} = -\frac{\pi}{4}$, we have $-0{,}39 < \emptyset_{23} < 0{,}39$, etc. It may be noted that with increasing of angle, the permissible variation range of the angle $\emptyset_{23}$ (or angle $\emptyset_{12}$) increase (up $\pm\frac{\pi}{2}$), in contrast for decrease of angle will be a regression of angle (up to 0).

4. CONCLUSIONS

In the present work we can retain following:

1. It is proceeded to experimental identification of mechanical properties of a polymeric composite with woven reinforcements;
2. Using these mechanical characteristics, one could calculate the values of the static strength tensor components;
3. Using the Goldenblat-Kopnov's anisotropic model could provide a method of optimization of the reinforcement winding angle of a composite shell with woven reinforcements taking into account anisotropy of resistance of this material;
4. From the Sylvester's criterion it is established variation ranges of the strength tensor components for better optimization of structural performance of composite shells with woven reinforcements.

Nomenclatures	
q	Uniform distributed internal hydrostatic loading
h	Shell thickness
R	Mean radius
M_k	External applied torsion moment
Greek Symbols	
$\boldsymbol{\sigma_{t1}^{+}}$	Strength limit in traction following direction 1
σ_{t2}^{+}	Strength limit in traction following direction 2
$\boldsymbol{\sigma_{C1}^{-}}$	Strength limit in compression following direction 1
$\boldsymbol{\sigma_{C2}^{-}}$	Strength limit in compression following direction 2
$\boldsymbol{\tau_{t45}^{+}}$	Shear strength limit in plan of angle 45° to direction 1
$\boldsymbol{\tau_{t45}^{-}}$	Shear strength limit in plan of angle 45° to direction 2
$\boldsymbol{\tau_{t0}}$	Shear strength limit in parallel to directions 1 and 2 planes
Π_{ik} , Π_{ikmn}	Static strength tensor components
σ_{11}	Normal longitudinal stress
σ_{22}	Radial stress
φ	Reinforcement winding angle

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