TWO-DIMENSIONAL CUTTING STOCK MANAGEMENT IN FABRIC INDUSTRIES AND OPTIMIZING THE LARGE OBJECT’S LENGTH

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ABSTRACT

Selection of cutting patterns in order to minimize production waste is an important issue in operations research, which has attracted many researchers. Solving this problem is very important in the industries that have priorities in minimizing the waste in the Pages section. In this paper, the two-dimensional cutting stock problem has been studied to reduce the cutting waste, with focus on men’s clothing (male pants size 42). In this method, regular and irregular shapes are enclosed by rectangles the goal is to minimize waste in total fabric. As solving these problems optimally are infeasible with current optimization algorithms because of the large solution space, the metaheuristic algorithm of simulated annealing (SA) was used. One of the main objectives of this research is to calculate the optimum length of fabric rolls, such that when several of such rolls are put together, the amount of cutting waste is minimized. To achieve this goal, we initially consider an unlimited length for each roll, and then obtain the optimum length of each roll. This research shows that the amount of required stock can be reduced in fabric cutting by using the SA algorithm. Moreover, if the length of pieces is not fixed, uncontrollable stock can be changed into controllable ones; e.g., stock could be concentrated in a form which is usable in future consumption.

Keyword: Two-dimensional Cutting Problem, Metaheuristic, Approximation, Simulated Annealing, Waste.

1. INTRODUCTION

Selecting the patterns of cuts is an important problem in operations research, and several researchers have performed studies toward producing products with minimal waste. This problem appears in various industries, such as producing car body, building structures, shipbuilding, manufacturing aircrafts, as well as paper, glass, leather, and textile industries.

In the production process, it is needed to cut large pieces into the smaller pieces. This often results in small pieces which cannot be used in any products and are considered as waste. Minimizing this waste is very important to reduce costs. Many industries face the problem of two-dimensional cutting of irregular shapes in the first phase of production. This problem is especially serious in the textile industry. In this case, no distance is needed between the pieces, but the wrap direction and the texture pattern show that the direction of the main sheet is usually fixed (or a rotation of 180° is allowed). Usually, a manufactured layout pattern is used several times, which consists of a few large pieces, related to the main parts of clothes (such as legs), in addition to many small pieces such as pockets, waste, etc.

Recently, a large number of algorithms have been developed to obtain the global optimum in simple and ideal models in engineering optimization problems. However, many of these problems are very complex and difficult to solve. So, new heuristic and powerful algorithms based on biological evolutionary process (Genetic Algorithm proposed by Holland [13] and Goldberg [14]), animal behavior (Tabu Search proposed by Glover [15]) and the physical annealing process (simulated annealing proposed by Kirkpatrick et al. [16]) are developed. We found SA suitable for solving the mentioned problem.

2. GENERAL MODELS OF CUTTING PROBLEMS

Assume that the main sheet is a rectangular shape A₀ with length L₀ and width W₀; i.e., dimensions (L₀, W₀), and R is a set of m rectangular parts with dimensions (Lᵢ, Wᵢ), satisfying the following conditions:

- In the case where rotation of parts is not permitted, the length and width of each part is less than or equal to the length and width of the main sheet; i.e.,
  \[ Lᵢ \leq L₀ \quad Wᵢ \leq W₀ \quad i = 1, \ldots, m \]
In the case where rotation of parts is permitted, the largest dimension of each piece is less than or equal to that of the main sheet, and similarly, the smallest dimension of each piece is less than or equal to that of main sheet; i.e.,
\[
\max (L_i, W_i) \leq \max (L_0, W_0) \quad \text{and} \quad \min (L_i, W_i) \leq \min (L_0, W_0)
\]

(In this work, rotation of parts is not permitted as is stated in the assumptions.)

Now the problem is to cut a main sheet \( A_0 \) such that the total value of the resulting parts is maximized, while, for each \( i \), at least one and at most \( b_i \) copies of piece \( i \) is generated. The general form of this problem is as follows [2]:
\[
\begin{align*}
\text{Max } Z &= \sum_{i=1}^{m} v_i \cdot x_i \\
\text{s.t.} & \quad 0 \leq X_i \leq b_i \quad \forall i \\
& \quad \text{Integer: } x_i \quad i = 1, \ldots, m
\end{align*}
\]

In this formulation, \( x_i \) is the decision variable indicating the number of copies of the \( i^{th} \) piece if the \( i^{th} \) item is cut from \( A_0 \).

In the case with demands, it is assumed that we have an unlimited number of large rectangular sheets \( A_0 \) with dimensions \((L_0, W_0)\) as well as a set \( R \) consisting of \( m \) small rectangular parts \( R = \{(L_i, W_i) \mid i = 1, \ldots, m\} \). In addition, indicate the minimum required number of copies of the \( i^{th} \) piece and the usage cost of the \( j^{th} \) cutting method are denoted, respectively, by \( N_i \) and \( C_j \). The resulting optimization problem will have the form [3]:
\[
\begin{align*}
\text{Min } Z &= \sum_{j=1}^{n} C_j \cdot x_j \\
\text{s.t.} & \quad \sum_{j=1}^{n} a_{ij} \cdot x_j \geq N_i \quad i = 1, \ldots, m \\
& \quad \text{Integer} \quad \text{and} \quad x_j \geq 0 \quad j = 1, \ldots, n
\end{align*}
\]

In this model, \( x_i \) indicates the number of main sheets which are cut according to the \( j^{th} \) cutting method, \( a_{ij} \) indicates the number of copies of the \( i^{th} \) piece produced by cutting a main sheet according to the \( j^{th} \) cutting, and \( m \) and \( n \) are the total numbers of piece types and cutting methods, respectively.

The above model is not sufficient to describe the cutting problem, because it does not contain explicit constraints on the layout of pieces in the sheets, forbidding the pieces from having overlaps in each sheet. Also there aren’t any variables showing the position of the pieces in each sheet. In this model at the beginning cutting methods should be identified so that the model could choose a combination of cutting methods to have the least possible waste.

### 2-1- Model Assumptions

The assumptions and constraints of the studied cutting problem are as follows:

- The required patterns and the main sheets have rectangular shape (or more precisely, it’s assumed that the required pieces are enclosed by rectangles).
- The main sheets are identical and are available with unlimited length.
- The main sheets are homogeneous and even, and there whole surface is usable and free of defects.
- The cutting of fabric is Guillotine and cutting of patterns is non-Guillotine. A Guillotine cut is one which starts from one side of the rectangular sheet and terminates at the opposite side [1].
- All examples have been developed for male pants size 42.
- Patterns in this algorithm are arranged as wrapping; which is, no rotation is permitted.
- The width of the sheet is specified, but it length is unlimited (Fabric width is considered 150 cm).
- There are no limitations on the minimum and maximum numbers of copies of each piece produced from a main blot.
- The free edge of fabric is included in rectangles, and the cutting lines have zero width.
- The coordinates of the located pieces are expressed with respect to the origin, which is at the lower-left corner of the main sheet.
3. METHOD OF CHANGING IRREGULAR SHAPES TO REGULAR ONES

In this phase, different parts of male pants of size 42 with irregular shapes are enclosed by rectangular shapes in order to be changed into regular shapes. Then, regular rectangular are arranged by different arrangement only in the wrap direction (without rotation), showing different lengths with different amounts of waste. The details of this procedure are explained in the following.

![Figure 1- changing irregular shape to regular one](image)

In the cutting problem, when items are cut from large pieces, some parts may not be used, because there are many choices for cutting of different patterns by the user. Therefore, different choices for cutting will result in different amounts of waste. In the cutting problem studied in operation research, the objective function is usually defined as minimizing the number of cutting patterns. But overall, the final objective is to minimize the total waste.

4. DEFINITION OF PARAMETERS

To provide a more accurate description of our approach to the cutting problem, these parameters are introduced to explain the cutting stock problem:

- \( i \) = The number of items (pants)
- \( j \) = The number of rolls (fabric sheets)
- \( j = 1, 2, \ldots, m \)
- \( l_i \) = The length of the \( i \)-th ordered item
- \( n_i = 2, 1 = i \)
- \( y \) = Width of fabric (fix and 150cm)
- \( S \) = Item’s area
- \( n \) = The number of ordered item
- \( m \) = The number of rolls being used
- \( W_j \) = Waste in each roll
- \( W \) = Total waste

Using the parameters above, we can calculate the cutting waste as

\[
W_j = (l_i * y) - (1 * S) \\
0 \leq W_j < S
\]

\( j = 1, 2, \ldots, m \)

As mentioned previously, there are large numbers of possible values for \( W_j \) in optimum result of a cutting model, while all these results satisfy the constraint
In the cutting stock problem, among all possible solutions, we are interested in the one which corresponds to the smallest number of rolls containing waste.

5. PROBLEM CONSTRAINTS

In this problem, two constraints are considered:

1) The cutting length: The maximum length of each roll is considered to be 5 meters.
   \[ l_i \leq 5 \]

2) Deterioration: The waste in the each step cannot be more than the previous step
   \[ W_i \geq W_{i+1} \]

In this paper, as noted in the assumptions, it is assumed that the lengths of fabrics from which the patterns (pieces) are cut are unlimited. This makes the problem different from those studied so far in the literature. As mentioned previously in Section 2, general cutting models are not sufficient to describe the cutting problem, since they do not include variables indicating the location of pieces and constraints on their arrangements. Instead, one needs to identify different combinations of cutting styles and choose a combination with minimum waste. However, since there are an extremely large number of possible cutting styles, it is not feasible to utilize and directly solve the aforementioned model. In this paper, in order to achieve a near-optimum solution, we have used simulated annealing, which is one of the most commonly-used metaheuristic algorithms.

6. SOLVING USING SIMULATED ANNEALING

Simulated annealing is a technique inspired by the natural process of annealing solids. The physical process of annealing is the cooling of a metal sufficiently slowly so that it adopts a low-energy, crystalline state. When the temperature of the metal is high, the particles within the metal are able to move around, changing the structure of the metal, freely. As the temperature is lowered, the particles are limited in the movements they can make as many movements have a high energy cost and are increasingly limited to only those configurations with lower energy than the previous state. Simulated annealing draws inspiration from the physical process, in a computational model of the physical system [12].

In the following, we present the simulated annealing method to solve the two-dimensional cutting stock problem of fabric.

6-1- Objective function definition

The objective is defined as minimizing the total waste value. The total waste value is calculated as follows:

\[
\text{Min } Z = \sum_{j=1}^{M} \left[ (l_i \cdot y) - (i \cdot S) \right]
\]

j = 1, 2, 000, m

6-2- Initial solution

An initial solution is needed to start the SA process. The quality of this initial solution has a significant effect on the searching process of SA. In other words, if the initial solution corresponds to a good quality of the objective function, the process to achieve an optimum or near optimum solution of the problem may have a better performance, and in many cases, a higher speed.

To find a good initial solution, we started to layout the parts from the reference point (left side and bottom angle of each rectangular in each fabric).
6-3- Neighborhood generation

In order to generate a neighborhood solution, a random state will be used. In this state, using random selection, the item orders are change algorithmically. The steps of this algorithm are explained below:

Step1. Number the available small items in initial solution from 1 to N.
Step2. Generate a uniform random number \( R_1 \) in the range \([2, N]\) defining the number of displacements of items.
Step3. Generate \( R_i \) uniform random numbers \( R_i : i = 1, ..., N \) in the range \([1, N]\), defining the indices of displaced items.
Step4. If the \( i^{th} \) and \((i + 1)^{th}\) item have the same length, avoid displacement; otherwise, swap the \( i^{th} \) and \((i + 1)^{th}\) items.

6-4- Simulated Annealing procedure

After determining the neighborhood solution, the value of the Boltzmann function will be computed as follows:

\[
\Delta = Z_2 - Z_1
\]

\( Z_1 = \) waste in initial solution

\( Z_2 = \) waste in neighborhood solution

Boltzmann function = \( e^{-\Delta/T} \)

If \( \Delta < 0 \), then choose neighborhood solution instead of the initial solution, but if \( \Delta \geq 0 \), choose neighborhood solution instead of initial solution with the Boltzmann probability.

6-5- Algorithm termination conditions

Two criteria are considered to specify the termination of the algorithm.

1- Reaching the limit of the cutting length. Algorithm will be stop when the length to be cut reaches 5 meters.
2- Algorithm will terminate when the resulting waste in one step is more than the previous step.

7. COMPUTATIONAL RESULTS

For solving two-dimensional cutting stock problems, simulated annealing (SA) was programmed in visual basic on a PII computer, and MS Excel was used for analyzing the data. The result was close to optimum.

We generated an initial pattern randomly. At each iteration, we generated another pattern using a random displacement method of \( n \) patterns [4]. If the new solution was better than the previous one, it would replace the previous solution. On the other hand, if the new solution was worse than the previous solution, it would replace the previous solution by the corresponding Boltzmann probability. This process continued until one of the termination condition was met. (We considered the number of pants in demand to be equal to 1000.)

Figure 1- Achieve to the first stop condition and stop the problem
In the studied examples, the algorithm was terminated based on the first condition before meeting the second termination condition of the algorithm. However, in order to evaluate the amount of waste in cutting process, we considered another variation where the first termination condition was ignored and we investigated a number of different cases.

As described in table 1, the waste was controlled in order to make a proper decision in case of a sudden change.

<table>
<thead>
<tr>
<th>Number of pants</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rolls</td>
<td>1,000</td>
<td>500</td>
<td>333</td>
<td>250</td>
<td>200</td>
<td>167</td>
<td>143</td>
<td>125</td>
<td>111</td>
<td>100</td>
</tr>
<tr>
<td>L (cm)</td>
<td>151</td>
<td>251</td>
<td>375</td>
<td>500</td>
<td>620</td>
<td>741</td>
<td>900</td>
<td>870</td>
<td>991</td>
<td>1,020</td>
</tr>
<tr>
<td>Si</td>
<td>19,650</td>
<td>37,650</td>
<td>56,250</td>
<td>75,000</td>
<td>93,600</td>
<td>112,150</td>
<td>130,500</td>
<td>148,050</td>
<td>166,000</td>
<td>188,000</td>
</tr>
<tr>
<td>Waste in one roll(cm²)</td>
<td>1,941</td>
<td>2,056</td>
<td>2,810</td>
<td>3,663</td>
<td>3,845</td>
<td>4,230</td>
<td>5,637</td>
<td>7,572</td>
<td>8,003</td>
<td>8,583</td>
</tr>
<tr>
<td>Total waste (cm²)</td>
<td>1,941,000</td>
<td>1,028,000</td>
<td>935,750</td>
<td>808,250</td>
<td>759,200</td>
<td>688,040</td>
<td>806,091</td>
<td>916,875</td>
<td>858,333</td>
<td>858,300</td>
</tr>
</tbody>
</table>

The algorithm was stopped because the situation will be worse in 7 pants. It means that reached to second stop condition.

The results based on the second termination condition are shown in Figure 2.

![Review the length and waste in 10 cases](image)

As shown in Figure 2, percentage of waste for 7 pants 0.61% is higher than that for 6 pants, and the algorithm stopped at 7 pants.

8. CONCLUSION

In this paper we studied the two-dimensional cutting stock problem to reduce cutting stock. Most of researchers have studied cutting stock problems with the aim of reducing waste in the sheet with specified length and width. But in this research, only the width is specified and an unlimited length is assumed for the fabric from which the pieces were cut. This turns uncontrollable wastes into controllable ones with displacement of the length, distinguishing the studied method is different from those studied until now. It is difficult and impractical to utilize a general model for the problem, since there is an extremely large number of combinations of cutting styles. In such problems, to achieve the near optimum solution, simulated annealing is one of the most effective metaheuristic algorithms
This research has been done on men’s clothing (male pants size 42) and the patterns were enclosed by rectangular shapes and irregular shapes changed to regular one. To obtain numerical result in this research, we assume unlimited length of fabric, and incontrollable waste was changed into controllable and concentrated ones, which can be used in future consumptions.

9. REFERENCES