CALCULATION OF MOMENTUM DISTRIBUTIONS OF ^{7}Be FRAGMENT FROM ^{8}B + ^{12}C REACTION USING THE GLAUBER THEORY

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ABSTRACT
The momentum distributions of ^{7}Be core fragment from ^{8}B + ^{12}C reaction system are computed in the framework of the Glauber Theory using the CSC_GM code. The contribution of each component of the angular momentum (for $m_l = 0, \pm1$) for the p-orbital to the longitudinal angular momentum distribution is also shown. The CSC_GM code used in the computations originated from the CPC Program Library, Queen's University of Belfast, N. Ireland. The CSC_GM code is a Fortran 90 program that was originally run on UNIX operating system. The code was modified and run on Windows xp. The projectile nucleus is assumed to have the structure of a core plus valence nucleon. The input data needed for the calculations are the core and target densities and the nucleon-nucleon profile function. Results are found to agree with the experimental data, especially at high incident energies.

Keywords: *Glauber model, Momentum distribution, Nucleon-removal cross section*

1. INTRODUCTION
The momentum distribution of a fragment is one of the quantities measured in the experimental study of unstable radioactive nuclei which has advanced considerably through the technique of using secondary radioactive beams [1,2,3]. Other relevant quantities also measured in this type of study are the various reaction cross sections that include the total reaction cross section, nucleon-removal cross sections, etc. These quantities play important role in revealing the nuclear structure of unstable nuclei, particularly the halo structure, proton and neutron skins [1,3]. Halo and skin are exotic nuclear properties or structures that are peculiar to only unstable radioactive nuclei. Sizes and density distributions (of both nuclear matter and charge) of unstable nuclei are therefore quite different from those of stable nuclei.In this paper momentum distributions of ^{7}Be core fragment from the ^{8}B + ^{12}C reaction system are calculated in the framework of the Glauber Theory using the CSC_GM code. The reaction system is described as: $^{8}B + {}^{12}C \rightarrow {}^{7}Be + {}^{12}C + p$. The projectile nucleus (^{8}B) is assumed to have the structure of a core nucleus (^{7}Be) plus a valence proton. Measurement of the momentum distribution of the core fragment is now a standard work for the study of unstable nuclei.The Glauber model is a microscopic reaction theory of high-energy collision based on the eikonal approximation and on the bare nucleon-nucleon interaction. It is now a standard tool to calculate the momentum distribution because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus [4,5].

2. PROBLEM FORMULATION
The reaction of a projectile nucleus P with a target nucleus T is considered. At the initial stage of the reaction, the projectile in the ground state, described with an intrinsic wave function Ψ_0, impinges with momentum $hK = (0, 0, hK)$ on the target in its ground state, described with an intrinsic wave function Θ_0. The center-of-mass wave function is removed from Ψ_0 (Θ_0). At the final stage of the reaction, the projectile goes to the state a specified by a wave function Ψ_a and the target goes to the state c specified by a wave function Θ_c. The state a is not necessarily a bound state but may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is hq. The scattering amplitude for this reaction is written in the Glauber theory as an integral over the impact parameter $\boldsymbol{b}$ between the projectile and the target [6] as

$$F_{ac}(q) = \frac{iK}{2\pi}\int db e^{-iq\cdot b}\left\langle \psi_a \Theta_c \left| 1 - \prod_{i\in P}\prod_{j\in T}\left(\mathbf{1} - \Gamma_{ij}\right) \right| \psi_0 \Theta_0 \right\rangle \tag{1}$$

The integrated cross section for this reaction is given by

$$\sigma_{ac} = \int \frac{dq}{K^2}\left|F_{ac}(q)\right|^2 \tag{2}$$

The profile function Γ *in* Eq. (1) is given by:

$$\Gamma(b) = \frac{1-i\alpha}{4\pi\beta}\sigma_{NN}e^{-b^2/2\beta} \qquad (3)$$

The parameters σ_{NN}, α, and β usually depend on either the proton–proton (neutron–neutron) or proton–neutron case. The argument of Γ_{ij} in Eq. (2) is $\boldsymbol{b} + \boldsymbol{s}^{\mathrm{P}} - \boldsymbol{s}^{\mathrm{T}}$, which stands for the impact parameter between ith and jth nucleons. Here $\boldsymbol{s}^{\mathrm{P}}$ ($\boldsymbol{s}^{\mathrm{T}}$) is the two-dimensional coordinates comprising the x- and y-components of the ith nucleon coordinate in the projectile (target) relative to its center-of-mass coordinate.

3. LONGITUDINAL MOMENTUM DISTRIBUTION

The one-nucleon removal reaction is contributed by both the elastic and inelastic processes with the inelastic process becoming dominant at high energies beyond a few hundred MeV/nucleon [4]. The longitudinal momentum distribution of the core fragment is therefore here calculated after the inelastic breakup of the projectile. Let the momentum of the core be $P = (P_{\parallel}, P_{\perp})$ and that of the nucleon going to the continuum state be $\hbar k$. Assuming that the core remains in its ground state the momentum distribution is calculated by the equation [7].

$$\frac{d\sigma_{-N}^{inel}}{dP} = \int \frac{dq}{K^2}\sum_{c\neq 0}\int dk\,\delta\left(P - \frac{A_c}{A_p}\hbar q + \hbar k\right)\left|F_{(k,0)c}(q)\right|^2 \qquad (4)$$

Since the momentum transfer received by the ejected valence nucleon is considered to be large, the final state interaction can be ignored. The continuum scattering wave function of the last nucleon is then approximated by a plane wave,

$$\varphi(r) = \frac{1}{(2\pi)^{2/3}}e^{-p\cdot r}, \qquad (5)$$

and equation (4) then reduces to:

$$\frac{\sigma_{-N}^{inel}}{dP} = \int d\,b_N\left(1 - e^{-2Im\chi_{NT}(b_N)}\right)$$

$$X\frac{1}{(2\pi\hbar)^3}\frac{1}{2j+1}\sum_{mm_s}\left|\iint dr e^{\frac{i}{\hbar}P\cdot r}\chi^{*}_{\frac{1}{2}m_s}e^{i\chi_{CT}(b_N-s)}\varphi_{nljm}(r)\right|^2, \qquad (6)$$

where b_N stands for the impact parameter of the valence nucleon with respect to the target, $\varphi_{nljm}(r)$ is the valence nucleon wave function. The core-target and nucleon-target phase-shift functions χ_{CT}, χ_{NT} are defined through the relevant densities ρ by

$$\int i\chi_{CT}(\boldsymbol{b}) = -\int d\boldsymbol{r}\int d\boldsymbol{r}'\rho_C(\boldsymbol{r})\rho_T(\boldsymbol{r}')\Gamma(\boldsymbol{b}+\boldsymbol{s}-\boldsymbol{s}'), \qquad (7)$$

$$\int i\chi_{NT}(\boldsymbol{b}) = -\int d\boldsymbol{r}\,\rho_T(\boldsymbol{r})\Gamma(\boldsymbol{b}-\boldsymbol{s}). \qquad (8)$$

The density ρ is given by

$$\rho(r) = \sum_i c_i e^{-a_i r^2} \qquad (9)$$

and is normalized to the mass number of a nucleus as $\int dr\,\rho(r) = A$.

Integrating Eq. (6) over the transverse momentum leads to the longitudinal momentum distribution:

$$\frac{\sigma_{-N}^{inel}}{dP_{\parallel}} = \int dP_{\perp} \frac{\sigma_{-N}^{inel}}{dP}$$

$$= \frac{1}{2\pi\hbar} \int db_N \left(1 - e^{-2Im\chi_{NT}(b_N)}\right) \int ds \left(1 - e^{-2Im\chi_{CT}(b_N - s)}\right)$$

$$X \int dz \int dz' e^{\frac{i}{\hbar}P_{\parallel}(z-z')} u_{nlj}^{*}(r) \frac{1}{4\pi} P_l(\hat{r'} \cdot \hat{r}), \quad (10)$$

where $r = (s, z)$ and $r' = (s, z')$ and P_l is the Legendre polynomial, $u_{nlj}(r)$ is the single-particle wave function. The integration of equation (10) over $P_{\parallel}$ gives σ_{-N}^{inel}.

The longitudinal momentum distribution is then expressed as a sum of contributions from the azimuthal components of the valence-nucleon wave function:

$$\frac{\sigma_{-N}^{inel}}{dP_{\parallel}} = \sum_{m_l=-l}^{l} \left(\frac{\sigma_{-N}^{inel}}{dP_{\parallel}}\right)_{m_l},$$

with

$$\left(\frac{\sigma_{-N}^{inel}}{dP_{\parallel}}\right)_{m_l} = \frac{1}{2\pi\hbar} \int db_N \left(1 - e^{-2Im\chi_{NT}(b_N)}\right) \int ds \left(1 - e^{-2Im\chi_{NT}(b_N - s)}\right)$$

$$X \frac{1}{2l+1} \left| \int dz e^{\frac{i}{\hbar}P_{\parallel}z} u_{nlj}^{*}(r) Y_{lm_l}(\hat{r}) \right|^2. \quad (11)$$

The momentum distribution of the core which comes out along the beam direction is calculated by setting $P_{\perp} = 0$ in equation (7).

$$\sum_{m_l=-l}^{l} \left.\frac{d\sigma_{-N}^{inel}}{dP}\right|_{P_{\perp}=0} = \frac{1}{(2\pi\hbar)^3} \frac{1}{2l+1} \int db_N \left(1 - e^{-2Im\chi_{NT}(b_N)}\right)$$

$$X \left| \int dr\, e^{\frac{i}{\hbar}P_{\parallel}z + i\chi_{CT}(b_N - s)} u_{nlj}(r) Y_{lm_l}(\hat{r}) \right|^2 \quad (12)$$

3.1 Methodology

The *Cross Section Calculations in the Glauber Model* (CSC_GM) code is a Fortran 90 program that was originally run on UNIX operating system. The code can be used to calculate the cross sections of various reactions for a core plus one valence-nucleon system in the framework of the Glauber model. The program has earlier been used to calculate the total and one-nucleon removal cross sections [8]. The code is slightly modified to enable the computation of the longitudinal momentum distribution of ^{7}Be core fragment in the ^{8}B + ^{12}C reaction system. The reaction system, the type of cross section to be calculated and the target and core densities are specified by the input data. Table 1 represents the input data for the ^{8}B + ^{12}C reaction system in the format of the *csc.inp* file. The first line, according to this format, gives the mass numbers of the target, projectile and core (A_T, A_P, and A_C), the second line gives the charge numbers of those nuclei (Z_T, Z_P, and Z_C). The code assumes $A_P - A_T = 1$. For a proton target, A_T = 1 and Z_T = 1. The third line defines the incident energy of the projectile per nucleon (in GeV). The fourth line defines the parameters of the nucleon–nucleon profile function, Eq. (3): σ_{NN} (in fm^2), α, and β (in fm^2). Values of these parameters are taken from Ref. [9]. The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line gives the number of Gaussians used to fit the core and target densities, and the following lines give the coefficients c_i and the ranges a_i (in fm^{-2}) as defined by Eq. (9). Results of the computations are written on a file *momdist.out*The multi-dimensional integration over the valence-nucleon coordinates is performed with a Monte Carlo technique. In the Monte Carlo integration, a set of configuration or integration points is generated according to a suitably chosen guiding function $w(\boldsymbol{x})$. The random walk with the Metropolis algorithm is taken to generate such points. The code generates the single-particle wave function from the parameters of the input file *wf.inp*. The radial part of the single-particle wave function, $R_{nlj}(r) = r u_{nlj}(r)$, which is used as the guiding function $w(x)$ is obtained by solving a Schrödinger equation with a potential $U(r)$ [1].

$$\frac{d^2R(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r) = 0 \tag{13}$$

$$U(r) = -V_0 f(r) + V_{ls}(\vec{l}\cdot\vec{s})r_o^2 \frac{1}{r}\frac{d}{dr} f(r) + V_{Coul} \tag{14}$$

where

$$f(r) = \left[1 + \exp\frac{r-R}{a}\right]^{-1} \text{ with } R = r_0 A_C^{1/3}, \tag{15}$$

where r_0 and a are the radius and diffuseness parameters in fm respectively. V_0 is the initial depth of the potential. V_{ls} = 17 MeV [10].

4. RESULTS AND DISCUSSION

The ^{8}B nucleus is described with a ^{7}Be + proton system. The input parameters which have to be filled in the files csc.inp and wf.inp are shown in Table 1. The momentum distributions expressed in Eq. (12) are written on the output file, *momdist.out*, as in Table 2.The longitudinal momentum distribution of the ^{7}Be for the reaction ^{8}B + ^{12}C at the energy of 1.44 GeV/nucleon is compared with experiment in Fig.1. The density of ^{7}Be is taken from[11].

Table 1A. csc.inp input file for the $^{8}B + ^{12}C$ system

S/N	INPUT PARAMETERS	VALUES
1	Mass numbers of target, projectile and core: (A_T; A_P; A_C)	12; 8; 7
2	Atomic numbers of target, projectile and core: (Z_T; Z_P; Z_C)	6; 5; 4
3	Incident Energy per nucleon (in GeV)	1.44
4	Profile function parameters	4.26; -0.07; .021008
5	*l* (angular momentum quantum) number)	1
6	Monte Carlo parameters (Ns, δ, irand)	500000; 2.5; -11213
7	*icond1* (initial condition 1)	0
8	*icond2, icond3*	2; 1
9	Number of Gaussians used to fit the core and target densities	2
10	Coefficient c_i, range a_i (in fm^{-2})	c_1=-1.23342874; a_1=0.462770142 c_2=1.38536085; a_2=0.373622826
11	Maximum angle (in degrees)	20

Table 1B. wf.inp input file for the ^{8}B + ^{12}C system

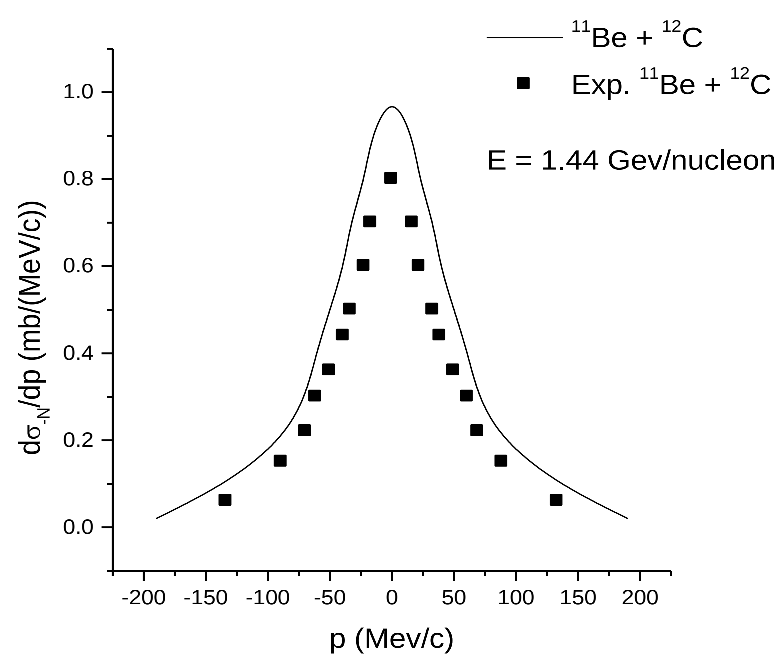
70.0	Initial depth V_0 of the optical potential (in MeV)
0.6	Difussness parameter a (in fm)
1.2	Radius parameter r_0 (in fm)
-0.137	Energy iegenvalue for the valence nucleon (in MeV)
1.5	j value for the valence nucleon orbit
1	Node number for the valence nucleon orbit

Table 2. *momdist.out* output file format.

P_l [MeV/c]	$d\sigma/dp$ [mb/(MeV/c)]
0.0	2.321504
10.0	2.014542
…	…

Table 3. *comp.out* output file format

m_l state	P_l [MeV/c]	$d\sigma_{-N}/dP_l$ [mb/(MeV/c)]
$m_l = 0$	0.0	3.715373x10^{-3}
	10.0	2.2067766x10^{-2}
	…	…
$m_l = 1$	0.0	2.428567x10^{-1}
	…	…
$m_l = -1$	…	…



F.g 1. The longitudinal momentum distribution of ^{7}Be from the ^{8}B+ ^{12}C reaction at the energy of 1.44 GeV/nucleon. The experimental data are taken from Ref. [12].

The experimental data is obtained from Ref. [12]. Only the inelastic breakup process is taking into account in calculating the momentum distribution of the core fragment as the factor dominates the elastic process at high energies[9]. The solid curve is the result of equation (10). The results clearly compare well the experimental data. The calculated momentum distribution is a little broad, compared to experiment. This suggests that the spatial extension of the valence-proton orbit may probably be too small to suggest a proton-halo structure of ^{8}B. This may be thought to be consistent with the small rms radius of the valence proton orbit of only 4.3 fm[6]. But this can be probably understood to be due to the presence of the Coulomb barrier. On the other hand the small separation energy of the valence-proton of only ε = 0.137 MeV[6] suggests that the projectile ^{8}B nucleus is a proton-halo candidate. The momentum distribution from each m_l state is written on the output file, *comp.out,* as in Table 3.

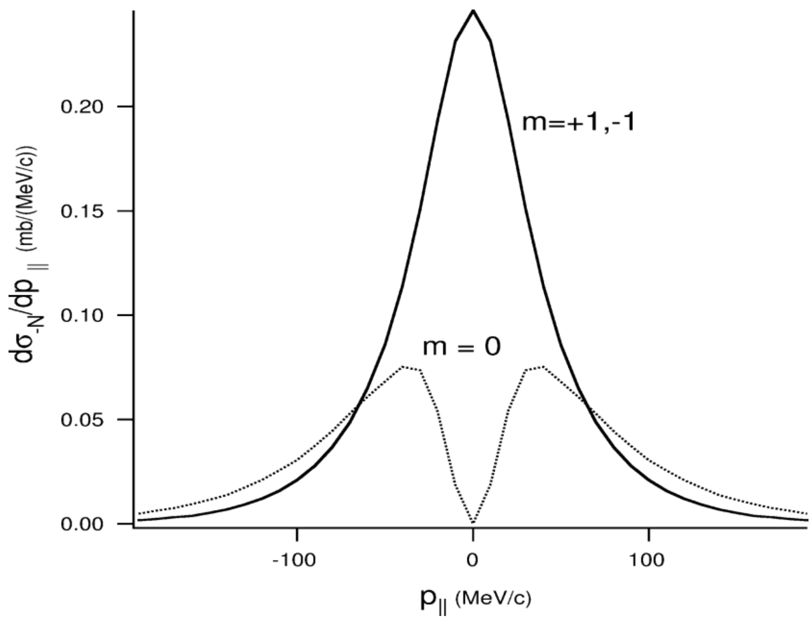


Fig. 2. The contribution of the single-particle orbits with different m_l values to the momentum distribution for the ^{8}B+ ^{12}C reaction

The contribution of each component of the single-particle orbits to the distribution, Eq. (11) is shown in Fig. 2. It is seen that the $m_l = \pm 1$ orbits have a large contribution to the longitudinal momentum distribution.

6. OVERALL CONCLUSIONS

A Fortran program was used to calculate the longitudinal momentum distribution of ^{7}Be core fragment in the $^8B + {}^{12}C$ reaction system in the framework of the Glauber theory. The results clearly consistent with the small separation energy of the valence proton in ^{8}B and compare well the experimental data. it was noted that the small rms radius of the proton orbit (4.3 fm) is due to the presence of the Coulomb barrier.The Glauber model is obviously suitable for high energy reactions. For reactions at lower energy, (less than a hundred MeV/nucleon), predictions of the Glauber model are rather poor. Contributing factors might be inappropriate choice of the effective interactions between the nucleon and the target for the low energy reactions.

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